BC: Q403 CHAPTER 10 – LESSON 1 (10.1)

DEF: A plane curve is a set C of ordered pairs $\left(x(t), y(t)\right)$, where $x(t)$ and $y(t)$ are continuous functions on an interval I.

DEF: Let C be the curve consisting of all ordered pairs $\left(x(t), y(t)\right)$, where $x(t)$ and $y(t)$ are continuous on an interval I. The equations $x = x(t)$ and $y = y(t)$, for t in I, are parametric equations for C with parameter t.

NOTES $\dfrac{dy}{dx}$:

NOTES $\dfrac{d^2y}{dx^2}$:

THM: The length of a *smooth* curve $y = f(x)$ from $x = a$ and $x = b$ is given by

THM: If a *smooth* curve C is given parametrically by $x = x(t)$, $y = y(t)$; $a \le t \le b$, and if C does not intersect itself, except possibly for $t = a$ and $t = b$, then the length L of C is

THM: Let a smooth curve C be given by $x = x(t)$, $y = y(t)$; $a \le t \le b$, and suppose C does not intersect itself, except possibly for $t = a$ and $t = b$. If $y(t) \ge 0$ throughout $[a, b]$, then the area S of the surface of revolution obtained by revolving C about the x-axis is

$$S = \int_a^b 2\pi y\,ds = \int_a^b 2\pi\big(y(t)\big)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

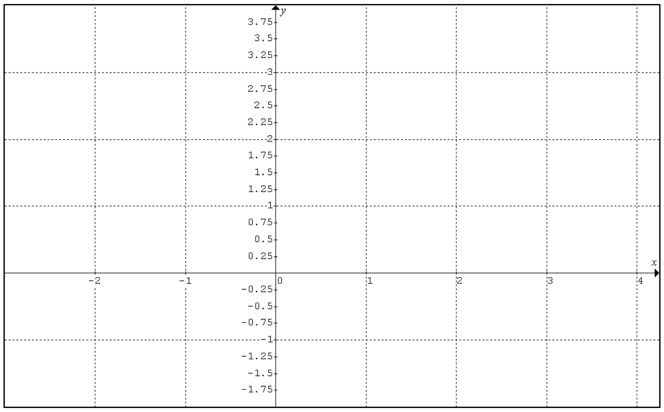
THM: Let a smooth curve C be given by $x = x(t)$, $y = y(t)$; $a \le t \le b$, and suppose C does not intersect itself, except possibly for $t = a$ and $t = b$. If $x(t) \ge 0$ throughout $[a, b]$, then the area S of the surface of revolution obtained by revolving C about the y-axis is

$$S = \int_a^b 2\pi x\,ds = \int_a^b 2\pi\big(x(t)\big)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

Example 1: Let C be the curve that has parametrization

$x = 2t$, $y = t^2 - 1$, $-1 \le t \le 2$.

a. Sketch the graph of C by hand by plotting several points and joining them with a smooth curve.
Indicate the orientation
b. Find the slopes of the tangent line and normal line to C at any point $P(x,y)$.
c. Obtain an equation for the curve in the form $y = f(x)$ for some function f.
d. Use a graphing utility to plot a graph of C. Set the viewing window so that it contains the entire graph.
e. Find the length of C .
f. Find $\frac{d^2y}{dx^2}$ and discuss its implications.



Example 2: A point moves in a plane such that its position $P(x,y)$ at time t is given by $x = a\cos t$, $y = a\sin t$; $t \geq 0$, where a is a constant greater than 0.

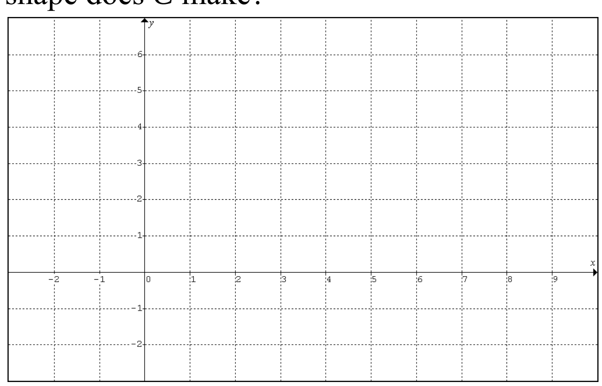
a. Describe the motion of the point.

b. Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$ for varying values of t.

c. Find the length of C from $t = 0$ to $t = 2\pi$.

Example 3: Sketch the graph of the curve C that has the parametrization:

$x = -2 + t^2$, $y = 1 + 2t^2$; $t \in \Re$. What geometric shape does C make?



Example 4: Let C be the curve with parametrization $x = e^{-t}$, $y = e^{2t}$; $t \in \Re$

a. Find dy/dx and the equation of the tangent line to C at the point when $t = \ln(2)$.

b. d^2y/dx^2 and discuss the concavity of the curve *C*.

c. Use a calculator to find the length of *C* from $t = 1$ to $t = 10$.

Example 5: Suppose the curve C defined as $x = \cos t$ and $y = 2 + \sin t$ for $0 \le t \le 2\pi$ is rotated about the *x*-axis. Without a calculator, find the area of the resulting figure and describe the shape.

Q402: Lesson 1 Homework

I. Textbook: Chapter 10.1: #9, 11, 16, 17, 26, 27, 30, 43

II. Supplemental

A. Find an equation in x and y whose graph contains the points on the curve C. Sketch the graph of C and indicate the orientation.

1. $x = t - 2$ $\quad y = 2t + 3$ $\quad 0 \le t \le 5$
2. $x = 4t^2 - 5$ $\quad y = 2t + 3$ $\quad t \in \Re$
3. $x = 2\sin t$ $\quad y = 3\cos t$ $\quad 0 \le t \le 2\pi$
4. $x = \sec t$ $\quad y = \tan t$ $\quad -\pi/2 < t < \pi/2$

B. Find the slopes of the tangent line and the normal line at the point on the curve that corresponds to $t = 1$.

5. $x = t^2 + 1$ $\quad y = t^2 - 1$ $\quad -2 \le t \le 2$
6. $x = e^t$ $\quad y = e^{-2t}$ $\quad t \in \Re$

C. Let C be the curve with the given parametrization, for t in $\Re$. Find the points on C at which the slope of the tangent line is m.

7. $x = -t^3$ $\quad y = -6t^2 - 18t$ $\quad m = 2$

D. (1) Find the points on the curve C at which the tangent line is either horizontal or vertical.
(2) Find d^2y/dx^2.

8. $x = 4t^2$ $\quad y = t^3 - 12t$ $\quad t \in \Re$

E. Find the length of the curve.

9. $x = 5t^2$ $\quad y = 2t^3$ $\quad 0 \le t \le 1$
10. $x = e^t \cos t$ $\quad y = e^t \sin t$ $\quad 0 \le t \le \pi/2$

F. Find the area of the surface generated by revolving the curve about the x-axis.

11. $x = t^2$ $\quad y = 2t$ $\quad 0 \le t \le 4$

G. Find the area of the surface generated by revolving the curve about the y-axis.
(Review Integration by Parts)
12. $x = e^t \sin t$ $\quad y = e^t \cos t$ $\quad 0 \le t \le \pi/2$

BC: Q403 CHAPTER 10 – LESSON 2 (10.2)

Consider a curve C in $\Re^2$

Parametric Equation for C:

Vector Equation for C:

Position Function:

Velocity Function:

Acceleration Function:

Speed:

Differential Equations Method of finding a position function:

Solve for the constant of integration

FTC2 Method of finding a position function:

EXAMPLE 1: (No Calculator)
A particle moves in the *xy*-plane so that any time t its coordinates are $x = t^2$ and $y = 4 - t^3$.
A. Find the speed of the particle at $t = 1$.
B. Find the acceleration vector at $t = 1$.

EXAMPLE 2: (No Calculator)
A particle moves in the *xy*-plane so that its velocity vector at time t is $v(t) = \left\langle t^2, \sin(\pi t) \right\rangle$ and the particle's position vector at time $t = 0$ is $(1,0)$.
A. Find the speed of the particle at time $t = 3$
B. Find the position vector of the particle when $t = 3$.

EXAMPLE 3: (Calculator Required)
A particle moves in the *xy*-plane so that it velocity at time t is $v(t) = \left\langle e^{\sqrt{t}} - t, \sin(\sqrt{t+1}) \right\rangle$ and the particle's position vector at time $t = 1$ is $(2,-3)$.
A. Find the position vector of the particle when $t = 4$.
B. Find the distance traveled by the particle on $1 \le t \le 4$.

EXAMPLE 4: (Calculator Required)

An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with

$$\frac{dx}{dt} = \cos\left(t^3\right) \text{ and } \frac{dy}{dt} = 3\sin\left(t^2\right)$$

for $0 \le t \le 3$. At time $t = 2$, the object is at position $(4, 5)$.

(a) Write an equation for the line tangent to the curve at $(4, 5)$.

(b) Find the speed of the object at time $t = 2$.

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.

(d) Find the position of the object at time $t = 3$.

EXAMPLE 5: (No Calculator)

A moving particle has position $(x(t),\ y(t))$ at time t. The position of the particle at time $t = 1$ is $(2, 6)$, and the velocity vector at any time $t > 0$ is given by $\left(1 - \frac{1}{t^2}, 2 + \frac{1}{t^2}\right)$.

(a) Find the acceleration vector at time $t = 3$.

(b) Find the position of the particle at time $t = 3$.

(c) For what time $t > 0$ does the line tangent to the path of the particle at $(x(t),\ y(t))$ have a slope of 8 ?

(d) The particle approaches a line as $t \to \infty$. Find the slope of this line. Show the work that leads to your conclusion.

CH10 LESSON 2 HOMEWORK

1 (No Calculator). The position of a moving particle in the *xy*-plane is given by parametric equations $x(t) = 9\cos t$ and $y(t) = 4\sin t$ for $t \geq 0$.

A. Find the speed of the particle at $t = \pi / 3$

B. Find the acceleration vector at $t = 3$.

2 (No Calculator). A particle moves in the *xy*-plane so that any time *t*, $t > 0$, its coordinates are $x = e^t \sin t$ and $y = e^t \cos t$. Find the velocity vector at $t = \pi$.

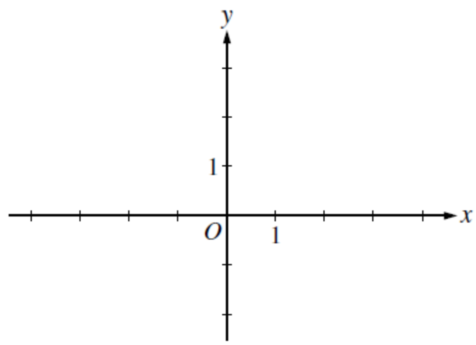
3 (No Calculator). The velocity vector of a particle moving in the *xy*-plane is given by $\vec{v} = \langle 2\sin t, 3\cos t \rangle$ for $t \geq 0$. At $t = 0$, the particle is at the point (1, 1). What is the position vector at $t = 2$?

4 (Calculator Required). The velocity vector of a particle moving in the *xy*-plane is given by $\vec{v} = \left\langle \sqrt{1+t^2}, \sin\left(e^t - 4\right) \right\rangle$ for $t \geq 0$. At $t = 0$, the particle is at the point (-3, 1). What is the position vector at $t = 2$?

HW #5 (Calculator Required)

A particle moves in the xy-plane so that its position at any time t, $0 \le t \le \pi$, is given by $x(t) = \frac{t^2}{2} - \ln(1 + t)$ and $y(t) = 3 \sin t$.

(a) Sketch the path of the particle in the xy-plane below. Indicate the direction of motion along the path.



(b) At what time t, $0 \le t \le \pi$, does $x(t)$ attain its minimum value? What is the position $(x(t), y(t))$ of the particle at this time?

(c) At what time t, $0 < t < \pi$, is the particle on the y-axis? Find the speed and the acceleration vector of the particle at this time.

HW #6 (No Calculator)

. A particle moves along the curve defined by the equation $y = x^3 - 3x$. The x–coordinate of the particle, $x(t)$, satisfies the equation $\frac{dx}{dt} = \frac{1}{\sqrt{2t+1}}$, for $t \geq 0$ with initial condition $x(0) = -4$.

(a) Find $x(t)$ in terms of t.

(b) Find $\frac{dy}{dt}$ in terms of t.

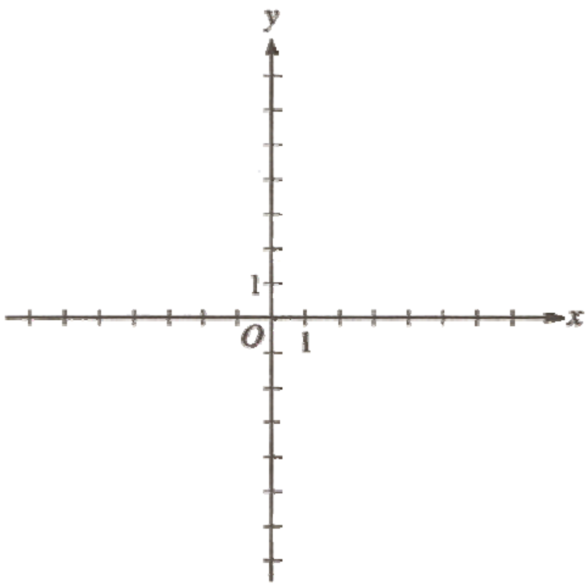
(c) Find the location and speed of the particle at time $t = 4$.

HW #7 (Calculator Required)

During the time period from $t = 0$ to $t = 6$ seconds, a particle moves along the path given by $x(t) = 3\cos(\pi t)$ and $y(t) = 5\sin(\pi t)$.

(a) Find the position of the particle when $t = 2.5$.

(b) On the axes provided below, sketch the graph of the path of the particle from $t = 0$ to $t = 6$. Indicate the direction of the particle along its path.



(c) How many times does the particle pass through the point found in part (a)?

(d) Find the velocity vector for the particle at any time t.

(e) Write and evaluate an integral expression, in terms of sine and cosine, that gives the distance the particle travels from $t = 1.25$ to $t = 1.75$.

BC: Q403 CHAPTER 10 – LESSON 3 (10.3)

Notes Outline for Polar Calculus

Polar Function: $r = f(\theta)$
For $r \geq 0$ and $\theta \geq 0$:
r at point P is the distance from the origin to the point P.
θ at point P is the counterclockwise angle between the *x*-axis and the line segment connecting the origin and a point

1. Graph the following given in polar form: $(2, \pi/3)$; $(-3, \pi)$; $r = 2$; $\theta = \pi/4$; $r = 2 + 2\cos\theta$

2. Covert each polar point to a Cartesian point: $(\sqrt{2}, \pi/4)$; (1,0); $(-3, 5\pi/6)$
Notes: $x = r\cos\theta$ and $y = r\sin\theta$, but why?

3. Convert each polar equation to a Cartesian equation:
Notes: $x^2 + y^2 = r^2$, but why?
a. $r\sin\theta = 0$ 10.5 #19
b. $r = 4\csc\theta$ 10.5 #21
c. $r\cos\theta + r\sin\theta = 1$ 10.5 #23
d. $r^2 = 4r\sin\theta$ 10.5 #25
e. $r^2 \sin 2\theta = 2$ 10.5 #27
f. $r = a\sin\theta$ BERMEL
g. $r = \cot\theta\csc\theta$ 10.5 #28

What is the domain of *f*(θ) to complete exactly one revolution of the curve? Is it $0 \leq \theta \leq \pi$ or is it $0 \leq \theta \leq 2\pi$? Each curve is different. Use polar MODE to graph. Use WINDOW to check from $\alpha = 0$ to $\beta = \pi$ or 2π.
See page 557: 11-20

4. → (a) Sketch $r = 2 + 2\cos\theta$
Derive formula for dy/dx: → (b) Find dy/dx for $r = 2 + 2\cos\theta$
Derive formula for area enclosed: → (c) Find the area enclosed by $r = 2 + 2\cos\theta$
→ (d) Find the area inside $r = 2 + 2\cos\theta$ but outside $r = 3$.
(e) Find the area outside $r = 2 + 2\cos\theta$ but inside $r = 3$.
(f) Find the area inside both $r = 2 + 2\cos\theta$ and $r = 3$.
*Derive formula for polar length: → (g) *Find the length of $r = 2 + 2\cos\theta$ on $0 \leq \theta \leq 2\pi$.

5. Convert from Cartesian to Polar
(-1 , 1); $(1, -\sqrt{3})$; (0, 3)

$y = x$

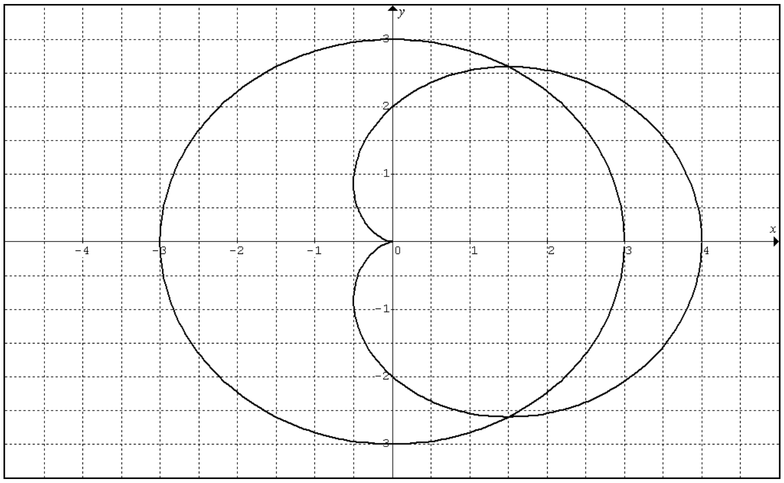
$x = 7$

$x^2 + y^2 = 4$

$\frac{x^2}{9} + \frac{y^2}{4} = 1$

$x^2 + (y-2)^2 = 4$

HOMEWORK in TEXTBOOK
Section 10.3: 41, 46, 47, 53, 56, 57, 58, 59
Section 10.3: 1 – 9 odd; 21 -29 odd, 16, 17



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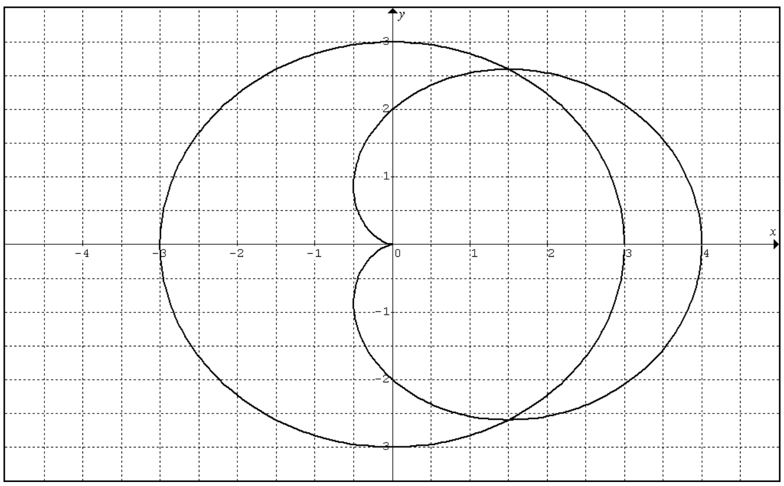

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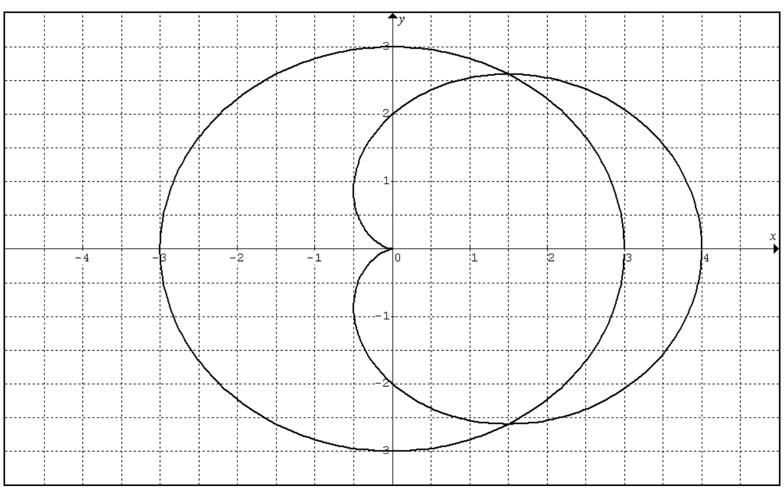

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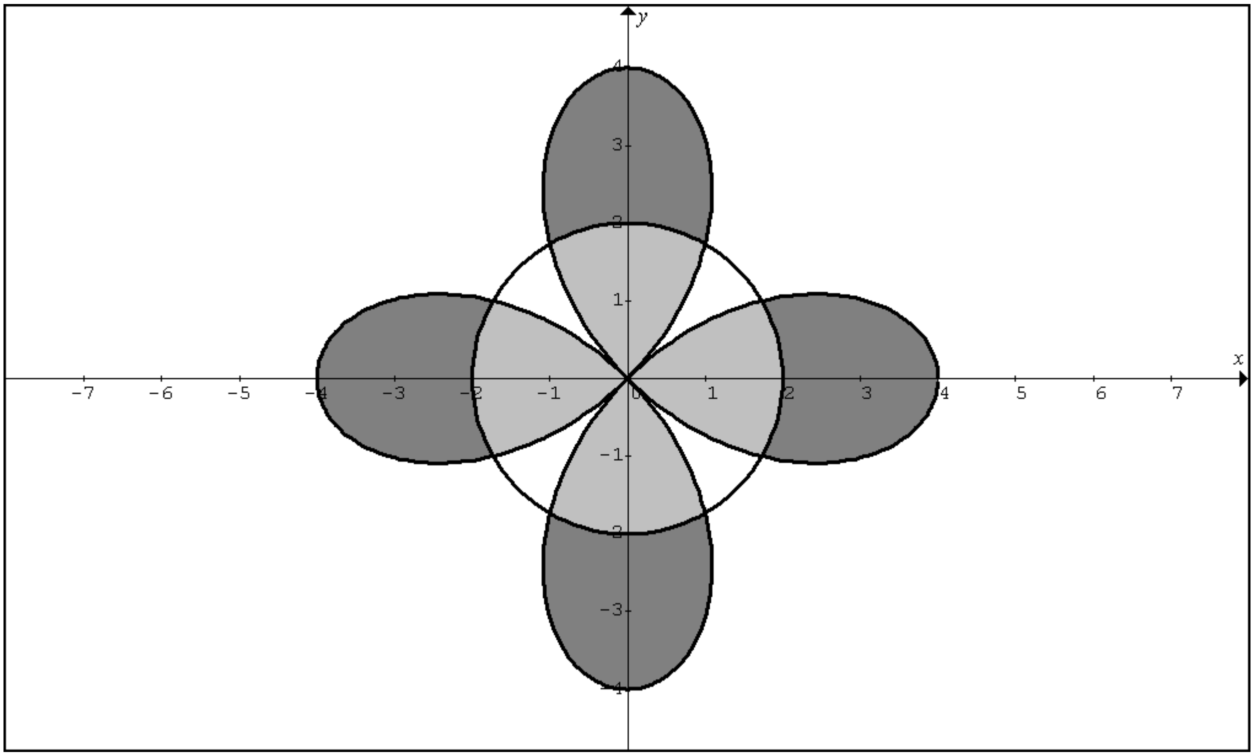
BC: Q403 CHAPTER 10 – LESSON 4 (REVIEW)

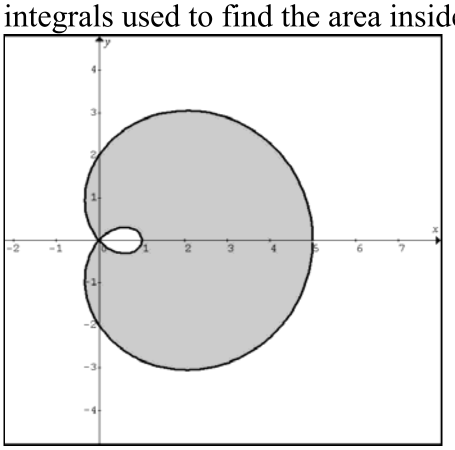


1. The diagram above shows the graphs of $r = 4\cos(2\theta)$ and $r = 2$. Set up, but do not evaluate, an expression involving one or more integrals, used to find the area of the light shaded region.

2. Revisit HW #47 Find the area within one loop of $r^2 = 4\cos(2\theta)$

3. Text Problem #48. Find the area inside the curve $r^2 = 2\sin(3\theta)$.

4. Consider $r = 3\cos(\theta) + 2$. Set up, but do not evaluate, an expression involving one or more integrals used to find the area inside the large loop but outside the small loop.



CHAPTER 10 [THE BASICS] REVIEW

1(NC). A curve is parametrized by $x = t^2 + 5$ and $y = e^{2t}$.

A. Find $\dfrac{dy}{dx}$ B. Find $\dfrac{d^2y}{dx^2}$

2(NC). Find the length of the curve parametrized by $x = \frac{1}{6}(4t+1)^{3/2}$ and $y = t^2$ on $1 \le t \le 5$.

3(Calc). A curve is generated by $x = 12t$ and $y = \frac{t^2}{2} + 4$ on $5 \le t \le 9$. Find the area of the surface generated by revolving the curve about the *y*-axis.

7. The position vector of a particle in the plane is given by $\vec{\mathbf{r}}(t) = [\ln(t+2)]\mathbf{i} + [t^2 - 2]\mathbf{j}$ on $-2 < t \le 2$

A(Calc). Draw the graph of the particle.

B(NC). Find the velocity and acceleration vectors.

8(NC). Solve the initial value problem for **r** as a vector function of *t*.

$\frac{dr}{dt} = \langle 3e^{3t}, 2t \rangle$, $\vec{\mathbf{r}}(0) = \langle 1, -4 \rangle$.

9(Calc). At time *t* = 1, a particle starts has the position (1,2) and continues to moves along a curve C. The velocity of a particle moving along the curve C is given by:
$\vec{v}(t) = \langle \ln\sqrt{t}, -\cos(e^t) \rangle$. Find the position of the particle at time *t* = 3.1.

12(Calc). Graph the polar curve given by $r = 1 + 2\cos(2\theta)$.

13(NC). Suppose a polar graph is symmetric about the x-axis and contains the point $\left(4, \frac{\pi}{6}\right)$.
Which of the following identifies another point that must be on the graph?

I. $\left(4, -\frac{\pi}{6}\right)$ II. $\left(4, \frac{5\pi}{6}\right)$ III. $\left(-4, \frac{5\pi}{6}\right)$

(A)I only (B)II only (C)III only (D) I and II (E) I and III

14(NC). Replace the polar equation $r = \sec^2(\theta)$ by an equivalent Cartesian equation.

15(NC). Find the slope of the polar curve $r = -2\cos(3\theta)$ at $\theta = \frac{\pi}{6}$

16(Calc). Find the area of the region enclosed by the oval limacon $r = 5 - 2\cos\theta$.

17(NC … check with Calc). Find the length of the polar curve given by $r = 5\sin^2\frac{\theta}{2}$ for