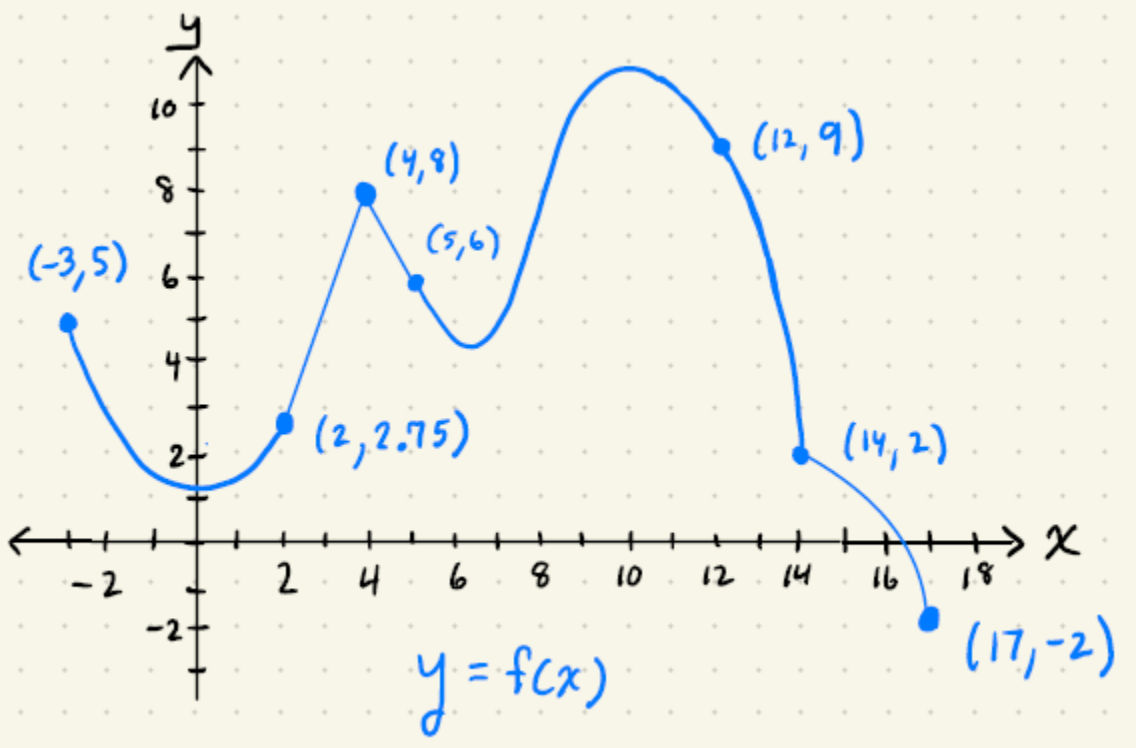
AP CALCULUS AB

UNIT: Q100

THE SPIRIT OF DERIVATIVES

ANALYSIS OF SLOPE

AB.Q100.LESSON1.GRAPH BLUE 1



Above is the graph of $y = f(x)$.

Notes:

Select integer coordinates have been labeled

The sections of the function f on $2 \le x \le 4$ and $4 \le x \le 5$ are linear.

f has a smooth slope transition at $x = 2$

$f'(5) = -2$

$f'(12) = -1.7$

QUESTIONS FOR GRAPH BLUE 1

A1. Find or estimate the domain of *f*

A2. Find or estimate the range of *f*

B1. Find or estimate the values of *x* for which the function *f* is continuous.

B2. Write the appropriate justification for *x*-values where the function *f* is not continuous.

C1. Find or estimate the values of *x* for which the function *f* is increasing. Justify with slope.

C2. Find or estimate the values of *x* for which the function *f* is decreasing. Justify with slope.

D1. Find or estimate the values of *x* for which the function *f* has a relative maximum. Justify with slope where applicable.

D2. Find or estimate the values of *x* for which the function *f* has a relative minimum. Justify with slope where applicable.

E1. Find or estimate the values of *x* for which the function *f* is concave upward. Justify with slope.

E2. Find or estimate the values of *x* for which the function *f* is concave downward. Justify with slope.

F1. Find the average slope over [-3, 17]

F2. Find the average $f'(x)$ over [4, 12]

G1. Find or estimate the slope of *f* at $x = 4.5$

G2. Find or estimate $f'(3)$

G3. Find or estimate $\frac{dy}{dx}$ at $x = 0$

G4. Find or estimate $f'(4)$

G5. Find or estimate $f'(13)$

H. Find or estimate the *x*-values where *f* has a horizontal tangent

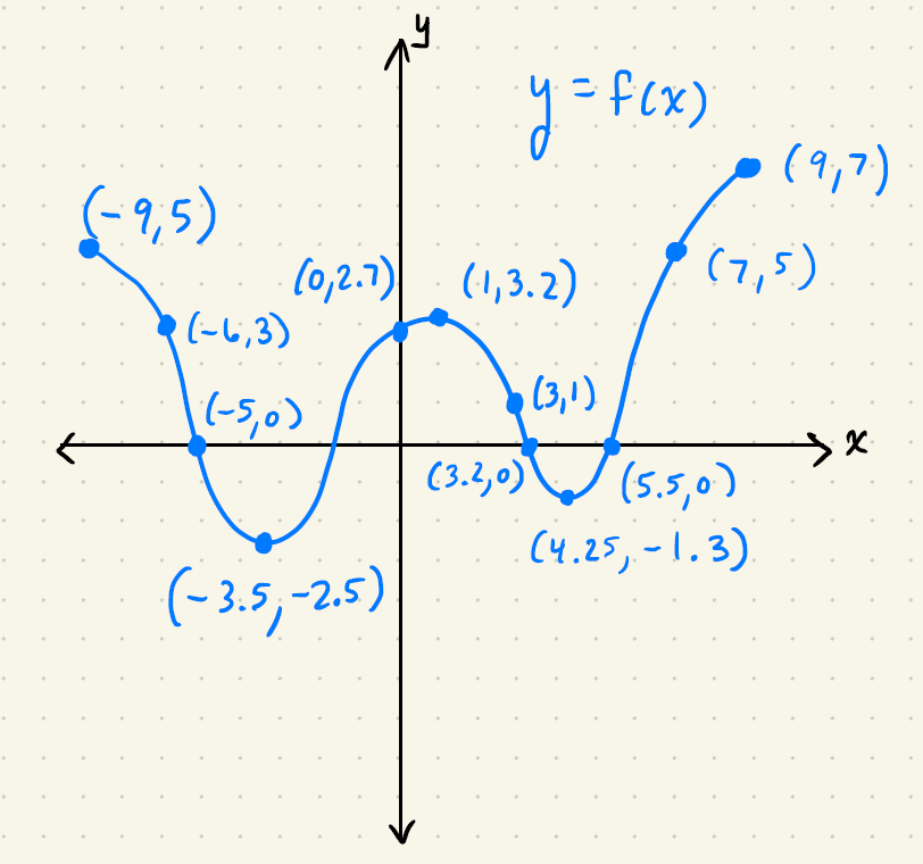
I. Write the appropriate justification for *x*-values where the function *f* does not have a slope.

J. Explain why the Mean Value Theorem applies on [-3, 4] and estimate or find the *x*-value(s) where the instantaneous slope is equal to the average slope on [-3, 4]

K. Write a point-slope equation of the line segment on [2,4]

L. Write a point-slope equation of the line tangent to *f* at $x = 12$.

AB.Q100.LESSON1.GRAPH BLUE 2



Above is the graph of $y = f(x)$.

Notes:

Select integer coordinates have been labeled

f is smooth on the domain.

f has local vertices at $x = -3.5, 1, 4.25$

$f'(0) = \frac{1}{9}$

QUESTIONS FOR GRAPH BLUE 2

A1. Find or estimate the domain of *f*

A2. Find or estimate the range of *f*

B1. Find or estimate the values of *x* for which the function *f* is continuous.

B2. Write the appropriate justification for *x*-values where the function *f* is not continuous.

C1. Find or estimate the values of *x* for which the function *f* is increasing. Justify with slope.

C2. Find or estimate the values of *x* for which the function *f* is decreasing. Justify with slope.

D1. Find or estimate the values of *x* on $-9 < x < 9$ for which the function *f* has a relative maximum. Justify with slope.

D2. Find or estimate the values of *x* on $-9 < x < 9$ for which the function *f* has a relative minimum. Justify with slope.

E1. Find or estimate the values of *x* for which the function *f* is concave upward. Justify with slope.

E2. Find or estimate the values of *x* for which the function *f* is concave downward. Justify with slope.

F1. Find the average slope over [-9, 9]

F2. Find the average $f'(x)$ over [0, 7]

G1. Find or estimate the slope of *f* at $x = -5$

G2. Find or estimate $f'(7)$

G3. Find or estimate $\frac{dy}{dx}$ at $x = -6$

G4. Find or estimate $f'(-3.5)$

G5. Find or estimate $f'(9)$

H. Find or estimate the *x*-values where *f* has a horizontal tangent

I. Write the appropriate justification for *x*-values where the function *f* does not have a slope.

J. Explain why the Mean Value Theorem applies on [-9, 9] and estimate or find the *x*-value(s) where the instantaneous slope is equal to the average slope on [-9, 9]

K. Write a point-slope equation of the line tangent to *f* at $x = 0$.

BASIC ESSENTIALS

Category I. Trigonometric and Transcendental Function Graphs

Commit the graph of the following functions to memory

(This includes possible zeroes, intercepts, and or asymptotes)

1. $y = \sin x$
2. $y = \cos x$
3. $y = \tan x$
4. $y = \sec x$
5. $y = e^x$
6. $y = \ln x$

Category II. Trigonometric Expressions

Commit the unit circle to memory in effort to quickly evaluate expressions such as the example below.

1. $\sin\left(\frac{5\pi}{6}\right)$
2. $\cos\left(\frac{11\pi}{6}\right)$
3. $\tan\left(\frac{4\pi}{3}\right)$
4. $\sec\left(\frac{2\pi}{3}\right)$
5. $\csc\left(\frac{7\pi}{6}\right)$

Category III. Trigonometric Equations

Commit the unit circle to memory in effort to solve equations such as the example below.

Solve each equation on the domain $[0, 2\pi]$ (answer in radians):

1. $\sin(x) = \frac{1}{2}$

2. $\tan(x) = -\frac{1}{\sqrt{3}}$

3. $\sin(x) + \cos(x) = 0$

4. $\cos^2(x) + \cos(x) = 0$

5. $\sin(x) - 2\cos(x)\sin(x) = 0$

Category IV. Simple Piecewise Equation Graphs

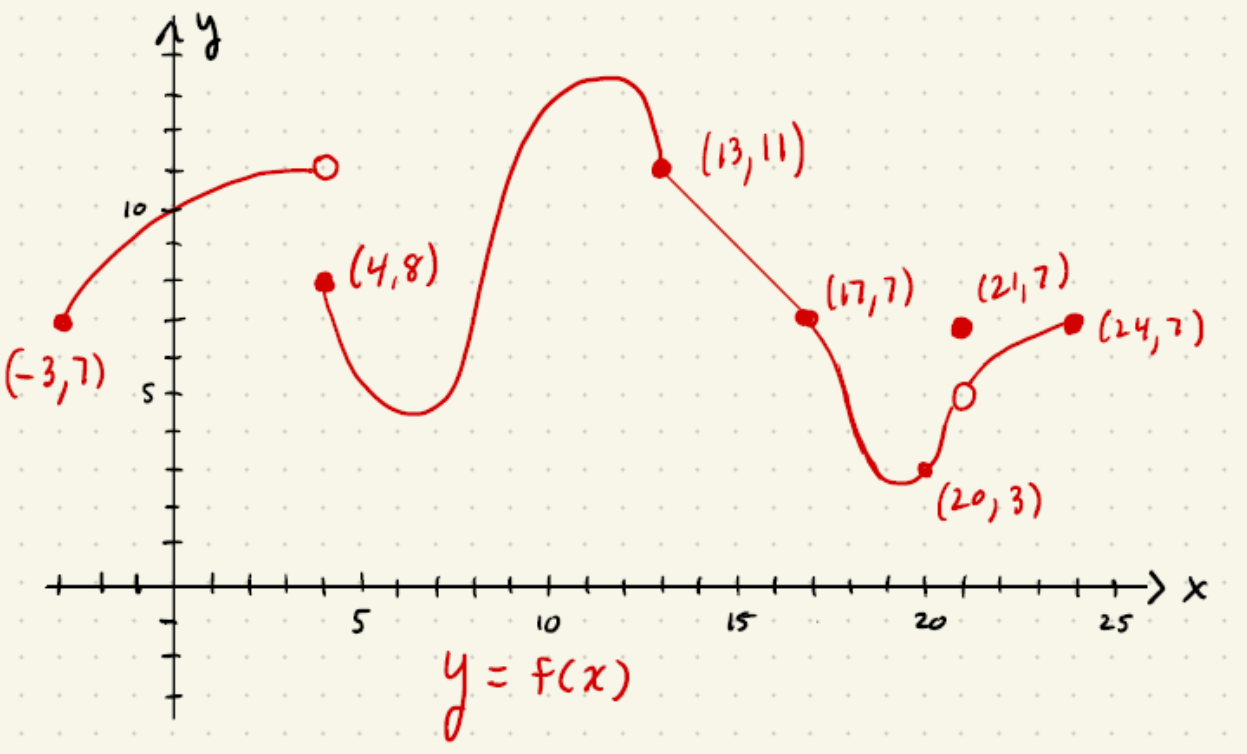
Be able to graph simple piecewise equations such as the one below.

1. Graph the function $f(x) = \begin{cases} 2x+3; & x < 0 \\ x^2; & 0 \le x < 2 \\ 1; & x \ge 2 \end{cases}$.

2 Graph the function $r(x) = \begin{cases} x+2; & x \ne -1 \\ 5; & x = -1 \end{cases}$

3. Graph the function $h(x) = \begin{cases} x^2; & x \le 0 \\ x; & x > 0 \end{cases}$

AB.Q100.LESSON2.GRAPH RED 3



Above is the graph of $y = f(x)$.

Notes:

Select integer coordinates have been labeled

The open circles are around integer coordinates

The section of f on $13 \le x \le 17$ is linear.

f has a smooth slope transition at $x = 17$

$f'(20) = 1.12$

QUESTIONS FOR GRAPH RED 3

A1. Find or estimate the domain of *f*

A2. Find or estimate the range of *f*

B1. Find or estimate the values of *x* for which the function *f* is continuous.

B2. Write the appropriate justification for *x*-values where the function *f* is not continuous.

C1. Find or estimate the values of *x* for which the function *f* is increasing. Justify with slope.

C2. Find or estimate the values of *x* for which the function *f* is decreasing. Justify with slope.

D1. Find or estimate the values of *x* for which the function *f* has a relative maximum. Justify with slope where applicable.

D2. Find or estimate the values of *x* for which the function *f* has a relative minimum. Justify with slope where applicable.

E1. Find or estimate the values of *x* for which the function *f* is concave upward. Justify with slope.

E2. Find or estimate the values of *x* for which the function *f* is concave downward. Justify with slope.

F1. Find the average slope over [4, 17]

F2. Find the average $f'(x)$ over [13, 17]

G1. Find or estimate the slope of *f* at $x = 15$

G2. Find or estimate $f'(-1.5)$

G3. Find or estimate $\frac{dy}{dx}$ at $x = 17$

G4. Find or estimate $f'(19.5)$

G5. Find or estimate $f'(13)$

H. Find or estimate the *x*-values where *f* has a horizontal tangent

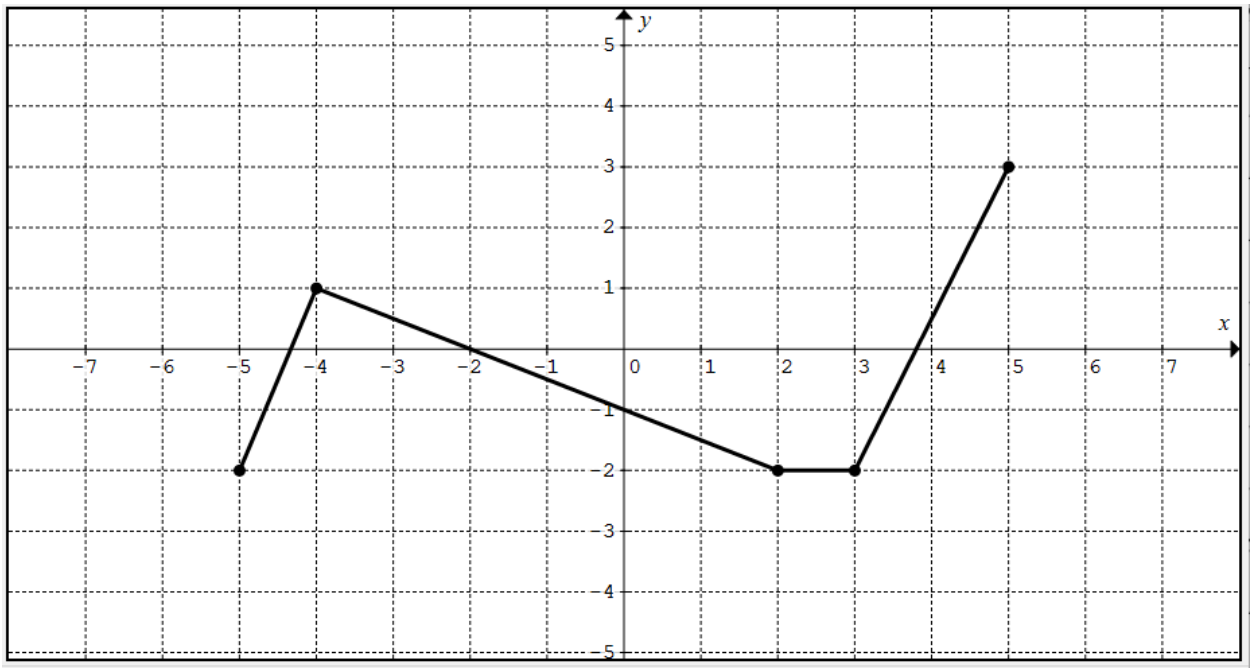
I. Write the appropriate justification for *x*-values where the function *f* does not have a slope.

J. Explain why the Mean Value Theorem applies on [4, 13] and estimate or find the *x*-value(s) where the instantaneous slope is equal to the average slope on [4, 13]

K. Write a point-slope equation of the line segment on [13,17]

L. Write a point-slope equation of the line tangent to *f* at $x = 20$.

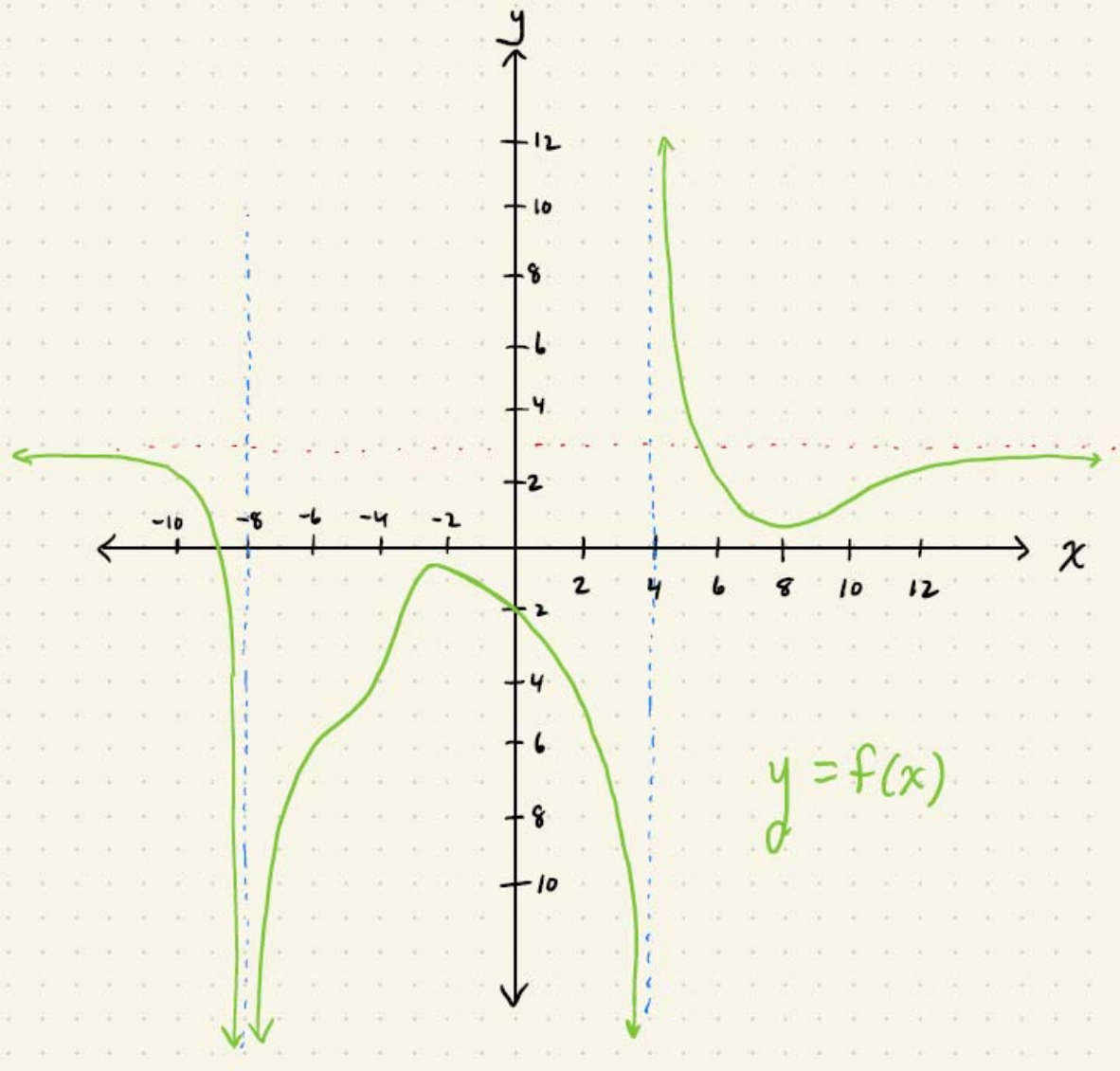
AB.Q100.LESSON2.GRAPH BLACK 4



Above is the graph of the function $y = f(x)$ which is comprised of four line segments.

A. Write a piecewise equation for $y = f(x)$.
B. Write a piecewise equation for the slope of $y = f(x)$.
C. Compute all zeros.
D. For what values of x is the function f increasing. Justify with slope.

AB.Q100.LESSON2.GRAPH GREEN 5



Above is the graph of the function $y = f(x)$.

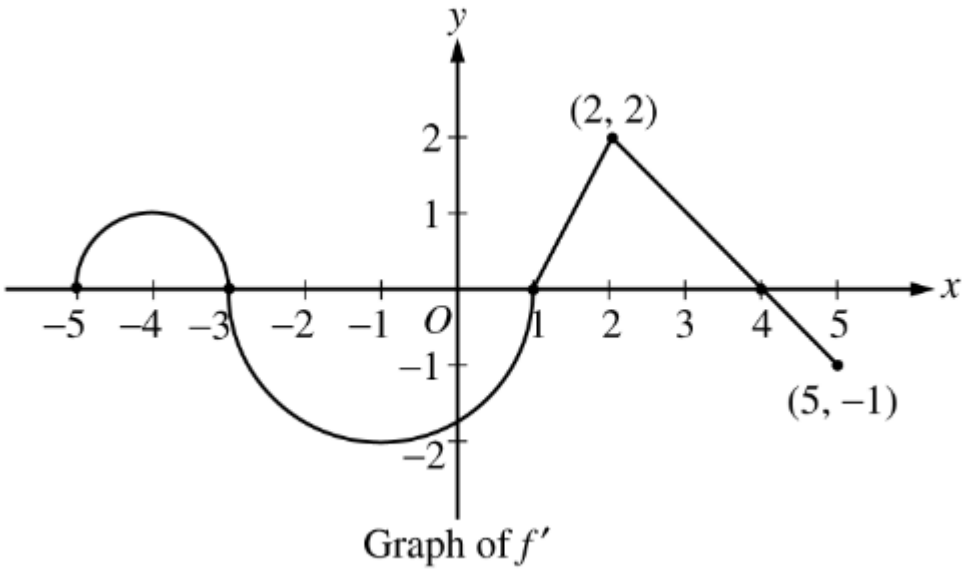
This graph shows asymptotes by using light dashed lines (which are not actually part of the function)

The function *f* has a relative maximum of -0.55 at $x = -2.5$.

The function *f* has a *y*-intercept of $y = -2$.

A. State the domain and range of the function *f*.
B. Estimate the *x*-values where the function *f* is increasing.
C. Estimate the *x*-values where the function *f* is concave upward. Justify with slope.
D. Estimate the *x*-value(s) where the function *f* has a relative minimum. Justify with slope.
E. Estimate $f'(-1)$.
F. Write complete limit statements for each of the graph's asymptotes.

AB.Q100.LESSON3.Deriviative Graph 1

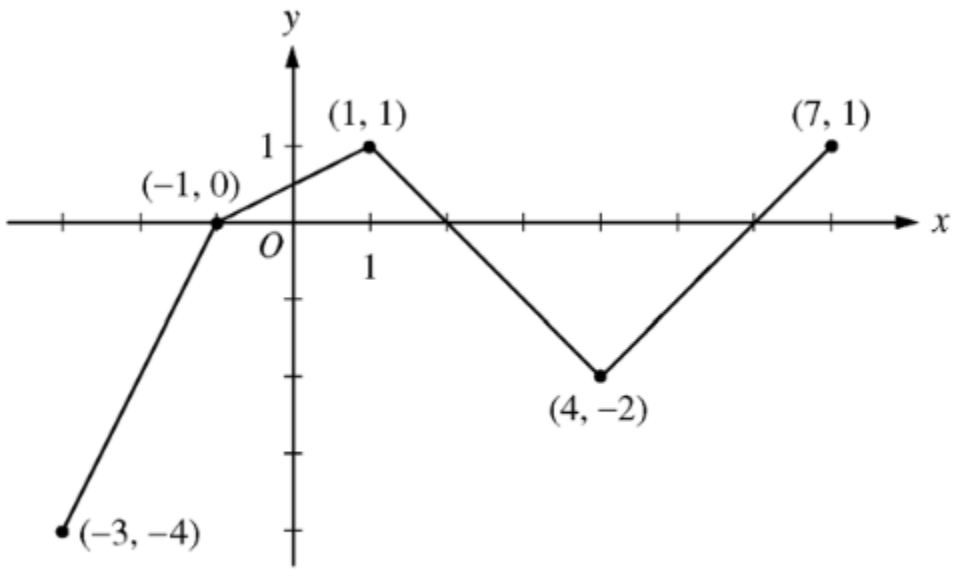


Graph of f'

Let *f* be a function defined on the closed interval $-5 \le x \le 5$ with $f(-1) = 3$. The graph of f', the derivative of *f*, consisting of two semi circles and two line segments, is shown above.

A. Find $f'(3)$
B. For what values of *x* is the function *f* decreasing?
C. For $-5 < x < 5$, find all values *x* at which *f* has a relative maximum. Justify your answer.
D. For $-5 < x < 5$, find all values *x* at which *f* is concave upward. Justify your answer.
E. Write an equation for the line tangent to *f* at $x = -1$
F. Write the piecewise equation for $y = f'(x)$.

AB.Q100.LESSON3.Deriviative Graph 2

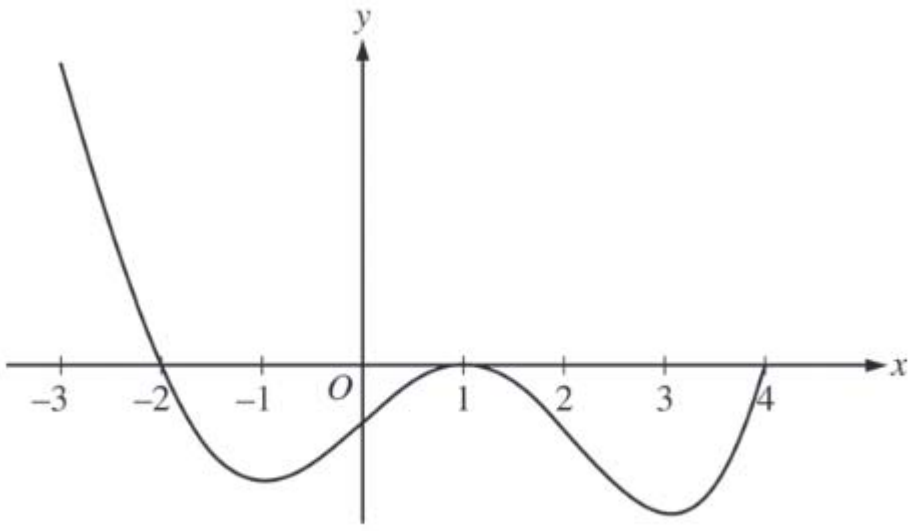


Graph of g'

Let g be a function defined on the closed interval $-3 \le x \le 7$ with $g(1) = 4$. The graph of g', the derivative of g, consisting of line segments, is shown above.

A. Find $g'(5)$
B. For what values of x is the function g increasing?
C. For $-3 < x < 7$, find all values x at which g has a relative minimum. Justify your answer.
D. For $-3 < x < 7$, find all values x at which g is concave downward. Justify your answer.
E. Write an equation for the line tangent to g at $x = 1$.
F. Write the piecewise equation for $y = g'(x)$.

AB.Q100.LESSON3.Deriviative Graph 3

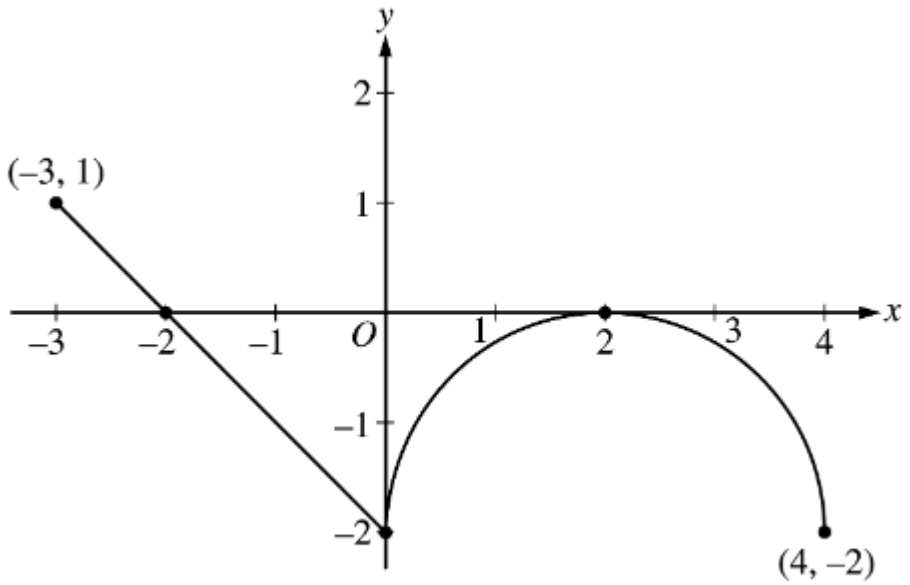


Graph of f'

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(1) = 3$. The graph of f' is shown above. The graph of $y = f'(x)$ has local vertices at $x = -1, 1, 3$ and zeros at $x = -2, 1, 4$.

A. Find $f'(-2)$
B. For what values of x is the function f decreasing?
C. For $-3 < x < 4$, find all values x at which f has a relative maximum. Justify your answer.
D. For $-3 < x < 4$, find all values x at which f is concave downward. Justify your answer.
E. Write an equation for the line tangent to f at $x = 1$

AB.Q100.LESSON3.Deriviative Graph 4

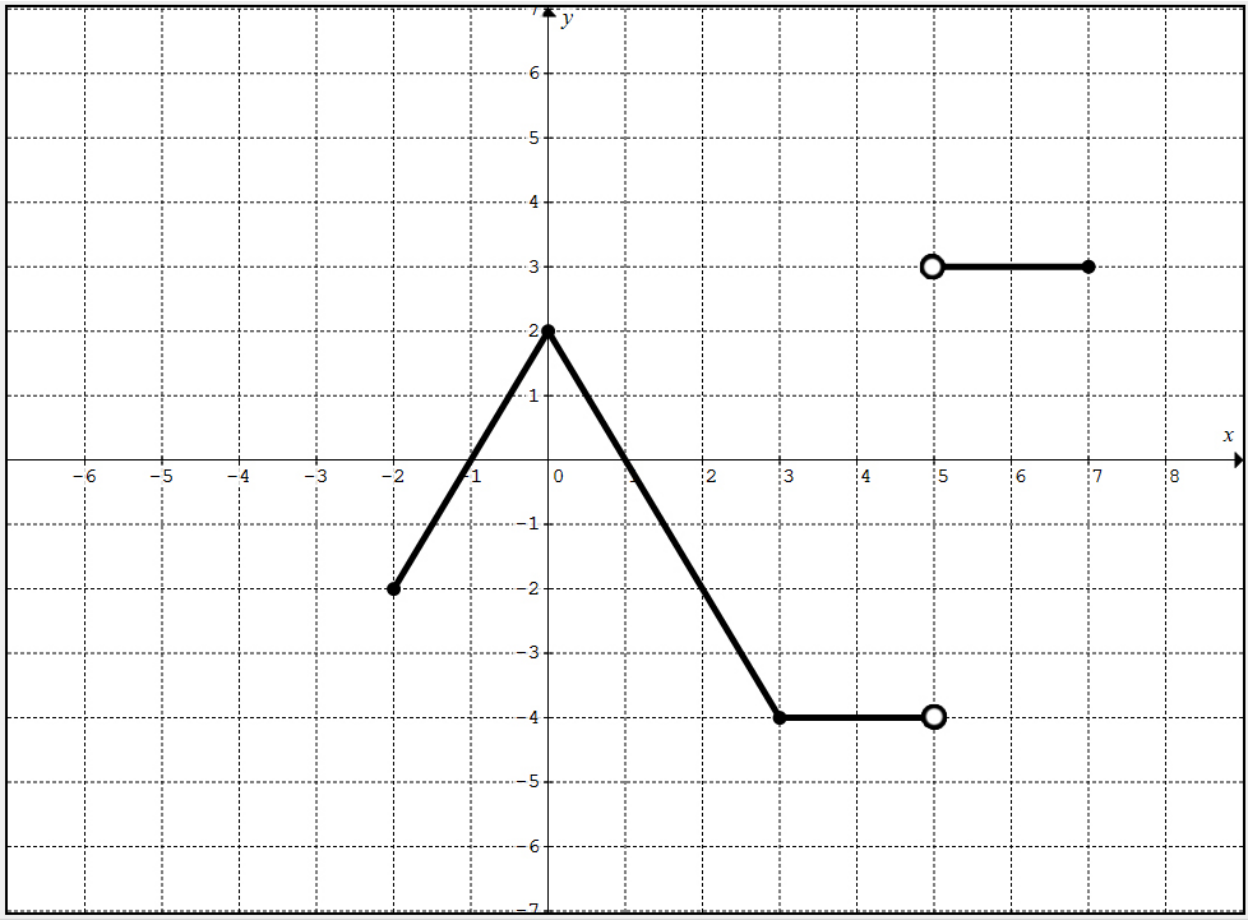


Graph of f'

Let *f* be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of *f,* consisting of a semi circle and a line segments, is shown above.

A. Find $f'(-0.25)$
B. For what values of *x* is the function *f* increasing?
C. For $-3 < x < 4$, find all values *x* at which *f* has a relative maximum. Justify your answer.
D. For $-3 < x < 4$, find all values *x* at which *f* is concave upward. Justify your answer.
E. Write an equation for the line tangent to *f* at $x = 0$
F. Write the piecewise equation for $y = f'(x)$.

AB.Q100.LESSON3.Deriviative Graph 5



Let h be a continuous function defined on the closed interval $-2 \le x \le 7$ with $h(0) = 0$. The graph of h', the derivative of h, consisting of line segments, is shown above.

A. Find $h'(4)$ and $h'(5)$
B. For what values of x is the function h increasing?
C. For $-2 < x < 7$, find all values x at which h has a relative minimum. Justify your answer.
D. For $-2 < x < 7$, find all values x at which h is concave downward. Justify your answer.
E. Write an equation for the line tangent to h at $x = 0$.
F. Write the piecewise equation for $y = h'(x)$.
G. Explain how it is possible for the graph of h (not seen) to be continuous when clearly h' is not continuous at $x = 5$.