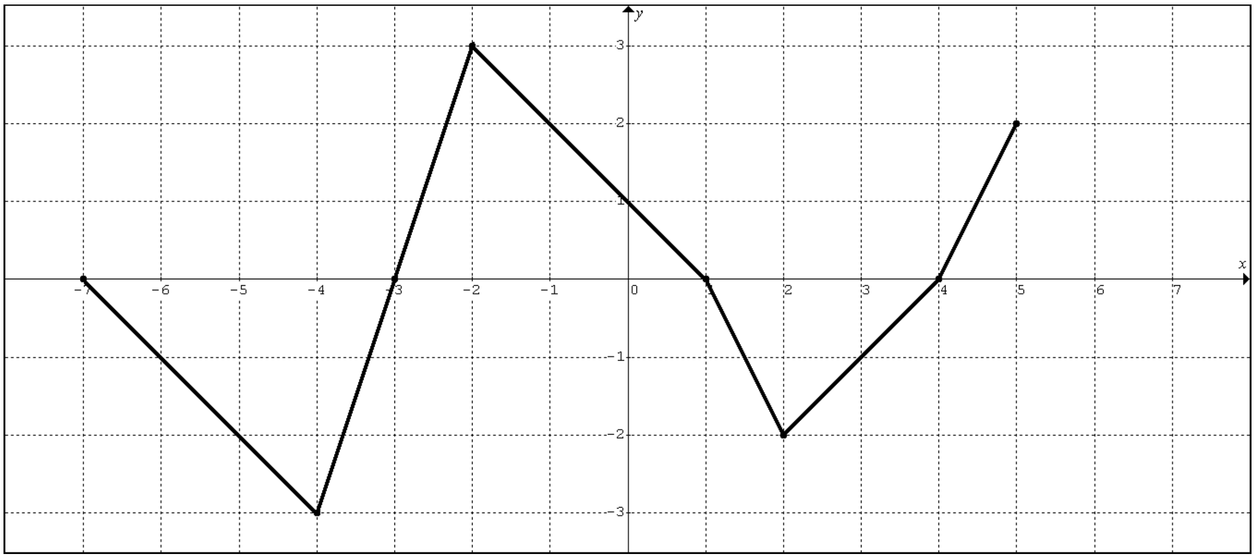
AB: Q304 UNIT WARM UP

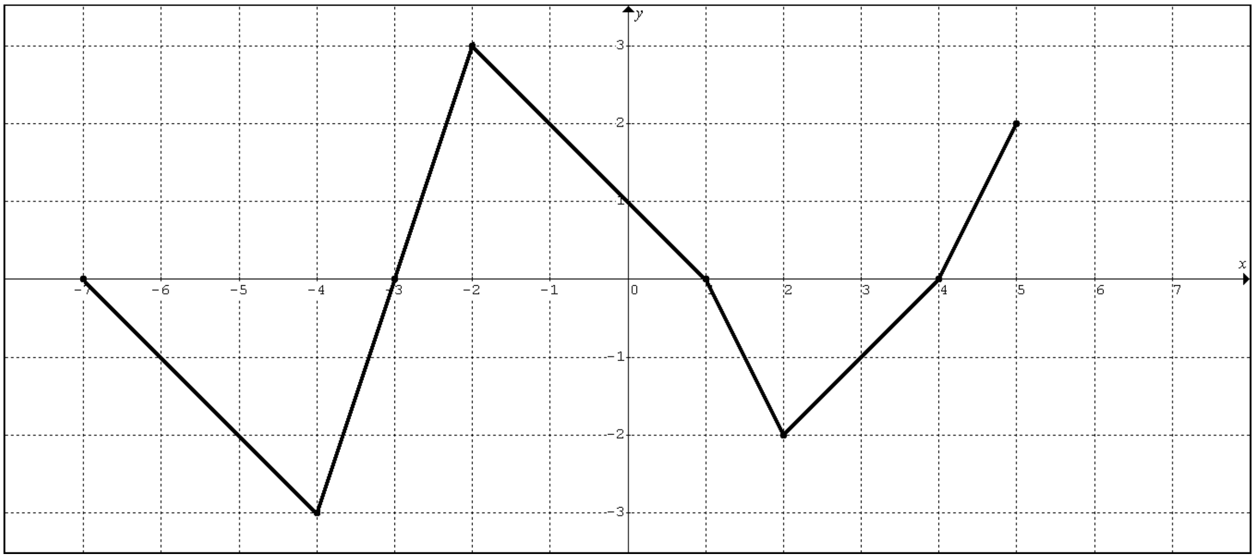


The graph above represents $f'(x)$ (the derivative of the function $f(x)$) on $-7 \le x \le 5$.
Suppose $f(-3) = 80$.

1. Evaluate $\int_{-7}^{5} f'(x)dx$

2. Find $f(5)$ and $f(-7)$

3. What is the average rate of change in $f(x)$ on the interval $-7 \le x \le 5$?



The graph above represents $f'(x)$ (the derivative of the function $f(x)$) on $-7 \le x \le 5$

Suppose $f(-3) = 80$.

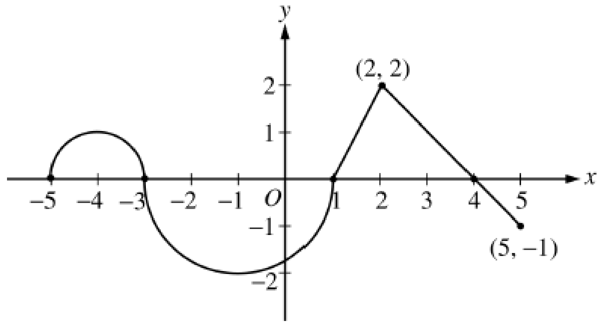
4. Find $f'(-1)$, $f''(-1)$, $f'(-4)$, and $f''(-4)$

5. For what value(s) of x will the function $f(x)$ have a local minimum? Justify.

6. For what value(s) of x will the function $f(x)$ be concave downward? Justify.

AB Q304 LESSON 1A

GUIDED PRACTICE 1: 2007FB #4



Graph of f'

4. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.

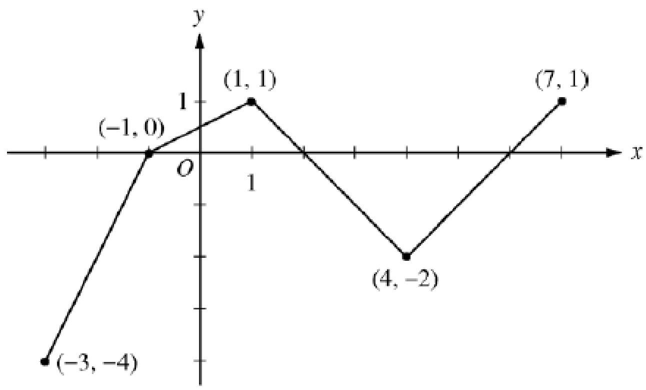
   (a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.

   (b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.

   (c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.

   (d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

GUIDED PRACTICE 2: 2008FB#5



Graph of g'

5. Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.

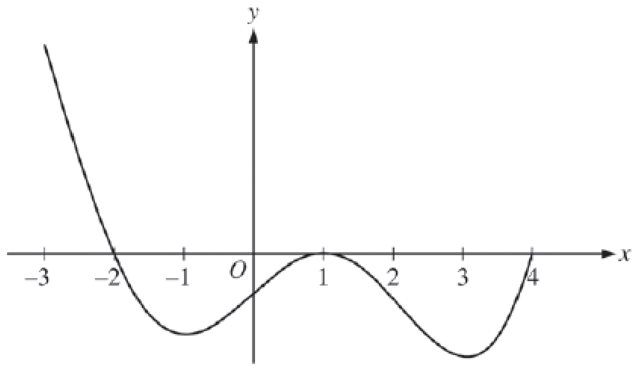
(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

GUIDED PRACTICE 3: 2015 #5



Graph of f'

5. The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

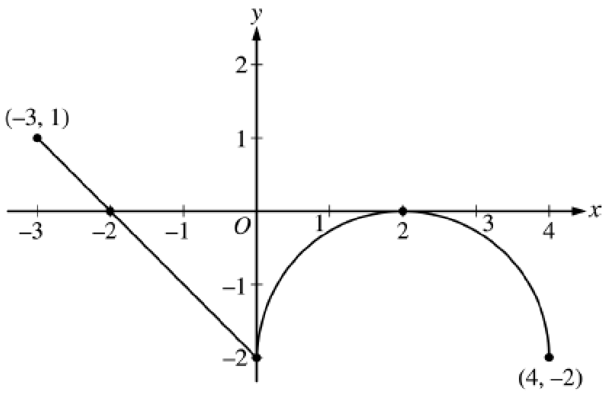
   (a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

   (b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

   (c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

   (d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

GUIDED PRACTICE 4: 2003 #4



Graph of f'

4. Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

   (a) On what intervals, if any, is f increasing? Justify your answer.

   (b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

   (c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

   (d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

AB Q304 LESSON 1B

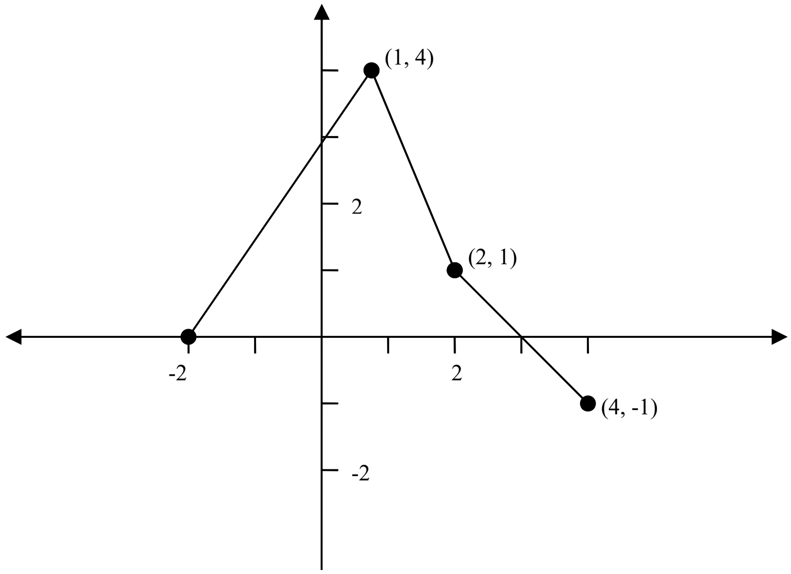
FUNDAMENTAL THEOREM OF CALCULUS PART 1

THE FUNDAMENTAL THEOREM OF CALCULUS PART 1 (Defines the Relationship)

Defining the relationship: $\int f(x)dx = g(x) + C$

Defining the relationship: $g(x) = \int_a^x f(t)dt$ (a is a constant)

GUIDED PRACTICE 1: 1999 #5 (adaptation)



EX1: The graph of the function *f*, consisting of three line segments, is given above. Let $g(x) = \int_1^x f(t)dt$.

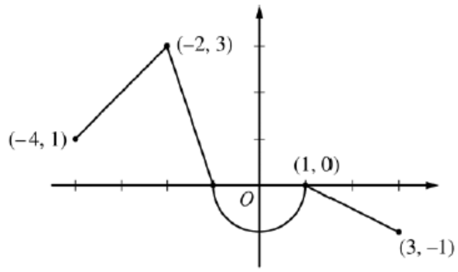
A. Compute $g(4)$ and $g(-2)$

B. Find the instantaneous rate of change of g, with respect to x, at $x = 1$.

C. Find the absolute maximum and minimum values of g on the closed interval [-2, 4]. Justify your answer.

D. The second derivative of g is not defined at $x = 1$ and $x = 2$. How many of these values are x-coordinates of points of inflection of the graph of g? Justify your answer.

GUIDED PRACTICE 2: 2012 #3



Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\, dt$.

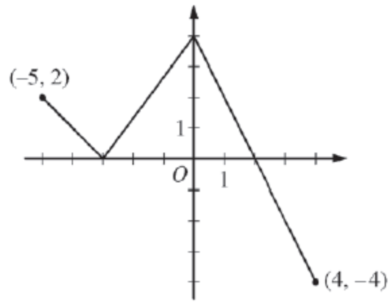
   (a) Find the values of $g(2)$ and $g(-2)$.

   (b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

   (c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

   (d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

GUIDED PRACTICE 3: 2014 #3 (AP SCORE THIS)



Graph of f

3. The function f is defined on the closed interval $[-5, 4]$. The graph of f consists of three line segments and is shown in the figure above. Let g be the function defined by $g(x) = \int_{-3}^{x} f(t)\, dt$.

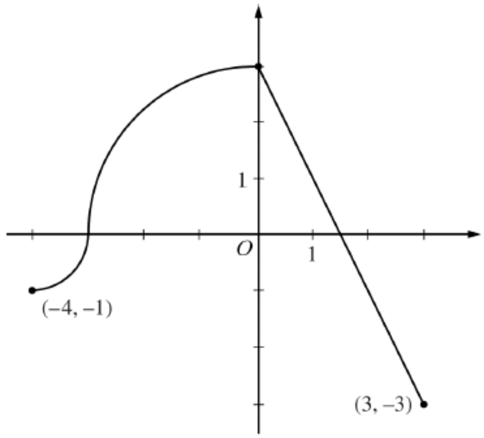
   (a) Find $g(3)$.

   (b) On what open intervals contained in $-5 < x < 4$ is the graph of g both increasing and concave down? Give a reason for your answer.

   (c) The function h is defined by $h(x) = \dfrac{g(x)}{5x}$. Find $h'(3)$.

   (d) The function p is defined by $p(x) = f\left(x^2 - x\right)$. Find the slope of the line tangent to the graph of p at the point where $x = -1$.

GUIDED PRACTICE 4: 2011 #4



Graph of f

4. The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t)\,dt$.

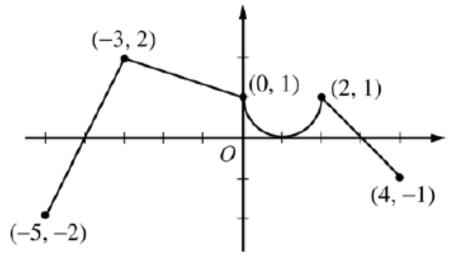
   (a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

   (b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

   (c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

   (d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

OTHER PRACTICE 1: 2004 #5



Graph of f

5. The graph of the function f shown above consists of a semicircle and three line segments. Let g be the function given by $g(x) = \int_{-3}^{x} f(t)\,dt$.

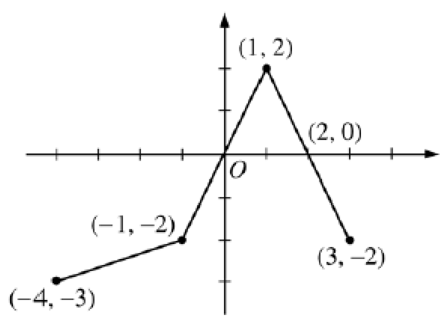
(a) Find $g(0)$ and $g'(0)$.

(b) Find all values of x in the open interval $(-5, 4)$ at which g attains a relative maximum. Justify your answer.

(c) Find the absolute minimum value of g on the closed interval $[-5, 4]$. Justify your answer.

(d) Find all values of x in the open interval $(-5, 4)$ at which the graph of g has a point of inflection.

OTHER PRACTICE 2: *VERY CHALLENGING* 2005FB #5



Graph of f

4. The graph of the function f above consists of three line segments.

(a) Let g be the function given by $g(x) = \int_{-4}^{x} f(t)\,dt$. For each of $g(-1)$, $g'(-1)$, and $g''(-1)$, find the value or state that it does not exist.

(b) For the function g defined in part (a), find the x-coordinate of each point of inflection of the graph of g on the open interval $-4 < x < 3$. Explain your reasoning.

(c) Let h be the function given by $h(x) = \int_{x}^{3} f(t)\,dt$. Find all values of x in the closed interval $-4 \le x \le 3$ for which $h(x) = 0$.

(d) For the function h defined in part (c), find all intervals on which h is decreasing. Explain your reasoning.

AB Q304 LESSON 2

APPROXIMATING THE DEFINITE INTEGRAL

WITH A RIEMANN SUM

Approximating a Definite Integral with a Riemann Sum

RECTANGLE APPROXIMATION

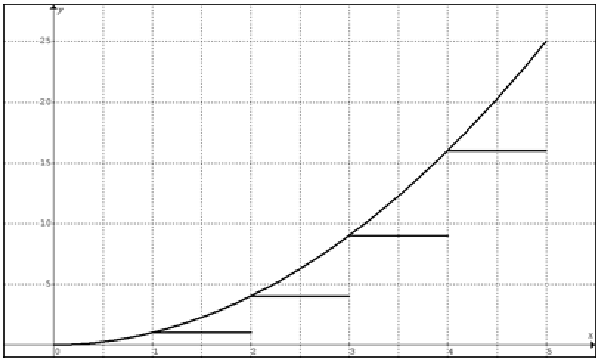
$\int_a^b f(x)dx \approx \sum_{k=1}^{n} f\left(c_k\right)\Delta x_k \; on\left[a,b\right]$ where $f(c_k)$ is the value of f at $x = c$ on the kth interval.

TRAPEZOIDAL APPROXIMATION

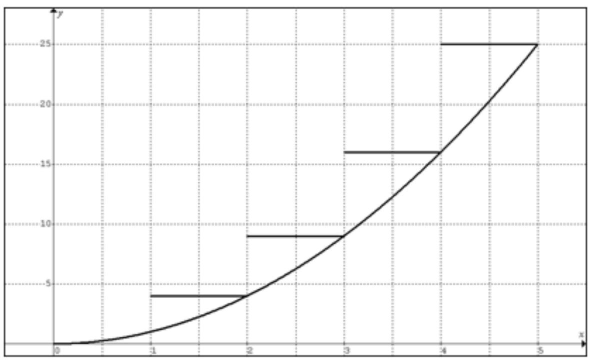
$\int_a^b f(x)dx \approx \frac{\Delta x}{2}\left(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n\right)$ where $y = f(x)$ and Δx is constant.

1. Consider the area under the curve (bounded by the x-axis) of $f(x) = x^2$ from $x = 1$ to $x = 5$.

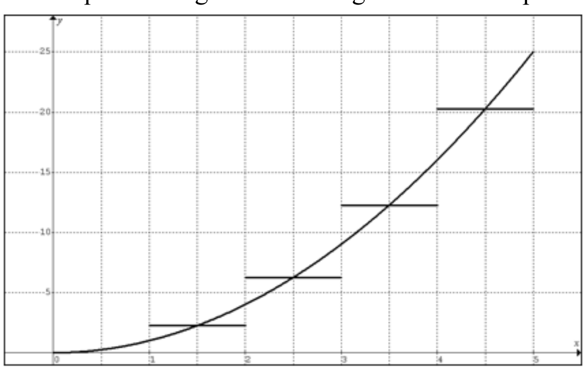
Use 4 equal rectangles whose heights are the left endpoint of each rectangle to approximate the area. (LRAM)



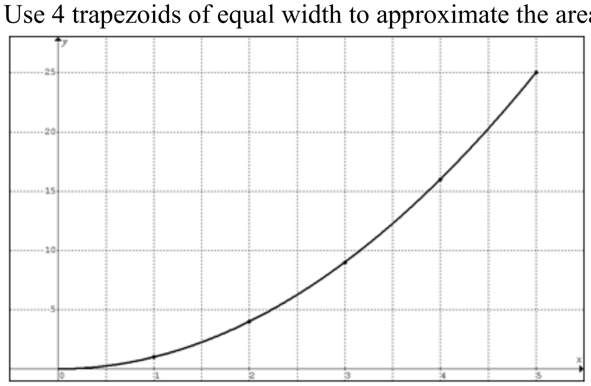
Use 4 equal rectangles whose heights are the right endpoint of each rectangle to approximate the area. (RRAM)



Use 4 equal rectangles whose heights are the midpoint of each rectangle to approximate the area. (MRAM)



Use 4 trapezoids of equal width to approximate the area. (TRAM)



2. Use LRAM, MRAM, and TRAM with $n = 3$ equal rectangles to estimate the $\int_{2}^{3.5} f(x)dx$ where values of the function $y = f(x)$ are as given in the table below.

x	2.0	2.25	2.5	2.75	3	3.25	3.5
y	3.2	2.7	4.1	3.8	3.5	4.6	5.2

3. Use a Trapezoidal approximation with $n = 3$ trapezoids to estimate the $\int_{2}^{5.5} f(x)dx$ where values of the function $y = f(x)$ are as given in the table below.

x	2.0	3	5	5.5
y	3.2	2.7	4.1	3.8

4.

t (hours)	$R(t)$ (gallons per hours)
0	9.6
3	10.4
6	10.8
9	11.2
12	11.4
15	11.3
18	10.7
21	10.2
24	9.6

The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function R of time t. The table above measured every 3 hours for a 24-hour period.

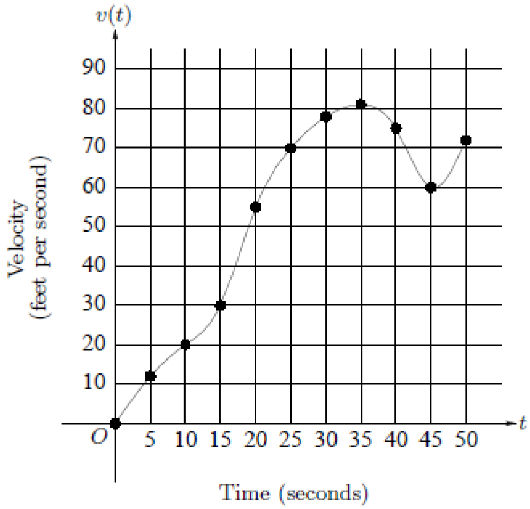
A. Using correct units, explain the meaning of the integral $\int_0^{24} R(t)dt$ in terms of water flow.

B. Use a trapezoidal approximation with 4 subdivisions of equal length to approximate $\int_0^{24} R(t)dt$.

C. Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate the average of $R(t)$.

D. The rate of water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}\left(768 + 23t - t^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the first 24-hour time period. Indicate the units of measure.

HW1: 1998 #3



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.

   (a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.

   (b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.

   (c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.

   (d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

HW2: 2004FB #3

t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

3. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

(a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

(b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

(c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.

(d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

HW3: 2008 #2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon $(t = 0)$ and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

   (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. $(t = 5.5)$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

   (c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

   (d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. $(t = 3)$, to the nearest whole number?

AB Q304 LESSON 3

FUNDAMENTAL THEOREM OF CALCULUS PART 1

(CHAIN RULE APPLIED)

ILLUSTRATIONS

Find $\frac{dy}{dx}$ for each equation below.

1. $y = \int_{7}^{x} \frac{1+t}{1+t^2}\,dt$

2. $y = \int_{0}^{x^2} e^{t^2}\,dt$

3. $y = \int_{x}^{6} \ln\left(1+t^2\right)dt$

4. $y = \int_{x^3}^{5} \frac{\cos t}{t^2+2}\,dt$

AB: Q304 LESSON 3 CUMULATIVE PRACTICE EXERCISE

x	-2	0	1	5	7	8
$f(x)$	14	5	2	-3	-10	9

Let $g(x)=\int_0^x f(t)dt$ where $f(x)$ is a differentiable function with specific values as shown above.

1. Estimate $g(8)$ using a trapezoidal approximation with four trapezoids.

2. Find $g'(1)$

3. Estimate $g''(1.3)$

4. If $h(x)=\int_5^x f(t)dt$, estimate $h(-2)$ using a left Riemann sum with three rectangles.

5. If $w(x)=\int_{-2}^{3x^2+2} f(t)dt$, find $w'(1)$.

6. If $n(x)=\int_x^0 f(t)dt$, find $n'(1)$.

AB. Q304 LESSON 3 (MVT PRACTICE)

Let $f(x) = \ln(x+2) + \sin(x)$ on $[-1, 4.5]$.

1. Find the average rate of change in f on [-1, 4.5].

2. Find the value(s) of x that satisfy the conclusion to the Mean Value Theorem for derivatives.

3. Find the average value of f on [-1, 4.5].

4. Find the value(s) of x that satisfy the conclusion to the Mean Value Theorem for integrals.

AB: Q304 CH5B PRACTICE EXAMINATION – **CALCULATOR** SECTION

(For integral approximations, show the integral for which you are approximating)

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63

1. An above ground swimming pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \le t \le 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The above table gives values of $P(t)$ for selected values of t. During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where

$R(t) = 0.25e^{0.05t^2}$

(a) Use a midpoint Riemann sum with three subintervals of equal length to approximate, to the nearest cubic foot, the total amount of water that was pumped into the pool during the time interval $0 \le t \le 12$ hours. Show the computations that lead to your answer.

(b) Calculate, to the nearest cubic foot, the total amount of water that leaked out of the pool during the time interval $0 \le t \le 12$ hours. Include the set up when expressing your answer.

(c) Use the results from parts (a) and (b) to approximate the volume of the water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.

(d) Write an equation, using an integral expression, for $A(t)$, the amount of water in the pool at time t.

(e) Find and interpret $A'(4)$, for $A(t)$ defined in part (d).

2.

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table above shows the water temperature as recorded every 3 days over a 15-day period.

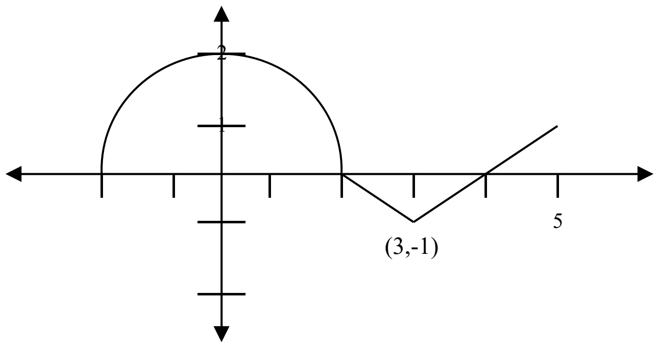
A. Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ days by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days. Use correct units when expressing your answer.

B. Using the correct units, explain the meaning of $W'(12)$.

C. Approximate $W'(12)$.

3. Suppose $g(3) = 7$ and $g'(x) = (\sin x)^x$. Estimate $g(0)$ to four decimal places. Show your reasoning.

AB: Q304 CH5B PRACTICE EXAMINATION – **NON CALCULATOR** SECTION



4. The graph of a function *f* consists of a semicircle and two line segments as shown above. Let *g* be the function given by $g(x)=\int_0^x f(t)dt.$

(a) Find $g(3)$.
(b) Find all values of *x* on the open interval (-2, 5) at which *g* has a relative maximum. Justify your answer.
(c) Write an equation for the line tangent to the graph of *g* at $x=3$.
(d) Find the *x*-coordinate of each point of inflection of the graph of *g* on the open interval (-2, 5). Justify your answer
(e) Find the absolute maximum value of *g* on [-2, 5].
(f) Let $B(x)=g(x)+\frac{1}{2}x$. Find all values of *x* on the open interval (-2, 5) at which *B* has a relative minimum. Justify your answer.

5. A particle starts at position $x = 5$ at time $t = 1$ and travels along a horizontal with velocity $x'(t)=t-2$.
A] What is the total distance traveled by the particle from time $t = 1$ to time $t = 4$? Show work.
B] Where is the particle located at time $t = 4$? Include an integral(s) set up and show work when expressing your answer.

6. Find $\frac{d}{dx}\int_8^{3x^2}\left(e^{t^3}-\cos^5 t\right)dt$.

7. Using the substitution $= x^2-3$, $\int_{-1}^{4} x\,(x^2-3)^5 dx$ is equal to which of the following?
(A) $2\int_{-2}^{13} u^5 du$

(B) $\int_{-2}^{13} u^5 du$

(C) $\frac{1}{2}\int_{-2}^{13} u^5 du$

(D) $\int_{-1}^{4} u^5 du$

(E) $\frac{1}{2}\int_{-1}^{4} u^5 du$

8. Evaluate $\int_{-2}^{1}\left(x^2-5x\right)dx$. Simplify your answer.

9. If $f(x)=2x^4$ on [-1,2], then find the value of *x* that satisfies the conclusion to the mean value theorem for integrals.