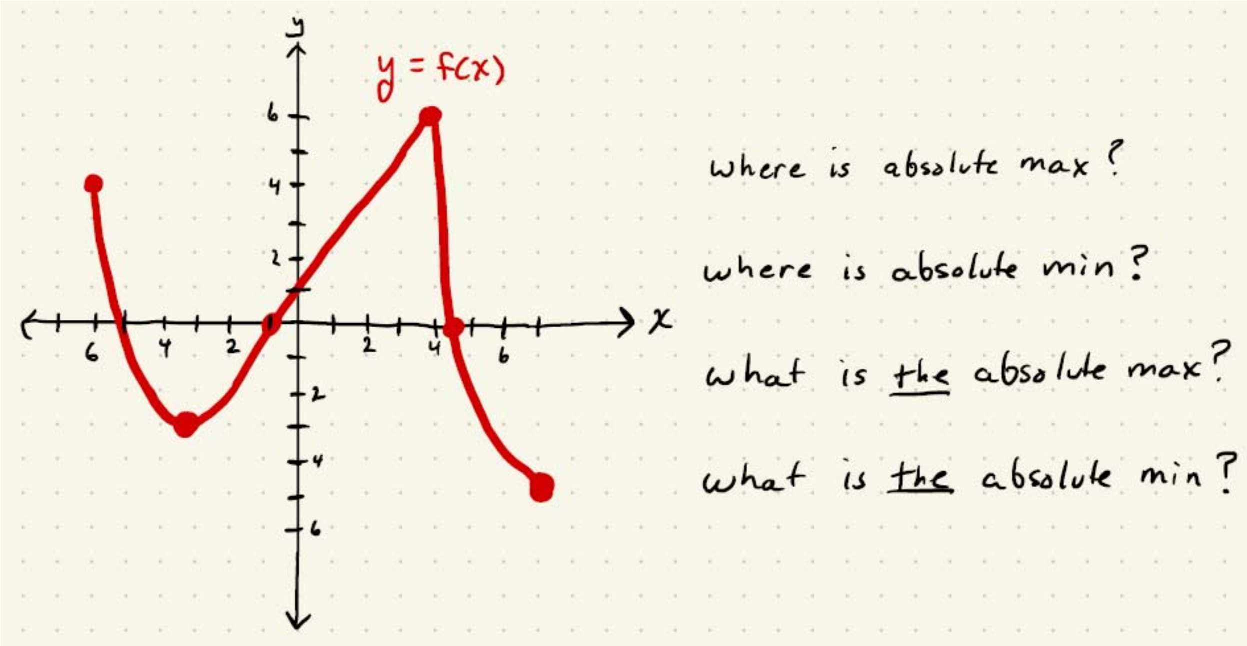
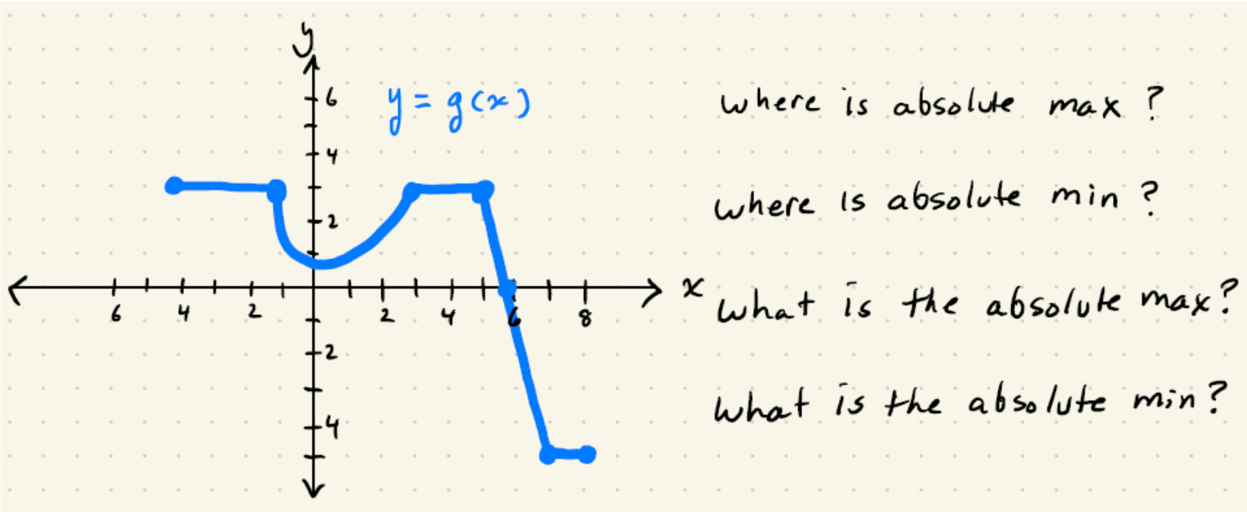
Q204.LESSON1.ABSOLUTE EXTREMES (VERSION 2.0)





EXTREME VALUE THEOREM

If a function is continuous on a closed interval $[a, b]$, then the function is guaranteed to have an absolute maximum and an absolute minimum value.

CLOSED INTERVAL TEST

Consider a function that is continuous on a closed interval [a, b].

Find all the critical x-values on the open interval (a, b).

Evaluate the function at each of the interior critical x-values and each of the endpoints.

The absolute maximum is the largest and the absolute minimum is the smallest of these function's values.

EXAMPLE: (NO TECHNOLOGY) Use the closed interval test to find the absolute maximum and the absolute minimum value of $f(x) = \ln\left(x^2 + x + 1\right)$ on the closed interval $-1 \le x \le 1$.

ONE CRITICAL ON AN OPEN INTERVAL THEOREM

Suppose a continuous function *f* has exactly one critical *x*-value on the open interval (a, b).

If the function has a local extreme at the critical point, then the local extreme is also the absolute extreme value.

EXAMPLE: (NO TECHNOLOGY) Provided any exists, find the absolute extreme values of $f(x) = 2\pi x^2 + \dfrac{2000}{x}$ on the open interval $0 < x < 10$.

OPTIMIZATION APPLICATION

EXAMPLE: (NO TECHNOLOGY) A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 100 running feet of chicken wire is available for the fence? Use calculus to justify your answer.

OPTIMIZATION APPLICATION

EXAMPLE: (TECHNOLOGY REQUIRED) A rectangle is to be inscribed under one arch of the cosine curve. What is the largest area the rectangle can have?

HOMEWORK

CLOSED INTERVAL TEST (NO TECHNOLOGY)

A. Use the closed interval test to find the absolute maximum and the absolute minimum value of $f(x)=2x^3-3x^2-12x+1$ on the closed interval $-2\le x\le 3$.

ONE CRITICAL ON OPEN THEOREM (NO TECHNOLOGY)

B. Provided it exists, find the absolute extreme value of $f(x)=2+\ln\left(\sin x+2\right)$ on the open interval $1<x<2$. Use calculus to justify your answer.

ONE CRITICAL ON OPEN THEOREM (TECHNOLOGY REQUIRED)

C. Provided it exists, find the absolute extreme value of $f(x)=x^2\sin\left(x+2.1\right)$ on the open interval $4<x<7$. Use calculus to justify your answer.

OPTIMIZATION APPLICATION (NO TECHNOLOGY)

1. What is the smallest perimeter possible for a rectangle whose area is 16 cm^2, and what are its dimensions? Use calculus to justify your answer.

2. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions? Use calculus to justify your answer.

OPTIMIZATION APPLICATION (TECHNOLOGY REQUIRED)

3. A rectangle is to be inscribed under the arch of the curve $y=4\cos(0.5x)$ from $x=-\pi$ to $x=\pi$. What is the largest area of this rectangle can have? Use calculus to justify your answer.

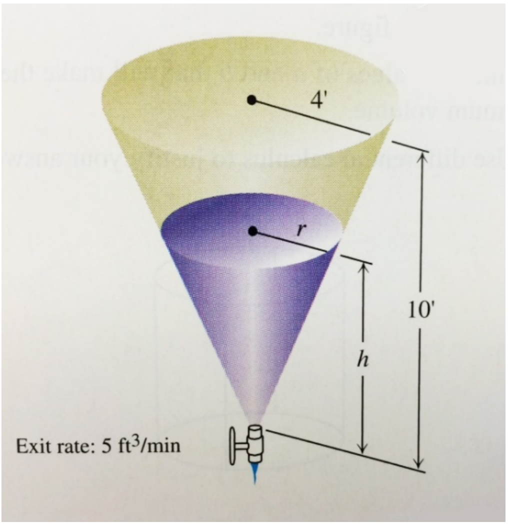
4. A rectangle has its base on the *x*-axis and is inscribed under the curve $y=\dfrac{4.6-x^2}{1+x^2}$.

What is the largest area of this rectangle can have? Use calculus to justify your answer.

RELATED RATES REVIEW (NO TECHNOLOGY)

5. Water drains from the conical tank shown in the figure at a rate of 5 ft^3/min.

How fast is the water level changing when $h = 6$ ft?



Q204.LESSON2.L’HOPITAL’S RULE (VERSION 2.0)

LINEARIZATION (SAME AS A TANGENT LINE TO A CURVE)

EX1. A. Find a linearization to $f(x) = 3x^3 - 2x^6 + 1$ at $x = 1$.

B. Use the linearization to approximate $f(1.03)$

EX2. A. Find a linearization to $f(x) = \cos^2(x) + 2x$ at $x = 0$.

B. Use the linearization to approximate $f(0.2)$

EX3. Consider the function $y = f(x)$

with $f(2) = 5$ and derivative $f'(x) = e^{(x-2)} + \sqrt{\cos(x-2) + 3x/2}$

A. Find a linearization to $y = f(x)$ at $x = 2$

B. Use the linearization to approximate $f(2.1)$

LESSON 2 HW – L'HOPITAL'S RULE

L'HÔPITAL'S RULE PRACTICE SET # 1

1) Find $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

2) Find $\lim_{x \to \pi} \frac{\csc x}{1 + \cot x}$

3) Find $\lim_{x \to 0^+} x \ln x$

4) Find $\lim_{x \to 0} \frac{\sin(x^2)}{x}$

5) Find $\lim_{x \to \pi/2} \left(\frac{\pi}{2} - x\right) \tan x$

6) Find $\lim_{x \to 1} \frac{x^3 - 1}{4x^3 - x - 3}$

7) Find $\lim_{x \to \infty} \frac{\log_2 x}{\log_2 (x+3)}$

* 8) Find $\lim_{x \to \infty} \frac{3x - 5}{2x^2 - x + 2}$

* 9) Find $\lim_{x \to \infty} \frac{4x^3 - 2x^2 + 5}{3x^3 - x}$

* 10) Find $\lim_{x \to \infty} \frac{3x^2 + 1}{5x}$

* CAN DO WITHOUT L'HOPITAL'S RULE

LESSON2 HW – LINEARIZATION

1. Let *f* be a function with $f(0) = 10.2$ and $f'(x) = 2\sin\left(\frac{\pi}{2} - x\right)$.

Use a linearization of *f* at $x = 0$ and use it to approximate $f(-0.3)$

2. Let $f(x) = \sqrt{x^2 + 9}$. Use a linearization of *f* at $x = -4$ and use it to approximate $f(-3.9)$.

3. Let $f(x) = \ln(x + 1)$. Use a linearization of *f* at $x = 0$ and use it to approximate $f(0.2)$.

4. Let *f* be a function with $f(0) = 2$ and $f'(x) = e^x \cos(x)$.

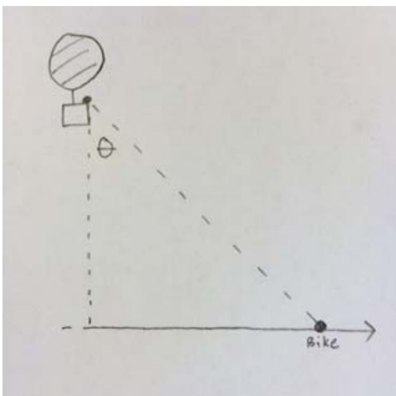
Use a linearization of *f* at $x = 0$ and use it to approximate $f(0.5)$

LESSON2 HW – RELATED RATES REVIEW

5. A man inside a hot-air balloon basket, directly above a level, straight path, is filming a bicyclist traveling along the path. The balloon is rising vertically at a constant rate of 1 m/sec. The cyclist is moving away from the point directly below the balloon at a constant rate of 5 m/sec. Consider the point in time when the balloon is 24 m above the ground and the bicycle is 7 m from the point directly below the balloon.

A. How fast is the distance between the bicycle and the balloon changing?

B. How fast is the angle θ of the balloon changing?



LESSON 2 L'HOPITAL'S RULE ADDITIONAL PRACTICE

L'HOPITAL'S RULE PRACTICE SET #2

① $\lim_{x \to 0} \dfrac{\sin x}{2x}$

② $\lim_{x \to 0} \dfrac{5x}{\tan x}$

③ $\lim_{x \to 5} \dfrac{\sqrt{x-1} - 2}{x^2 - 25}$

④ $\lim_{x \to -3} \dfrac{x^2 + 2x - 3}{2x^2 + 3x - 9}$

⑤ $\lim_{x \to 0} \dfrac{\sin x - x}{\tan x - x}$

⑥ $\lim_{x \to 0} \dfrac{x + 1 - e^x}{x^2}$

⑦ $\lim_{x \to 0} \dfrac{x - \sin x}{x^3}$

⑧ $\lim_{x \to \pi/2} \dfrac{1 - \sin x}{\cos x}$

⑨ $\lim_{x \to \pi/2} \dfrac{1 + \sin x}{\cos^2 x}$

⑩ $\lim_{x \to 0^+} \dfrac{\ln x}{\cot x}$

⑪ $\lim_{x \to 0} \dfrac{x\cos x + e^{-x}}{x^2}$

* ⑫ $\lim_{x \to \infty} \dfrac{2x^2 + 3x + 1}{5x^2 + x + 4}$

⑬ $\lim_{x \to 0} \dfrac{\sin^{-1}(2x)}{\sin^{-1}(x)}$

⑭ $\lim_{x \to -\infty} \dfrac{3 - 3^x}{5 - 5^x}$

⑮ $\lim_{x \to 1} \dfrac{2x^3 - 5x^2 + 6x - 3}{x^3 - 2x^2 + x - 1}$

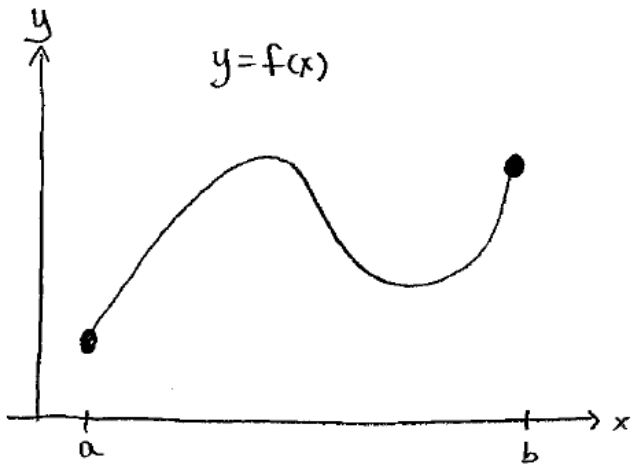
⑯ $\lim_{x \to 0} \dfrac{x - \tan^{-1} x}{x \sin x}$

* can do without L'HOPITAL'S Rule

Q204.LESSON3.MEAN VALUE THEOREM (VERSION 2.0)

MEAN VALUE THEOREM

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exist at least one point $x = c$ in (a, b) such that:



INTERMEDIATE VALUE THEOREM

If f is continuous on the closed interval $[a, b]$, then f takes on every value between $f(a)$ and $f(b)$.

If f' is continuous on the closed interval $[a, b]$, then f' takes on every value between $f'(a)$ and $f'(b)$.

MVT: TECHNOLOGY REQUIRED

EX1. Consider the function: $f(x) = x\cos(x^2)$ on [1.5, 3]

Find the value(s) of x on $1.5 < x < 3$ that satisfy the conclusion to the Mean Value Theorem.

Round Decimals to Three Decimal Places

MVT: NO TECHNOLOGY PERMITTED

EX2. Consider the function $g(x) = x^3 + 1$ on $-2 \le x \le 4$.

Find the value(s) of x on $-2 < x < 4$ that satisfy the conclusion to the Mean Value Theorem.

MVT: TECHNOLOGY REQUIRED

PRACTICE1: Consider the function: $f(x) = x^{-2x} + x^3$ on [1.8, 3.3]

Find the value(s) of *x* on $1.8 < x < 3.3$ that satisfy the conclusion to the Mean Value Theorem.

Round Decimals to Three Decimal Places

MVT: NO TECHNOLOGY PERMITTED

PRACTICE2: Consider the function $g(x) = 4 + \sqrt{x-1}$ on $1 \le x \le 5$.

Find the value(s) of x on $1 < x < 5$ that satisfy the conclusion to the Mean Value Theorem.

NOTES: MVT + IVT

1. Consider a function $y = f(x)$ which is both continuous and *twice* differentiable on $3 \le x \le 8$

Select values of x and $f(x)$ are shown in the table below.

x	3	4	8
$f(x)$	6	12	3

Prove that $f'(x) = 0$ at least one time on $3 \le x \le 8$.

2. Consider a function $y = f(x)$ which is both continuous and differentiable on $2 \le x \le 5$

Select values of x and $f(x)$ are shown in the table below.

x	2	4	5
$f(x)$	3	16	16

Prove that $f'(x) = 0$ at least one time on $2 \le x \le 5$.

MVT: TECHNOLOGY REQUIRED

HW1: Consider the function: $f(x) = e^{\sin x + 3} - \sqrt{x}$ on [4, 9]

Find the value(s) of x on $4 < x < 9$ that satisfy the conclusion to the Mean Value Theorem.

Round Decimals to Three Decimal Places

HW2: Consider the function: $f(x) = \cos\left(\sin\left(x^2\right)\right)$ on [0, 1.2]

Find the value(s) of x on $0 < x < 1.2$ that satisfy the conclusion to the Mean Value Theorem.

Round Decimals to Three Decimal Places

MVT: NO TECHNOLOGY PERMITTED

HW3: Consider the function $g(x) = x^3 + 4x$ on $-3 \le x \le 6$.

Find the value(s) of x on $-3 < x < 6$ that satisfy the conclusion to the Mean Value Theorem.

HW4: Consider the function $g(x) = x^3 - 2x^2 + x + 3$ on $-1 \le x \le 1$.

Find the value(s) of x on $-1 < x < 1$ that satisfy the conclusion to the Mean Value Theorem.

HW5: Explain why the MVT does not apply to $f(x) = \dfrac{1}{(x-1)^2}$ on $0 \le x \le 2$.

HW6: Explain why the MVT does not apply to $f(x) = x^{2/3}$ on $-8 \le x \le 8$.

MISCELLANEOUS: NO TECHNOLOGY (7 – 12)

7. Consider a function $y = f(x)$ which is both continuous and twice differentiable on $10 \le x \le 15$ Select values of x and $f(x)$ are shown in the table below.

x	10	12	15
$f(x)$	-6	-20	-1

Prove that $f'(x) = 0$ at least one time on $10 \le x \le 15$.

8. Let f be the function given by $f(x) = x^3 - 7x + 6$.

A. Write an equation of the tangent to the graph of f at $x = -1$.

B. Use the tangent line (linearization) to approximate $f(-1.02)$

C. Find the average rate of change of f on the interval [1, 3].

D. Find the number c on the interval (1, 3), such that the instantaneous rate of change in f at $x = c$ is equal to the average rate of change found in part C.

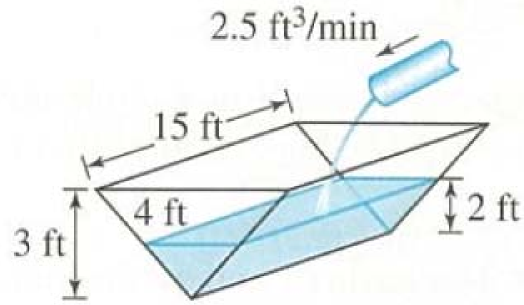
9. Consider the continuous and twice differentiable function $y = f(x)$ with selected values shown in the chart below.

X	-4	-3	-2	-1	0	1	2
$f(x)$	-139	72	41	2	-3	-4	17

What is the minimum number of times that $f'(x) = 0$ on the domain [-4, 2]. Explain your reasoning.

10.

Filling a Trough A trough is 15 ft long and 4 ft across the top as shown in the figure. Its ends are isosceles triangles with height 3 ft. Water runs into the trough at the rate of $2.5\ \text{ft}^3/\text{min}$. How fast is the water level rising when it is 2 ft deep?



11.

Particle Motion A point moves smoothly along the curve $y = x^{3/2}$ in the first quadrant in such a way that its distance from the origin increases at the constant rate of 11 units per second. Find dx/dt when $x = 3$.

12.

Largest Rectangle A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?