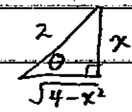
Lesson 2 Practice Solutions

#42:

$$\int \frac{8\,dx}{x^2\sqrt{4-x^2}} = \int \frac{8(2\cos\theta)\,d\theta}{4\sin^2\theta\sqrt{4-4\sin^2\theta}} = 2\int \frac{\cancel{\cos\theta}\,d\theta}{\sin^2\theta\,\cancel{\cos\theta}}$$

let $x = 2\sin\theta$ $\quad dx = 2\cos\theta\,d\theta$



$$= 2\int \csc^2\theta\,d\theta = -2\cot\theta + C$$

$$= -2\,\frac{\sqrt{4-x^2}}{x} + C$$

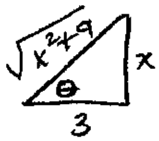
Supp #35

$$\int \frac{dx}{\sqrt{9+x^2}} = \int \frac{3\sec^2\theta\,d\theta}{\sqrt{9+9\tan^2\theta}} = \int \frac{3\sec^2\theta\,d\theta}{3\sqrt{1+\tan^2\theta}} = \int \sec\theta\,d\theta$$

Let $x = 3\tan\theta$

$dx = 3\sec^2\theta\,d\theta$

$$= \ln|\sec\theta + \tan\theta| + C$$



$$= \ln\left|\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right| + C$$

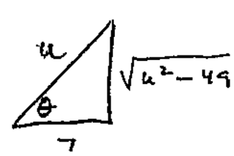
$$= \ln\left|\sqrt{x^2+9} + x\right| + C$$

Supp #37

$$\int \frac{dx}{\sqrt{4x^2-49}} = \int \frac{dx}{\sqrt{(2x)^2-7^2}} = \frac{1}{2}\int \frac{du}{\sqrt{u^2-7^2}} = \frac{7}{2}\int \frac{\sec\theta\tan\theta\,d\theta}{\sqrt{49\sec^2\theta-49}}$$

Let $u = 7\sec\theta$

$du = 7\sec\theta\tan\theta\,d\theta$



$$= \frac{1}{2}\int \frac{\sec\theta\,\cancel{\tan\theta}\,d\theta}{\cancel{\tan\theta}} = \frac{1}{2}\int \sec\theta\,d\theta = \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$$

$$= \frac{1}{2}\ln\left|\frac{u}{7} + \frac{\sqrt{u^2-49}}{7}\right| + C = \frac{1}{2}\ln\left|\frac{2x}{7} + \frac{\sqrt{4x^2-49}}{7}\right| + C$$

$$= \frac{1}{2}\ln\left|2x + \sqrt{4x^2-49}\right| + C$$

MISC: $\int \sin^3 x \, dx = \int \sin^2 x \cdot \sin x \, dx = \int (1-\cos^2 x)\sin x \, dx$

$u = \cos x \quad du = -\sin x \, dx$

$= -\int (1-u^2)\,du = -\left[u - \frac{u^3}{3} + c\right] = -u + \frac{u^3}{3} + c$

$= -\cos x + \frac{\cos^3 x}{3} + c$

Supp # 39

$\int \frac{x^3}{\sqrt{1-x^2}}\,dx = \int \frac{\sin^3\theta \cos\theta \, d\theta}{\sqrt{1-\sin^2\theta}} = \int \sin^3\theta \, d\theta$

Let $x = \sin\theta$

$dx = \cos\theta \, d\theta$

⊛ $\int \sin^3\theta \, d\theta = \int (\sin^2\theta)\cdot\sin\theta \, d\theta = \int (1-\cos^2\theta)\cdot\sin\theta \, d\theta$

$u = \cos\theta \quad du = -\sin\theta \, d\theta \quad d\theta = du/-\sin\theta$

$= -\int (1-u^2)\,du = -u + \frac{u^3}{3} + c = -\cos\theta + \frac{\cos^3\theta}{3} + c$

$= \boxed{-\sqrt{1-x^2} + \frac{(1-x^2)^{3/2}}{3} + c}$ ← my answer

$= \sqrt{1-x^2}\left(-1 + \frac{1}{3}\sqrt{1-x^2}^{\,2}\right) + c = \sqrt{1-x^2}\left[-1 + \frac{1}{3}(1-x^2)\right] + c$

$= \sqrt{1-x^2}\left[-\frac{2}{3} - \frac{x^2}{3}\right] = \sqrt{1-x^2}\left[\frac{-2-x^2}{3}\right] = \boxed{\frac{-(x^2+2)\sqrt{1-x^2}}{3} + c}$

CALCULATOR'S ANSWER

6.5 # 5

$$\int \frac{x-12}{x^2-4x}\,dx = \int \frac{A}{x}dx + \int \frac{B}{x-4}\,dx = \int \frac{3}{x}dx - \int \frac{2}{x-4}\,dx$$

$A(x-4) + B(x) = x - 12$

$$= 3\ln|x| - 2\ln|x-4| + c$$

$A + B = 1$, $-4A = -12$ } $A = 3$, $B = -2$

$$= \boxed{\ln \frac{|x|^3}{(x-4)^2} + c}$$

6.5 # 7.

$$\int \frac{2x^3}{x^2-4}\,dx = \int 2x\,dx + \int \frac{8x}{x^2-4}\,dx = x^2 + 4\int \frac{du}{u}$$

$$= x^2 + 4\ln|x^2-4| + c$$

$$= \boxed{x^2 + \ln(x^2-4)^4 + c}$$

$u = x^2 - 4$

$du = 2x\,dx$

$$\begin{array}{r} 2x \\ x^2-4\,\overline{)\,2x^3} \\ -(2x^3-8x) \\ \hline 8x \end{array}$$

6.5 # 9.

$$\int \frac{2}{x^2+1}\,dx = 2\int \frac{1}{1+x^2}\,dx = \boxed{2\tan^{-1}(x) + c}$$

6.5 # 11,

$$\int \frac{7\,dx}{2x^2-5x-3} = \int \frac{A}{(2x+1)}\,dx + \int \frac{B}{(x-3)}\,dx$$

$A(x-3) + B(2x+1) = 7$

$A + 2B = 0$

$-3A + B = 7$

$+6A - 2B = -14$

$-7A = -14$

$A = -2$ $B = 1$

$$= \int \frac{-2}{2x+1}\,dx + \int \frac{1}{x-3}\,dx$$

$$= -\ln|2x+1| + \ln|x-3| + c$$

$$= \boxed{\ln\left|\frac{x-3}{2x+1}\right| + c}$$

6.5 # 13.

$$\int \frac{8x-7}{2x^2-x-3}\,dx = \int \frac{A}{(2x-3)}\,dx + \int \frac{B}{(x+1)}\,dx$$

$A(x+1) + B(2x-3) = 8x - 7$

$A + 2B = 8$

$A - 3B = -7$

$-A + 3B = 7$

$5B = 15$

$B = 3$ $A = 2$

$$= \int \frac{2}{2x-3}\,dx + \int \frac{3}{x+1}\,dx$$

$$= \ln|2x-3| + 3\ln|x+1| + c$$

$$= \boxed{\ln\left[|2x-3| \cdot |x+1|^3\right] + c}$$