BC.Q404.REVIEW ASSESSMENTS (Part 4)

CH 6 (REVISITED) – DIFFERENTIAL EQUATIONS

(20 points)

NO CALCULATOR

NAME: KEY

DATE:

BLOCK:

I (*print name*)__________________________________ certify that I wrote and fully understand **all** marks made in this assessment. I did not write anything that I do not understand. I would now, having completed this assessment, be able to make similar (but equally accurate) responses if asked complete the same exact assessment on my own.

Signature:

1. Evaluate A. $\int e^{5x}dx$ B. $\int e^{\frac{x}{3}}dx$ C. $\int 2\sin(7x)dx$ D. $\int 2\cos\left(\frac{x}{9}\right)dx$

A. $\frac{1}{5}e^{5x} + C$

B. $3e^{x/3} + C$

C. $-\frac{2}{7}\cos(7x) + C$

D. $18\sin\left(\frac{x}{9}\right) + C$

2. (No Calculator)

Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = 6(14 - y)$ with $f(0) = 2$.

$\frac{dy}{dx} = 84 - 6y$

A. Find $\frac{d^2y}{dx^2}$ in terms of y.

B. Use part A to find $f''(0)$.

C. Solve the differential equation by separating the variables.

A. $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(84 - 6y\right) = -6 \cdot \frac{dy}{dx} = -6\left(84 - 6y\right)$ or $-6\left(6(14-y)\right)$

B. $f''(0) = f''(0, 2) = -6\left(84 - 6(2)\right) = -432$

C. $\int \frac{dy}{14 - y} = \int 6\, dx$

$-\ln|14 - y| = 6x + C$

very Big Deal!

$\ln|14 - y| = -6x + C$

$|14 - y| = Ce^{-6x}$

$14 - y = C^* e^{-6x}$

$y = 14 - C^* e^{-6x}$

$2 = 14 - C^* e^{0}$ 1

$\therefore C^* = 12$

$$\boxed{y = 14 - 12e^{-6x}}$$

3. (No Calculator)
Let $y = H(t)$ be the particular solution to the differential equation
$\frac{dH}{dt} = \frac{1}{2}(10 - H)$ with $H(0) = 2$.

$H'(0,2) = \frac{1}{2}(10-2) = 4$

A. Use a linearization centered at $t = 0$ to approximate the value of $H(0.75)$
B. Solve the differential equation by separating the variables.

A] $L(t) = H(0) + H'(0,2)(t-0)$

$L(t) = 2 + 4(t)$

$H(0.75) \approx L(0.75) = 2 + 4(0.75) = 5$

B] $\int \frac{dH}{10-H} = \int \frac{1}{2}\,dt$

$-\ln|10-H| = \frac{1}{2}t + c$

again very big deal

$\ln|10-H| = -\frac{1}{2}t + C$

$|10-H| = Ce^{-t/2}$

$10 - H = c^* e^{-t/2}$

$H = 10 - c^* e^{-t/2}$

$2 = 10 - c^* \cancel{e^0}$ 1

$\therefore c^* = 8$

$$\boxed{H = 10 - 8e^{-\frac{t}{2}}}$$

4. (No Calculator)
Let $y = f(t)$ be the particular solution to the differential equation

$$\frac{dy}{dt} = \frac{1}{3}y(12-y) \text{ with } f(0)=5.$$

$$\left.\frac{dy}{dt}\right|_{t=0,\, y=5} = \frac{1}{3}(5)(12-5) = \frac{35}{3}$$

A. Find $\lim_{t\to 0^+} \frac{f(t)-5}{\sin(t/3)}$

B. Solve the differential equation without separating the variables.
LOGISTIC (Memorize Solution)

C. What is the value of y when f is growing the fastest?

D. What is the value of t when f is growing the fastest?

A. $\lim_{t\to 0^+} \frac{f(t)-5}{\sin(t/3)} \left\{\frac{0}{0}\right\} = \lim_{t\to 0^+} \frac{f'(t)}{\frac{1}{3}\cos\left(\frac{t}{3}\right)} = \frac{\frac{35}{3}}{\frac{1}{3}} = \boxed{35}$

L'HOPITALS RULE

$\lim_{t\to 0^+} f(t)-5 = 0$

AND

$\lim_{t\to 0^+} \sin(t/3) = 0$

B. $y = \frac{L}{1+c^* e^{-KLt}}$

$L = 12 \quad K = \frac{1}{3}$

$y = \frac{12}{1+c^* e^{-4t}}$

$5 = \frac{12}{1+c^* \cancel{e^{0}}^{\,1}}$

$\therefore c^* = \frac{12}{5} - 1$

$c^* = \frac{7}{5}$

$$\boxed{y = \frac{12}{1+\frac{7}{5}e^{-4t}}}$$

C] $y = \frac{12}{2} = 6$ when y is growing the fastest

Half the limit

D] $6 = \frac{12}{1+\frac{7}{5}e^{-4t}}$ algebra

$1+\frac{7}{5}e^{-4t} = 2$

$\frac{7}{5}e^{-4t} = 1$

$e^{-4t} = \frac{5}{7}$

$t = \frac{\ln(5/7)}{-4}$

$$\left.\frac{dy}{dx}\right|_{(2,0)} = \frac{1}{4}$$

5. (No Calculator)

Let $\frac{dy}{dx} = \frac{(1-y)}{x^2}$ where $y = f(x)$ is the particular solution to the differential equation with the condition $f(2) = 0$.

A. Use the line tangent to f at $x = 2$ to approximate $f(1)$

B. Use Euler's method starting at $(2,0)$ with step size $\Delta x = -0.5$ to approximate $f(1)$.

C. Solve the differential equation by separating the variables.

A]

$$L(x) = f(2) + f'(2,0)(x-2)$$

$$L(x) = 0 + \frac{1}{4}(x-2)$$

$$f(1) \approx L(1) = \frac{1}{4}(1-2) = -\frac{1}{4}$$

B]

$\frac{(x_0,\ y_0)}{(2,\ 0)}$ $\quad \hat{y}_1 = y_0 + f'(x_0, y_0)\Delta x = 0 + \frac{1}{4}(-0.5) = -\frac{1}{8}$ or 0.125

$\frac{(x_1,\ \hat{y}_1)}{(1.5,\ -\frac{1}{8})}$ $\quad \hat{y}_2 = \hat{y}_1 + f'(x_1, \hat{y}_1)\Delta x = -\frac{1}{8} + \left(\frac{1-(-\frac{1}{8})}{(3/2)^2}\right)(-\frac{1}{2}) = -\frac{1}{8} - \frac{1}{4}$

$$= -\frac{3}{8}$$

C]

$$\frac{dy}{1-y} = \frac{dx}{x^2} \longrightarrow \int \frac{dy}{1-y} = \int x^{-2}\,dx$$

$$-\ln|1-y| = -x^{-1} + c$$

$$\ln|1-y| = x^{-1} + c$$

$$1-y = c^* e^{x^{-1}}$$

$$y = 1 - c^* e^{x^{-1}}$$

$$0 = 1 - c^* e^{(2)^{-1}}$$

$$c^* = \frac{1}{e^{(2)^{-1}}} \text{ or } \frac{1}{e^{1/2}} \text{ or } e^{-1/2}$$

$$\boxed{y = 1 - e^{-1/2} e^{\frac{1}{x}}}$$

$$\boxed{y = 1 - e^{\left(\frac{1}{x} - \frac{1}{2}\right)}}$$

Variety of ways to write this

6. Let $y = f(x)$ be the particular solution to the differential equation $\frac{dy}{dx} = \cos x + 2y^2$ with $f(0) = -3$.

A. Find $f'(0)$

B. Find $f''(0)$.

C. $\lim_{x \to 0} \frac{f(x) - 19x + 3}{5x^2}$

A] $f'(0) = f'(0, -3) = \cos(0) + 2(-3)^2 = 19$

B] $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\cos x + 2y^2\right) = -\sin x + 4y\frac{dy}{dx}$

$f''(0) = f''\left(x=0, y=-3, \frac{dy}{dx}=19\right) = -\sin(0) + 4(-3)(19) = 0 + 4(-3)(19)$

$= -228$

C] $\lim_{x \to 0} \frac{f(x) - 19x + 3}{5x^2} \left\{\frac{0}{0}\right\} = \lim_{x \to 0} \frac{f'(x) - 19}{10x} \left\{\frac{0}{0}\right\} = \lim_{x \to 0} \frac{f''(x)}{10} = \frac{-228}{10}$

L'HOPITAL'S RULE

L'HOPITAL'S RULE

$= -22.8$

$\lim_{x \to 0} f(x) - 19x + 3 = 0$ AND $\lim_{x \to 0} 5x^2 = 0$

$\lim_{x \to 0} f'(x) - 19 = 0$ and $\lim_{x \to 0} 10x = 0$