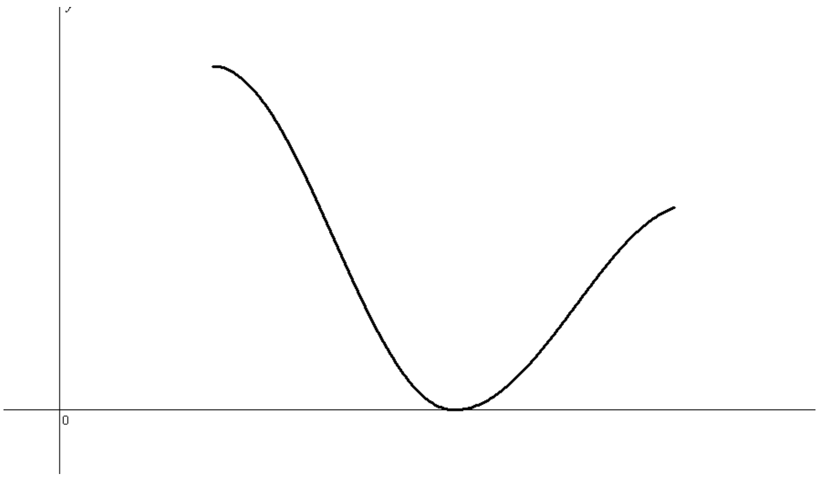
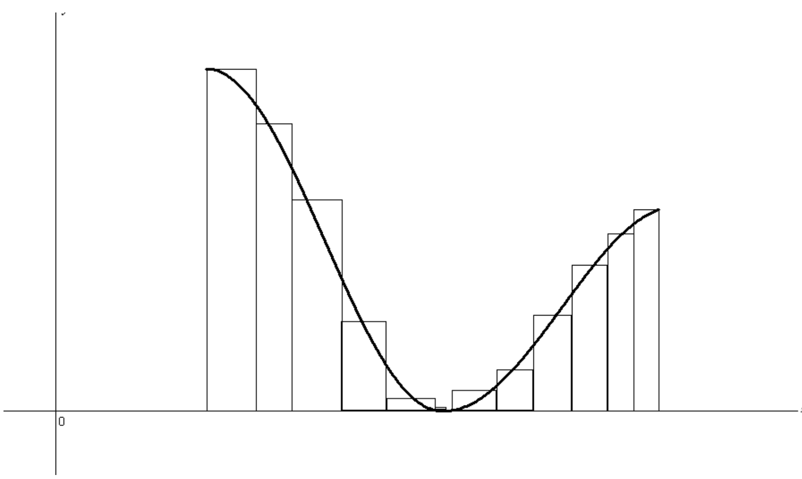
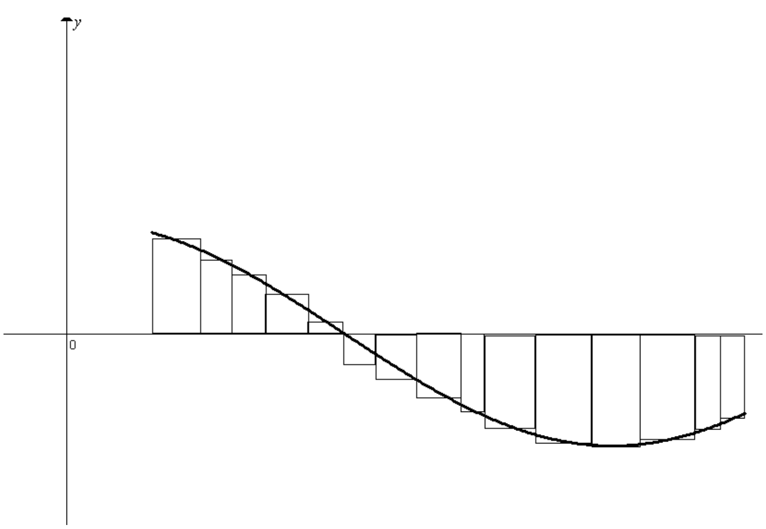
CALCULUS AP AB – Q303CH5A: (Lesson 1-A) AREA and INTEGRAL

Area – Integral Connection and Riemann Sums







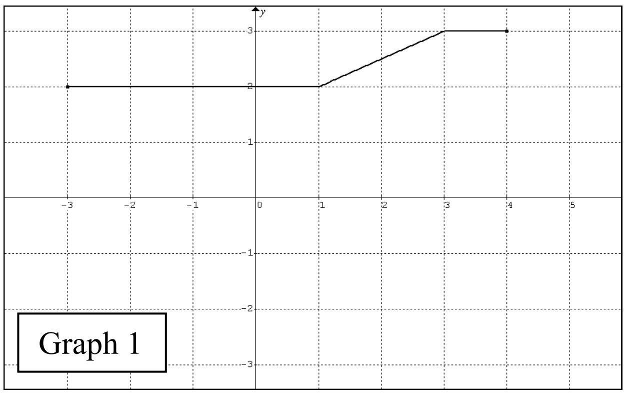
I. INTEGRAL AND AREA – BY HAND (APPEAL TO GEOMETRY)

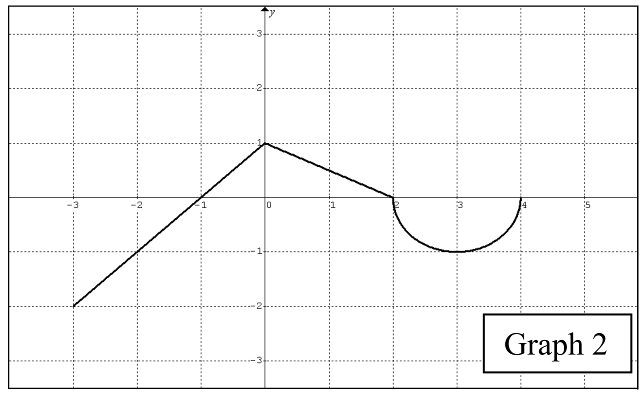
NOTES: Below are graphs that each represent a different $f(x)$ from $x = -3$ to $x = 4$.

A] Find the Area bounded by the graph of f and the x-axis.

B] Evaluate the integral $\int_{-3}^{4} f(x)dx$. Express the integral as it relates to a collection of areas.

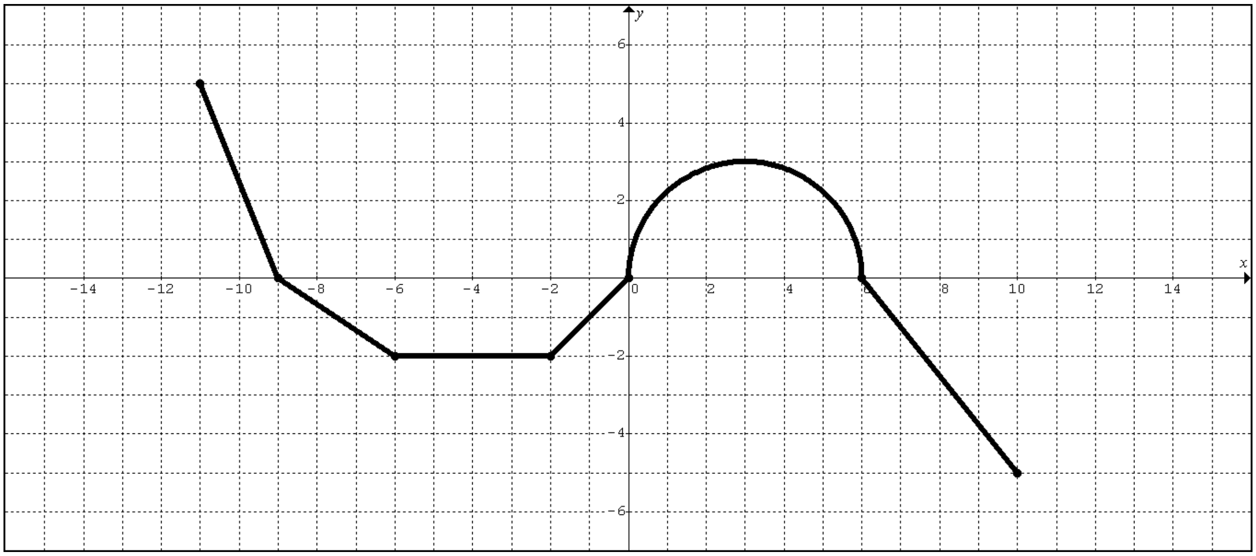
C] Express the Area as it relates to an (or set of) integral(s).





PRACTICE

1. The graph of $f(x)$ is made up of line segments and semi-circles as shown in the graph below. Evaluate A – E.



A. Find the total area bounded by the graph of $f(x)$ and the x-axis.

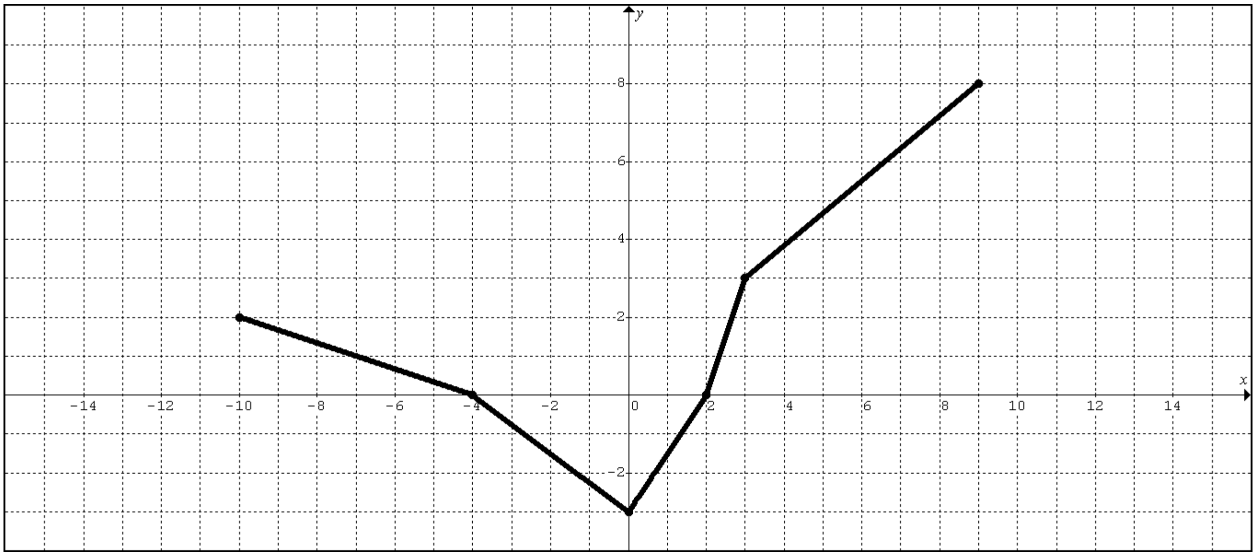
B. $\int_{-11}^{10} f(x)dx$

C. $\int_{0}^{10} f(x)dx$

D. $\int_{-9}^{-2} f(x)dx$

E. $\int_{0}^{-9} f(x)dx$

2. The graph of $f(x)$ is made up of line segments and semi-circles as shown in the graph below. Evaluate A – E.



A. Find the total area bounded by the graph of $f(x)$ and the x-axis.

B. $\int_{-10}^{9} f(x)dx$

C. $\int_{-4}^{2} f(x)dx$

D. $\int_{-4}^{-10} f(x)dx$

E. $\int_{2}^{2} f(x)dx$

3. Evaluate each integral by appealing to geometry.

A. $\int_{-2}^{4} (1 - x)dx$

B. $\int_{-2}^{4} 2dx$

II. CH5 – (INTEGRAL PROPERTIES):

Suppose that *f* and *h* are continuous functions and that

$\int_1^9 f(x)dx = -1$, $\int_7^9 f(x)dx = 5$, and $\int_7^9 h(x)dx = 4$.

Find each integral below:

$$\int_1^9 -2f(x)dx$$

$$\int_7^9 [f(x)+h(x)]dx$$

$$\int_7^9 [2f(x)-3h(x)]dx$$

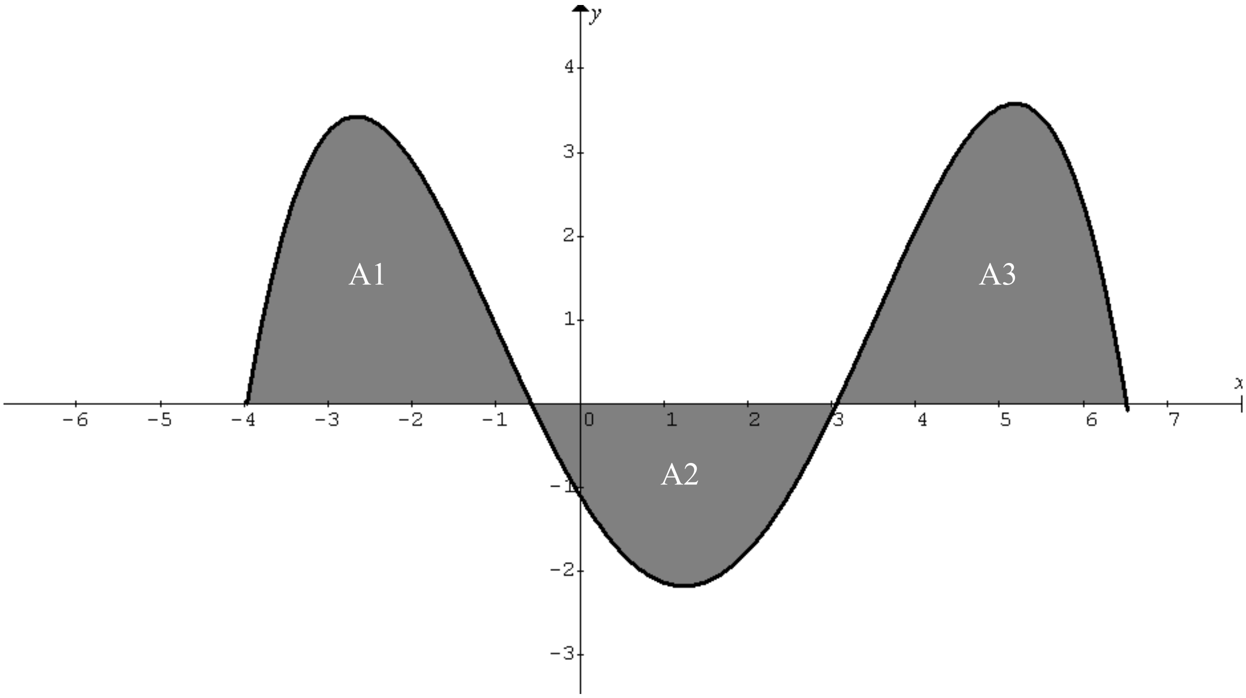
$$\int_9^1 f(x)dx$$

$$\int_1^7 f(x)dx$$

$$\int_9^7 [h(x)-f(x)]dx$$

BC: Q301 CH5A – LESSON (1A) HOMEWORK

1.



In the diagram above, the values of the areas A1, A2, and A3 bounded by the graph of $f(x)$ and the x-axis, are 7, 5, and 8 square units respectively. $f(x)$ has zeros at -4, -0.6, 3, and 6.5.

Calculate the following definite integrals:

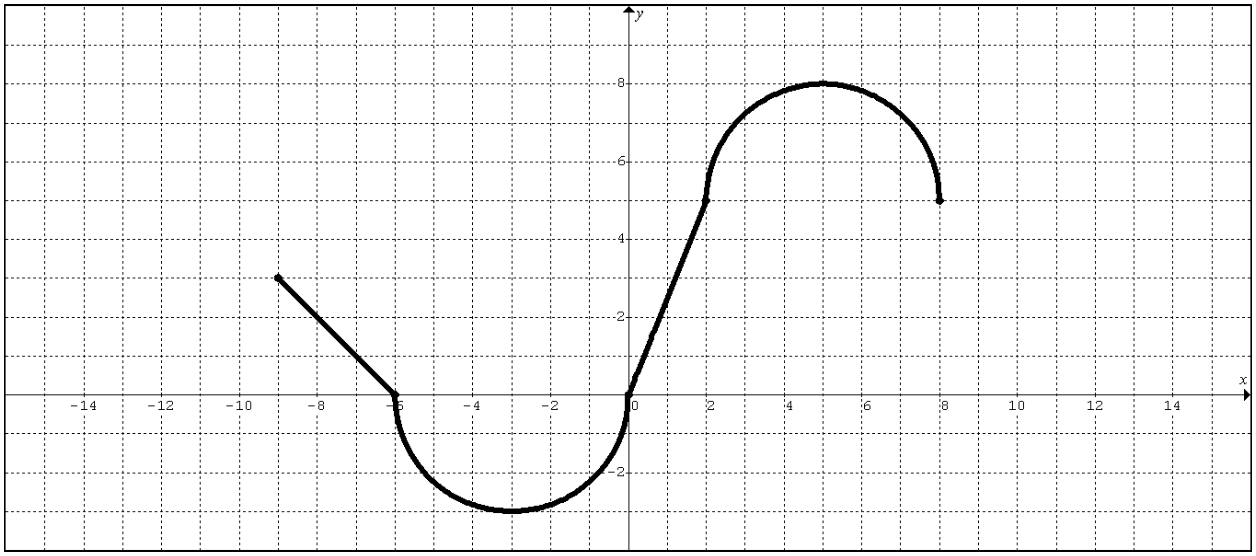
A. $\int_{-0.6}^{3} f(x)dx$

B. $\int_{-4}^{6.5} f(x)dx$

C. $\int_{3}^{-4} f(x)dx$

D. $\int_{-4}^{-0.6} f(x)dx - \int_{-0.6}^{3} f(x)dx + \int_{3}^{6.5} f(x)dx$

2. The graph of $f(x)$ is made up of line segments and semi-circles as shown in the graph below. Evaluate A – E.



A. Find the total area bounded by the graph of $f(x)$ and the x-axis.

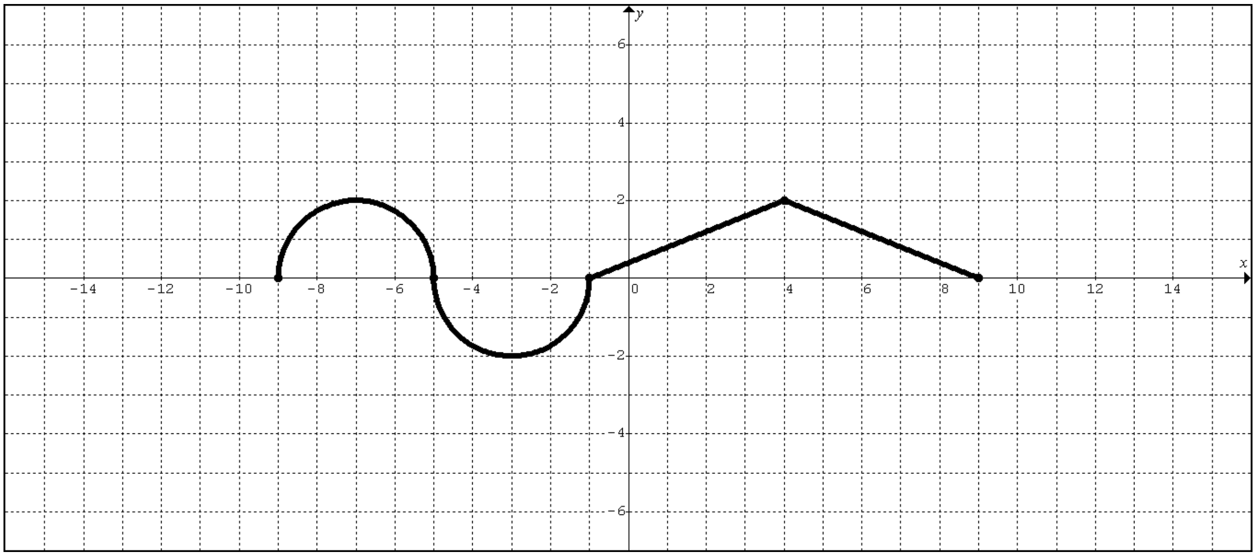
B. $\int_{-9}^{8} f(x)dx$

C. $\int_{-3}^{2} f(x)dx$

D. $\int_{0}^{-9} f(x)dx$

E. $7+\int_{-8}^{-6} f(x)dx$

3. The graph of $f(x)$ is made up of line segments and semi-circles as shown in the graph below. Evaluate A – E.



A. Find the total area bounded by the graph of $f(x)$ and the x-axis.

B. $\int_{-9}^{9} f(x)dx$

C. $\int_{-9}^{-1} f(x)dx$

D. $\int_{-3}^{4} f(x)dx$

E. $10-\int_{2}^{2} f(x)dx$

4. Evaluate the following by appealing to geometry.

A. $\int_{-2}^{1}(x-2)dx$

B. $\int_{-2}^{6}-4dx$

5. Suppose that *f* and *g* are continuous functions and that …

$\int_{1}^{2} f(x)dx = -4$, $\int_{1}^{5} f(x)dx = 6$, and $\int_{1}^{5} g(x)dx = 8$

Evaluate each of the following integrals:

A. $\int_{2}^{2} g(x)dx$

B. $\int_{5}^{1} g(x)dx$

C. $\int_{1}^{2} 3f(x)dx$

D. $\int_{2}^{5} f(x)dx$

E. $\int_{1}^{5}[f(x)-g(x)]dx$

F. $\int_{1}^{5}[4f(x)-g(x)]dx$

CALCULUS AP AB – Q303CH5A: (Lesson 1-B) AREA and INTEGRAL

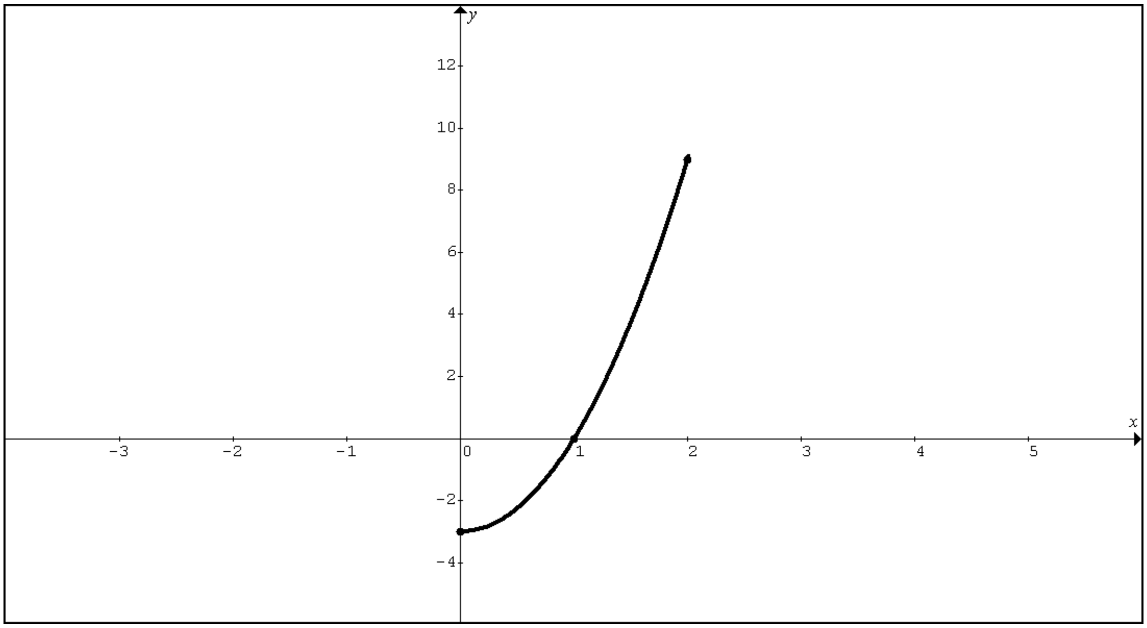
FTC2: Fundamental Theorem of Calculus (Part II) – Evaluation Method

I. [**FTC2-INTEGRAL**] Evaluate the following integrals (a) BY HAND and (b) BY TI89

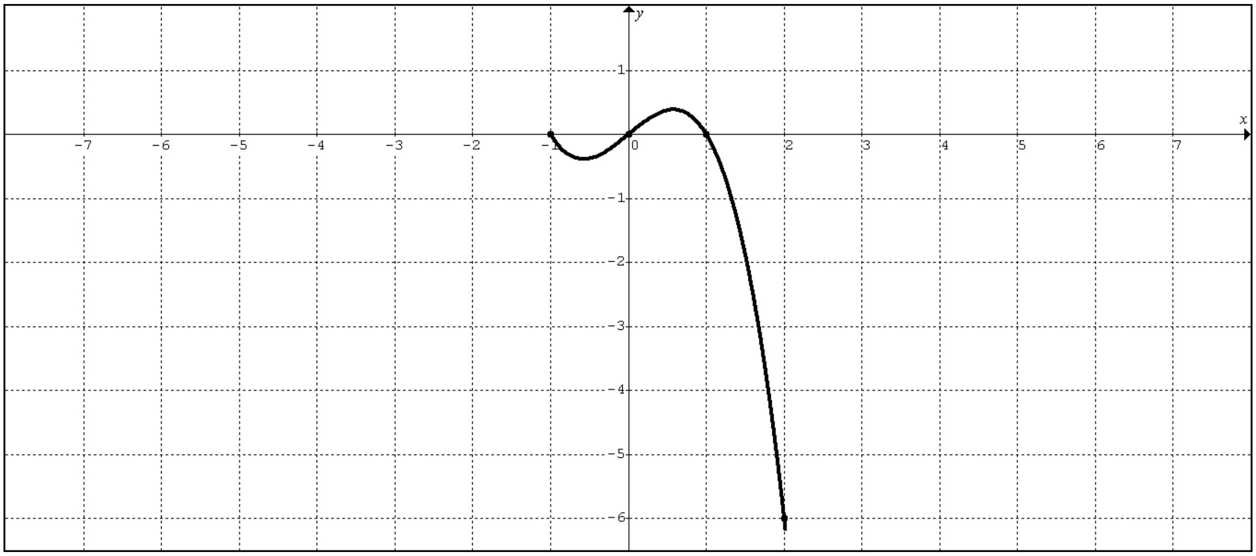
II. [**FTC2- AREA**]

1. Find the area bounded by $y = 3x^2 - 3$ and the x-axis on the interval [0, 2]

(a) BY HAND (b) BY TI89



2. Find the area bounded by $y = x - x^3$ and the x-axis on the interval [-1, 2].
(a) BY HAND (b) BY TI89



III. [**FTC2-AVERAGE VALUE**]:

1. (No Calculator) Find the average value of $f(x) = 3x^2 - 3$ on [0, 2]

2. (No Calculator) Find the average value of $y = x - x^3$ on [-1, 2]

IV. [**FTC2-UB**]: (No Calculator)
Rewrite and evaluate each integral using the appropriate u-substitution and u-bounds.

1. $\int_{-1}^{2} 45x^2\left(5x^3+1\right)^2 dx$

2. $\int_{0}^{\pi} \sin(5x)dx$

V. [**FTC2–ID**]: (Technology Required) Fundamental Theorem of Calculus "in disguise"

1. Suppose $f'(x) = e^{-x^2}$ and $f(1) = 3$. Find $f(2)$.

2. Suppose $f'(x) = (e^x)^{\cos(x)}$ and $f(2) = 1$. Find $f(0)$.

BC Q301 CH5A LESSON 1-B HW:

FTC2 INTEGRAL– BY HAND:
Section 5.3: 21, 23, 25, 27
Section 5.4: 27, 29, 31, 33, 35, 37

FTC2 INTEGRAL– BY TI89:
Section 5.4: 49, 50

FTC2 AREA– BY TI89:
Section 5.4: 41, 43, 45, 47

(APPEAL TO GEOMETRY) AVERAGE VALUE – BY HAND:
Section 5.3: 15, 16

FTC2 AVERAGE VALUE – BY HAND:
Section 5.3: 32, 34, 35

FTC2 AVERAGE VALUE – BY TI89:
Section 5.3: 11, 13

FTC2-UB

1. Rewrite and evaluate $\int_{\frac{3}{4}\pi}^{\frac{\pi}{2}} \cos(2x)\, dx$ using an appropriate u-substitution and u-bounds.

2. Rewrite and evaluate $\int_{1}^{2} (2x-1)^7\, dx$ using an appropriate u-substitution and u-bounds.

FTC2-ID

1. Suppose $f'(x) = (2+\sin x)^x$ and $f(-1) = 5$. Find $f(4)$

2. Suppose $f'(x) = (2+\ln x)^x$ and $f(2) = 5$. Find $f(0.5)$

AB.Q303 – FTC2: Fundamental Theorem of Calculus (Part II)
Lesson 2A: APPLICATIONS

A. Area Connection (Lesson 1-A)

B. Evaluation Method [FTC2] (Lesson 1-B)

C. FTC2 APPLICATIONS

C1: **Displacement/Position/Total Distance (Lesson 2A)**

C1: Displacement/Position/Total Distance (Lesson 2A)

A particle moves along the x-axis such that its position at time t is given as $x(t)$.
The velocity of the particle at time t is given as $x'(t) = v(t)$.

Review:

- The particle is moving to the right when $v(t) > 0$
- Acceleration is positive when $v'(t) > 0$
- The particle is getting faster (speed increasing) when $v(t)$ and $a(t)$ share the same sign.

New:

LESSON 2A NOTES

1. An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

A. What is the total distance traveled by the object over the time period $0 \leq t \leq 4$?
B. What is the position of the object at time $t = 4$?
C. What is the average velocity over the time period $0 \leq t \leq 4$?
D. What is the average acceleration over the time period $0 \leq t \leq 4$?

LESSON 2A NOTES

2. A particle moves along the *x*-axis so that its velocity at time *t* is given as

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time $t = 0$, the particle is at position $x = 1$.

A. What is the total distance traveled by the particle from time t = 0 until time t = 3?

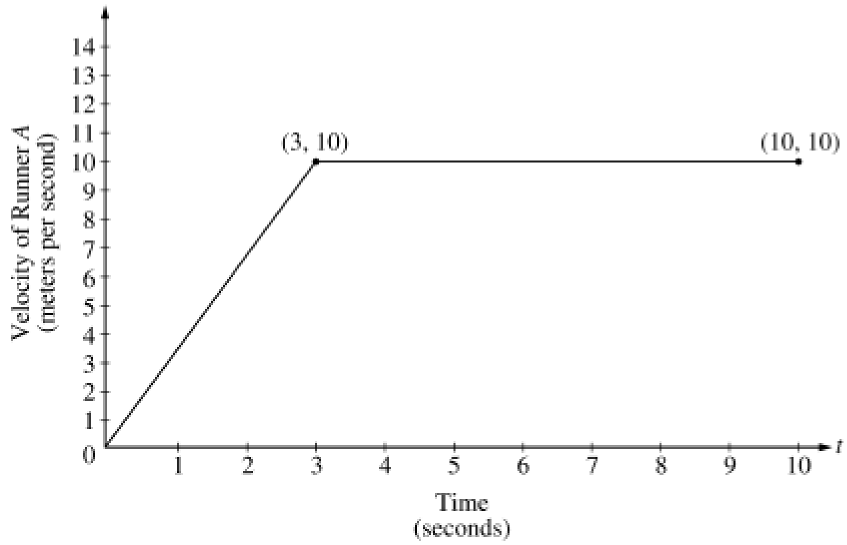
B. What is the position of the particle at time $t = 3$?

C. What is the average velocity from time $t = 0$ until time $t = 3$?

D. What is the average acceleration from time $t = 0$ until time $t = 3$?

LESSON 2A NOTES

3. AP:2000#2



2. Two runners, A and B, run on a straight racetrack for $0 \le t \le 10$ seconds. The graph above, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.

   (a) Find the velocity of Runner A and the velocity of Runner B at time $t = 2$ seconds. Indicate units of measure.

   (b) Find the acceleration of Runner A and the acceleration of Runner B at time $t = 2$ seconds. Indicate units of measure.

   (c) Find the total distance run by Runner A and the total distance run by Runner B over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

LESSON 2A HW

1. A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t = 0$, the particle is at $y = -1$.

A. Find the total distance traveled by the particle between time $t = 0$ and time $t = 2$.

B. What is the position of the particle at time $t = 2$?

C. What is the average velocity over the time period $0 \leq t \leq 2$?

D. What is the average acceleration over the time period $0 \leq t \leq 2$?

LESSON 2A HW

2. AP:2011#1

1. For $0 \le t \le 6$, a particle is moving along the x-axis. The particle's position, $x(t)$, is not explicitly given. The velocity of the particle is given by $v(t) = 2\sin\left(e^{t/4}\right) + 1$. The acceleration of the particle is given by $a(t) = \frac{1}{2}e^{t/4}\cos\left(e^{t/4}\right)$ and $x(0) = 2$.

   (a) Is the speed of the particle increasing or decreasing at time $t = 5.5$? Give a reason for your answer.

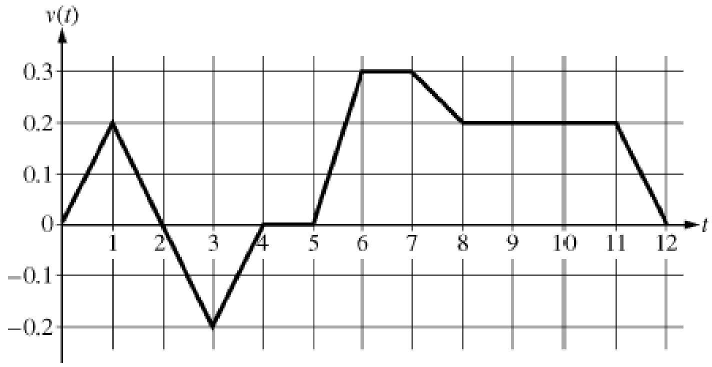
   (b) Find the average velocity of the particle for the time period $0 \le t \le 6$.

   (c) Find the total distance traveled by the particle from time $t = 0$ to $t = 6$.

   (d) For $0 \le t \le 6$, the particle changes direction exactly once. Find the position of the particle at that time.

LESSON 2A HW

3. AP:2009#1



1. Caren rides her bicycle along a straight road from home to school, starting at home at time $t = 0$ minutes and arriving at school at time $t = 12$ minutes. During the time interval $0 \le t \le 12$ minutes, her velocity $v(t)$, in miles per minute, is modeled by the piecewise-linear function whose graph is shown above.

   (a) Find the acceleration of Caren's bicycle at time $t = 7.5$ minutes. Indicate units of measure.

   (b) Using correct units, explain the meaning of $\int_0^{12} |v(t)|\,dt$ in terms of Caren's trip. Find the value of $\int_0^{12} |v(t)|\,dt$.

   (c) Shortly after leaving home, Caren realizes she left her calculus homework at home, and she returns to get it. At what time does she turn around to go back home? Give a reason for your answer.

   (d) Larry also rides his bicycle along a straight road from home to school in 12 minutes. His velocity is modeled by the function w given by $w(t) = \frac{\pi}{15}\sin\left(\frac{\pi}{12}t\right)$, where $w(t)$ is in miles per minute for $0 \le t \le 12$ minutes. Who lives closer to school: Caren or Larry? Show the work that leads to your answer.

AB.Q303 – FTC2: Fundamental Theorem of Calculus (Part II)
Lesson 2B: APPLICATIONS

A. Area Connection (Lesson 1-A)

B. Evaluation Method [FTC2] (Lesson 1-B)

C. FTC2 APPLICATIONS

C2: **Accumulating Quantity/(Rate In – Rate Out) (Lesson 2B)**

C2: Accumulating Quantity/(Rate In – Rate Out) (Lesson 2B)

LESSON 2B NOTES

1. The rate at which people enter an amusement park on a given day is modeled by the function E defined by $E(t) = \dfrac{15600}{t^2 - 24t + 160}$.

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by $L(t) = \dfrac{9890}{t^2 - 38t + 370}$.

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours in which the park is open. At time $t = 9$, there are no people in the park.

A. How many people have entered the park by 5:00 pm ($t = 17$)? Round to the nearest whole number.

B. The price of admissions to the park is \$15 until 5:00 pm. After 5:00 pm the price of admissions to the park is \$11. How many dollars are collected in admissions to the park on the given day? Round to the nearest whole number.

C. Let $H(t)$ be the number of people in the park at time t. Find $H(17)$.

D. Write a function, involving an integral expression, for $H(t)$, the number of people in the park at time t.

LESSON 2B NOTES

2. Traffic flow is defined as the rate at which cars pass through an intersection, measured in cars per minute. The traffic flow at a particular intersection is modeled by the function *F* defined by $F(t) = 82 + 4\sin\left(\frac{t}{2}\right)$ for $0 \le t \le 30$, where *F*(t) is measured in cars per minute and *t* is measured in minutes.

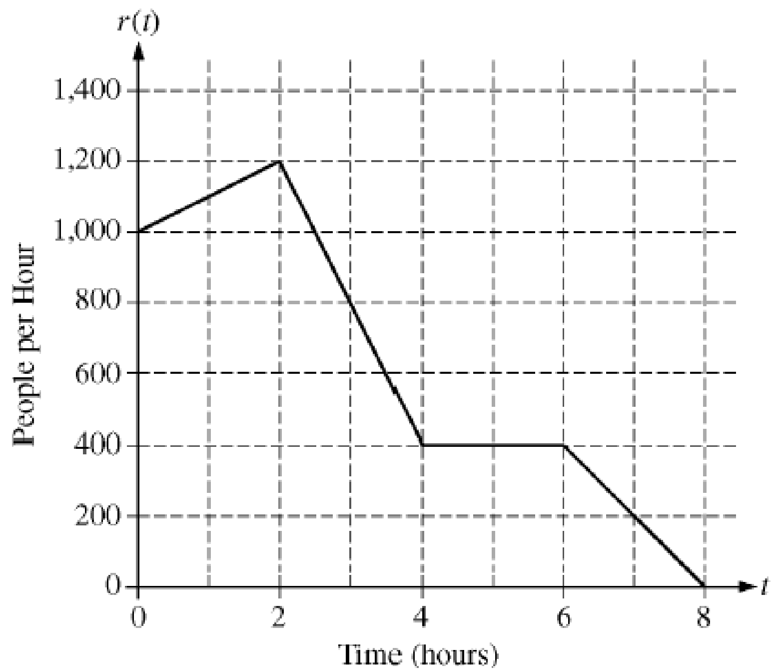
A. To the nearest whole number, how many cars pass through the intersection over the 30-minutes period?

B. What is the average traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

C. What is the average rate of change of the traffic flow over the time interval $10 \le t \le 15$? Indicate units of measure.

LESSON 2B NOTES

3. AP:2010#3



3. There are 700 people in line for a popular amusement-park ride when the ride begins operation in the morning. Once it begins operation, the ride accepts passengers until the park closes 8 hours later. While there is a line, people move onto the ride at a rate of 800 people per hour. The graph above shows the rate, $r(t)$, at which people arrive at the ride throughout the day. Time t is measured in hours from the time the ride begins operation.

   (a) How many people arrive at the ride between $t = 0$ and $t = 3$? Show the computations that lead to your answer.

   (b) Is the number of people waiting in line to get on the ride increasing or decreasing between $t = 2$ and $t = 3$? Justify your answer.

   (c) At what time t is the line for the ride the longest? How many people are in line at that time? Justify your answers.

   (d) Write, but do not solve, an equation involving an integral expression of r whose solution gives the earliest time t at which there is no longer a line for the ride.

LESSON 2B HW

1. The tide removes sand from Sandy Point Beach at a rate modeled by the function R, given by
$R(t) = 2 + 5\sin\left(\frac{4\pi t}{25}\right)$.

A pumping station adds sand to the beach at a rate modeled by the function S, given by
$S(t) = \frac{15t}{1+3t}$.

Both $R(t)$ and $S(t)$ have units of cubic yards per hour and t is measures in hours for $0 \le t \le 6$. At time $t = 0$, the beach contains 2500 cubic yards of sand.

A. How much of the sand will the tide remove from the beach during this 6-hour period? Indicate units of measure.
B. Write a function, involving an expression, for $Y(t)$, the total number of cubic yards of sand on the beach at time t.
C. Find the rate at which the total amount of sand on the beach is changing at time $t = 4$.
D. Find the total number of cubic yards of sand on the bank at time $t = 4$.

LESSON 2B HW

2. AP:2003FB#2

2. A tank contains 125 gallons of heating oil at time $t = 0$. During the time interval $0 \le t \le 12$ hours, heating oil is pumped into the tank at the rate

$$H(t) = 2 + \frac{10}{(1 + \ln(t + 1))} \text{ gallons per hour.}$$

During the same time interval, heating oil is removed from the tank at the rate

$$R(t) = 12 \sin\left(\frac{t^2}{47}\right) \text{ gallons per hour.}$$

(a) How many gallons of heating oil are pumped into the tank during the time interval $0 \le t \le 12$ hours?

(b) Is the level of heating oil in the tank rising or falling at time $t = 6$ hours? Give a reason for your answer.

(c) How many gallons of heating oil are in the tank at time $t = 12$ hours?

(d) At what time t, for $0 \le t \le 12$, is the volume of heating oil in the tank the least? Show the analysis that leads to your conclusion.

LESSON 2B HW

3. AP:2002FB#2

2. The number of gallons, $P(t)$, of a pollutant in a lake changes at the rate $P'(t) = 1 - 3e^{-0.2\sqrt{t}}$ gallons per day, where t is measured in days. There are 50 gallons of the pollutant in the lake at time $t = 0$. The lake is considered to be safe when it contains 40 gallons or less of pollutant.

(a) Is the amount of pollutant increasing at time $t = 9$? Why or why not?

(b) For what value of t will the number of gallons of pollutant be at its minimum? Justify your answer.

(c) Is the lake safe when the number of gallons of pollutant is at its minimum? Justify your answer.

(d) An investigator uses the tangent line approximation to $P(t)$ at $t = 0$ as a model for the amount of pollutant in the lake. At what time t does this model predict that the lake becomes safe?

LESSON 2B HW

4. AP:2010#1

1. There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \le t \le 9$.

(d) How many cubic feet of snow are on the driveway at 9 A.M.?