**AB. Q302. Chapter 6:  Lesson 1  (6.1, 6.2, 6.4)**
**[Separable Differential Equations]**

## SOLVE EACH DIFFERENTIAL EQUATION

1. $\dfrac{dy}{dx} = -\dfrac{x}{y}$ where $y = -3$ when $x = 4$.

2. $f'(x, y) = \dfrac{xy}{8}$ where $y(0) = 5$.

3. $f'(x, y) = e^{x-y}$ where $y(1) = \ln(e + 2)$

4. $\dfrac{dy}{dx} = 5y$ where $y = 9$ when $x = 0$.

5. $\dfrac{dy}{dx} = 4(6 + y)$ where $y = 9$ when $x = 0$.

6. $\dfrac{dy}{dx} = \dfrac{1}{3}(5 - y)$ where $y = 8$ when $x = 0$.

**ADDITIONAL PRACTICE**

1. $\dfrac{dy}{dx} = 7(3 - y)$ where $y = 1$ when $x = 0$.

2. $\dfrac{dy}{dx} = 3\sqrt{xy}$ where $y = 256$ when $x = 4$.

3. $\dfrac{dy}{dx} = \cos^2 y$ where $y = 0$ when $x = 0$.

4. $\dfrac{dy}{dx} = \dfrac{4\sqrt{y}\,\ln x}{x}$ where $y = 4$ when $x = e$.

AB: Q302 LESSON 1 HOMEWORK

Solve each differential equation (show all work on a separate sheet of paper):

## COMMON GROWTH/DECAY MODELS

1. $\dfrac{dy}{dt} = 5(10 - y)$ where $y = 7$ when $t = 0$.

2. $\dfrac{dy}{dt} = \dfrac{1}{2}(5 - y)$ where $y = 8$ when $t = 0$.

3. $\dfrac{dy}{dt} = 8(2 + y)$ where $y = 1$ when $t = 0$.

4. $\dfrac{dy}{dt} = -4(3 - y)$ where $y = 6$ when $t = 0$.

5. $\dfrac{dy}{dt} = 0.4y$ where $y = 6$ when $t = 0$.

## MATH MODELS

1. $\dfrac{dy}{dx} = \dfrac{x}{y}$ where $y = 2$ when $x = 1$.

2. $\dfrac{dy}{dx} = \dfrac{y}{x}$ where $y = 2$ when $x = 2$.

3. $\dfrac{dy}{dx} = xy$ where $y = 2$ when $x = 1$.

4. $\dfrac{dy}{dx} = \dfrac{1}{xy}$ where $y = -4$ when $x = 1$.

5. $\dfrac{dy}{dx} = (y + 5)(x + 2)$ where $y = 1$ when $x = 0$.

6. $\dfrac{dy}{dx} = (\cos x)e^{(y+\sin x)}$ where $y = 0$ when $x = 0$.

7. $\dfrac{dy}{dx} = -2xy^2$ where $y = \dfrac{1}{4}$ when $x = 1$
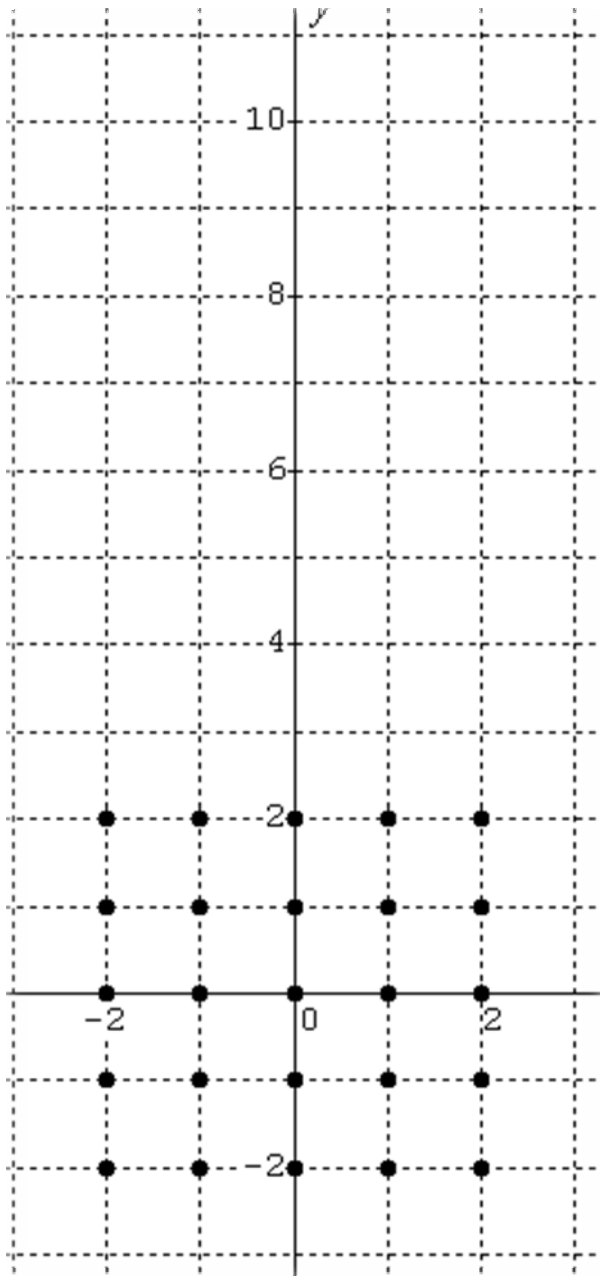
**AB. Q302 Chapter 6: Lesson 2 [Slope Fields and Linear Approximation]**

1. Consider the derivative equation: $\dfrac{dy}{dx} = 3x^2$

Let $y = f(x)$ be the solution to the differential equation that passes through the point (1,2).

A. Sketch the slope field on the given points.

B. Sketch the solution curve $y = f(x)$ that passes through the point (1,2).

C. Write an equation of the line tangent to the solution curve $y = f(x)$ at the point (1,2).

D. Use the linearization found in part C to estimate $f(2)$.

E. Find the general solution to the differential equation.

F. Find the particular solution $y = f(x)$ to the differential equation that passes through the point (1,2).

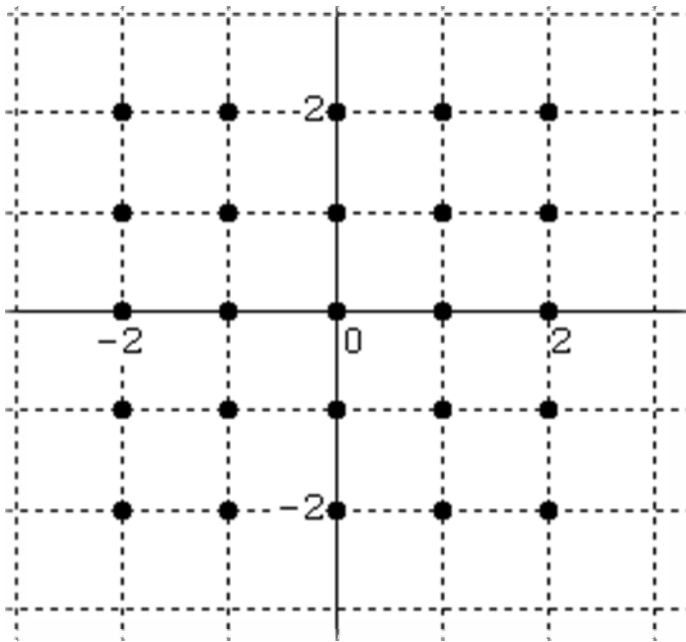G. Use the particular solution found in part F to find the exact value of $f(2)$.

H. Find $\dfrac{d^2 y}{dx^2}$ at the point (-1, 5) for the solution curve that passes through the point (-1, 5) and indicate the implications of its numeric value on this solution curve and subsequent linear approximations.

2. Consider the derivative equation: $\dfrac{dy}{dx} = -\dfrac{x}{y}$

Let $y = f(x)$ be the solution to the differential equation that passes through the point (1,-1).

A. Sketch the slope field on the given points.

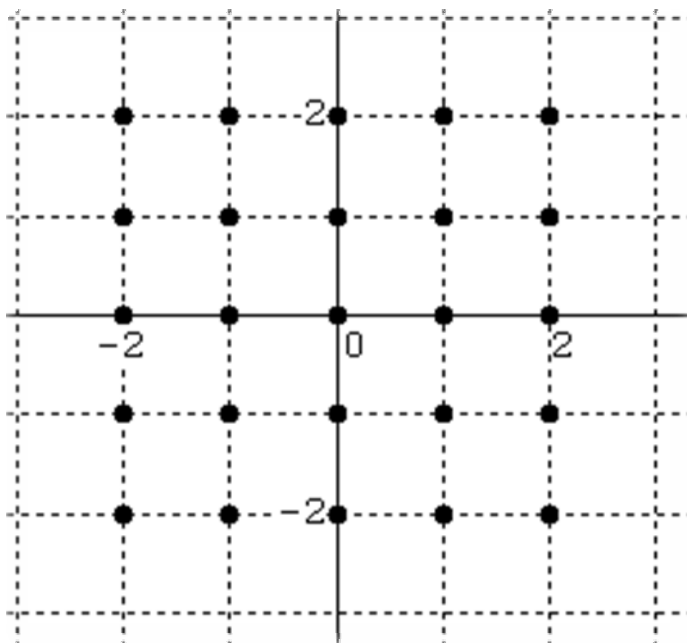B. Sketch the solution curve that passes through the point (1,-1).

C. Write an equation of the line tangent to the solution curve $y = f(x)$ at the point (1,-1).

D. Use the linearization found in part C to estimate $f(0)$.

E. Find the general solution to the differential equation.

F. Find the particular solution $y = f(x)$ to the differential equation that passes through the point (1,-1).

G. Use the particular solution found in part F to find the exact value of $f(0)$.

H. Find $\dfrac{d^2 y}{dx^2}$ at the point (-1, 5) for the solution curve that passes through the point (-1, 5) and indicate the implications of its numeric value on this solution curve and subsequent linear approximations.

3. Consider the derivative equation: $\dfrac{dy}{dx} = x + y$

A. Sketch the slope field on the given points.

B. Sketch the solution curve $y = h(x)$ that passes through the point (0,2).

C. Sketch the solution curve $y = g(x)$ that passes through the point (0,-1).

Let $y = f(x)$ be the solution to the differential equation at the point (-2, 2).

D. Write an equation of the line tangent to the solution curve $y = f(x)$ at the point (-2,2).

E. Use the linearization found in part D to estimate $f(-1)$.

F. Find $\dfrac{d^2 y}{dx^2}$ at the point (-1, 5) for the solution curve that passes through the point (-1, 5) and indicate the implications of this numeric value on this solution curve and subsequent linear approximations.

LESSON 2 CLASS PRACTICE SET:

1. Let $dy/dx = y^3 + 2x$ with solution $y = f(x)$. If $f(1) = 2$, find $f'(1)$ and $f''(1)$.

2: Let $y = f(x)$ be a function with $f(1) = 4$ such that for all points $(x, y)$ on the graph of $y = f(x)$ the slope is given by $\dfrac{3x^2 + 1}{2y}$.

    (a) Find the slope of the graph of $y = f(x)$ at the point where $x = 1$.

    (b) Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$ and use it to approximate $f(1.2)$

    (c) Find $f''(1)$ and explain the implications of its value to the linear approximation found in part (b).

    (d) Find $y = f(x)$ by solving the separable differential equation $\dfrac{dy}{dx} = \dfrac{3x^2 + 1}{2y}$ with initial condition $f(1) = 4$.

3: Consider the differential equation given by $\dfrac{dy}{dx} = \dfrac{xy}{2}$.

    (a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.
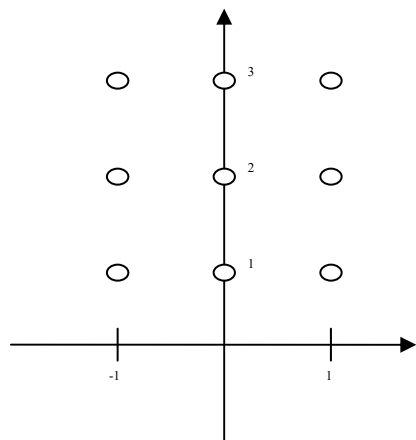


    (b) Let $y = f(x)$ be a particular solution to the given differential equation with initial condition $f(0) = 3$. Use a linear approximation centered at $x = 0$ to approximate $f(0.2)$. Show the work that leads to your answer.

    (c) Find $f''(0)$ and explain the implications of its value to the approximation found in part (b)

    (d) Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(0) = 3$.

LESSON 2 HOMEWORK

#1. No Calculator

Consider the differential equation $\dfrac{dy}{dx} = e^y(3x^2 - 6x)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through (1, 0).

A. Write an equation for the line tangent to the graph of $f$ at the point (1, 0). Use the tangent line to approximate $f(1.2)$.

B. Find $f''(1)$ and explain the implications of its value to the approximation found in part (A).

C. Find $y = f(x)$, the particular solution to the differential equation that passes through (1, 0).

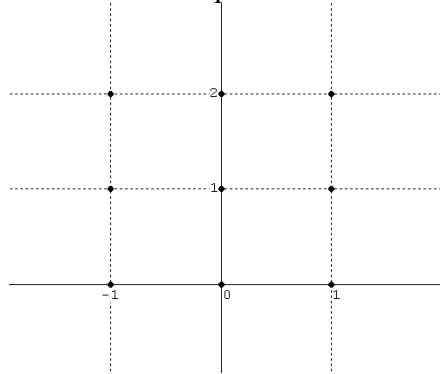#2 (No Calculator). Consider the differential equation: $\dfrac{dy}{dx} = x(2 - y)$

A. Find the particular function $y(x)$ if $y(0) = 5$.

B. Find $\lim\limits_{x \to \infty} y(x)$ for $y(x)$ found in part A.

C. Use a linearization, centered at $x = 0.5$ to approximate $y(1.5)$ if $y(0.5) = 2.5$

D. Find $\dfrac{d^2y}{dx^2}$ at the point (0.5, 2.5). Will the approximation in part "C" be an over or under estimate? Justify.

F. Draw the slope field for the 9 indicated points.



#3 (No Calculator). Find the general solution to each differential equation.

A. $\dfrac{dy}{dx} = \dfrac{x}{y^2}$ 　　　　　B. $\dfrac{dy}{dx} = \dfrac{xy^2}{1 + x^2}$

#4 (No Calculator).

Let $\dfrac{dy}{dx} = 100y - 5y^2$ with solution $y = f(x)$. If $f(0) = 3$, find $f'(0)$, and $f''(0)$.

## GROWTH/DECAY MODELS – WORD PROBLEMS

**Biological Growth**:  The number cells in a certain bacteria grow at a rate directly proportional to the existing number of cells in the population.   There are 500 cells initially and the number of cells doubles every 2.5 hours.
(A)  Find a model for the number of cells in terms of time $t$ and constant of proportionality $k$.
(B)  Find $k$ and simplify your model.  Provide a quick sketch.
(C) How many cells will there be after 10 hours?
(D) How long would it take the initial population to grow to 16,000 bacteria?

**Newton's Law of Cooling:** The rate of change in temperature of water in a cup is directly proportional to the difference between the room's temperature and the actual temperature of the water. A cup of water with temperature $100^{\circ}$C is placed in a room with constant temperature $20^{\circ}$C. The water cools to $60^{\circ}$C in one minute.

(A) Find a model for the temperature of the water in terms of time $t$ and constant of proportionality $k$.

(B) Find $k$ and simplify your model. Provide a quick sketch.

(C) What is the temperature of the water after 2 minutes?

(D) How long will it take for the temperature to reach $25^{\circ}$C?

**Bermy Fish**:  The length of a Bermy fish grows at a rate that is directly proportional to the difference between its theoretical maximum length of 12 inches and its actual length.  A Bermy fish is hatched at 0.5 inches.  A typical Bermy fish has a length of 6 inches in 20 weeks.

(A)  Find a model for the length of a Bermy fish in terms of time $t$ and constant of proportionality $k$.

(B)  Find $k$ and simplify your model.  Provide a quick sketch.

(C)  What is the length of a typical Bermy fish be after 50 weeks? (Expression only)

(D)  How long will it take a typical hatched Bermy fish to grow to be 9 inches? (Expression only)

AB.Q302.LESSON3:HOMEWORK

## GROWTH/DECAY WORD PROBLEMS

**1 WOLVES**: Let $P(t)$ represent the number of wolves in a population at time $t$ years. The rate at which the wolf population grows is directly proportional to $800 - P(t)$ where the constant of proportionality is $k$.

A. If there were 500 wolves initially, find the function $P(t)$ in terms of $t$ and $k$.
B. If there are 700 wolves in 2 years, find $k$ and write the simplified model.
Provide a quick sketch.
C. How many wolves (to the nearest whole number) does the model predict there will be at time $t = 6$ years?
D. Find $\lim_{t \to \infty} P(t)$.

**2 MEL SNAKE**: The length of a Mel snake grows at a rate that is directly proportional to the difference between its theoretical maximum length of 40 inches and its actual length. A Mel Snake is hatched at 2 inches and has a length of 21 inches in 4 months.
(A) Find a model for the length of a Mel Snake in terms of time $t$ and constant of proportionality $k$.
(B) Find $k$ and simplify your model. Provide a quick sketch.
(C) How long will it take a typical hatched Mel snake to grow to be 35.25 inches?
Note: $35.25 = \dfrac{141}{4}$

**3 NEWTON'S LAW OF COOLING**: The rate of change in temperature of an egg in running water is directly proportional to the difference between the running water's temperature and the actual temperature of the egg. An egg with temperature $98^{\circ}$C is placed under running water with constant temperature $18^{\circ}$C. The egg cools to $38^{\circ}$C in five minutes.
(A) Find a model for the temperature of the water in terms of time $t$ and constant of proportionality $k$.
(B) Find $k$ and simplify your model. Provide a quick sketch.
(C) How long will it take for the temperature to reach $20^{\circ}$C? (Expression Only)

**4 JAYS AMAZING GREEN GLOB**: Let $G(t)$ represent the volume of Jay's Amazing Green Glob which grows at the rate $\dfrac{dG}{dt} = -\ln\left(\dfrac{1}{10}\right)(200 - G)$ cm$^3$/min once it has been opened from its plastic container. Suppose Aaron opens a container of Jay's Amazing Green Glob which has an initial volume of 100 cubic centimeters.
(A) What will be the volume of the glob 2 minutes after it is opened?
(B) What is the maximum volume the glob can grow to be?

## AB: Q302 – CHAPTER 6:  PRACTICE EXAM

**#1 (No Calculator)** Let $\dfrac{dy}{dx} = 6(14 - y)$  where  $y(0) = 2$.

A. Solve the differential equation by separating the variables

B.  Find $\dfrac{d^2 y}{dx^2}$ in terms of $y$ and use it to find $\dfrac{d^2 y}{dx^2}$ at $y = 4$.

**#2 (No Calculator)** Let $\dfrac{dy}{dx} = 6(14 + y)$  where  $y(0) = 2$.
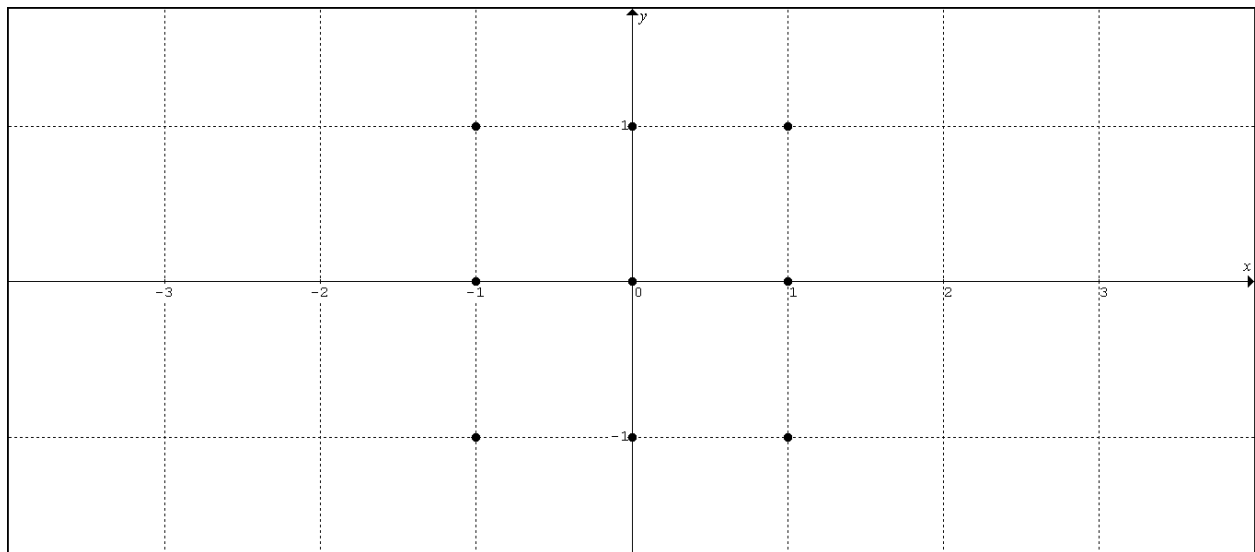
A. Solve the differential equation by separating the variables

B.  Find $\dfrac{d^2 y}{dx^2}$ in terms of $y$ and use it to find $\dfrac{d^2 y}{dx^2}$ at $y = 4$.

**#3 (No Calculator)** Let $\dfrac{dy}{dx} = 3x^2 y^2$  where  $y(0) = \dfrac{1}{3}$.

A. Solve the differential equation by separating the variables.

B.  Draw a slope for the indicated point in the graph below.



**#4 (No Calculator)**

Let $\dfrac{dy}{dx} = x^4 - 3y^2 + 6$ where  $y = f(x)$ is the particular solution to the differential equation with

the initial condition $f(0) = 1$.  Find $f'(0)$ and $f''(0)$.

#5 (No Calculator) Consider the differential equation $\dfrac{dy}{dx} = (3 - y)\cos(x)$.

Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function $f$ is defined for all real numbers.

(a)  A portion of the slope field of the differential equation is given below.  Sketch the solution curve through the point (0, 1).
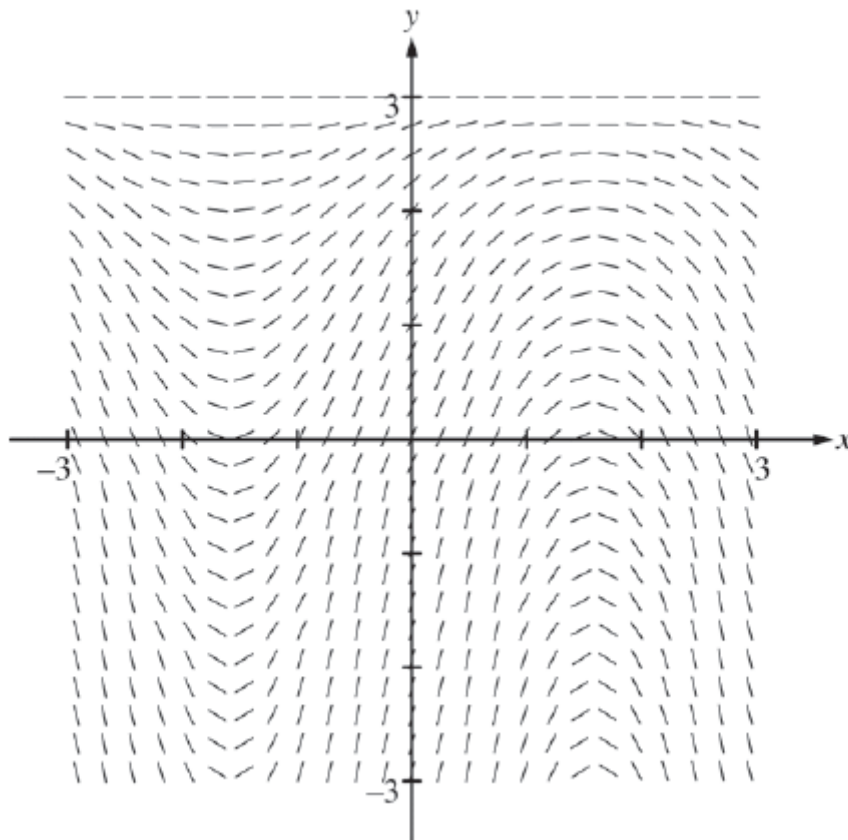


(b)  Write an equation for the line tangent to the solution curve in part (a) at the point (0, 1).  Use the equation to approximate $f(0.2)$.

(c)  Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = 1$.

#6. Consider the differential equation $\dfrac{dy}{dx} = x^2 - \dfrac{1}{2}y$

Let $y = h(x)$ be the particular solution to the differential equation with $h(0) = 2$.

A. Use a linearization centered at $x = 0$ to approximate $h(1)$.

B. Is the graph of $y = h(x)$ concave upward or concave downward at $x = 0$? Justify.

#7. Consider the differential equation $\dfrac{dy}{dx} = y^2(2x + 2)$

Let $y = f(x)$ be the particular solution to the differential equation with $f(0) = -1$.

A. Use a linearization centered at $x = 0$ to approximate $f\left(\dfrac{1}{2}\right)$.

B. Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

#8. (No Calculator) Let $R(t)$ represent the number of rats in a farm house at time $t$ days. The rate at which the rat population grows is directly proportional to the number of rats present where the constant of proportionality is $k$.

A. If there were 24 rats initially, find the function $R(t)$ in terms of $t$ and $k$.
B. If there are 72 rats in 2 days, find $k$.
C. How many rats would we expect to find after 8 days?

#9. (No Calculator) Let $T(t)$ represent the temperature (degrees Celsius) of tea in a cup at time $t$ minutes, with $t \geq 0$. The temperature $T(t)$ is changing at a rate directly proportional to the difference between the room temperature ($20°C$) and the temperature of the tea, i.e. $\dfrac{dT}{dt} = k(20 - T)$ where the constant of proportionality is $k$.

A. If the tea is poured with an initial temperature of 100 degrees Celsius, find $T(t)$ in terms of $t$ and $k$.
B. If the tea cools to 60 degrees Celsius in 3 minutes, find $k$.
C. What will be the temperature of the tea after 6 minutes?
D. At what time will the temperature reach 30 degrees Celsius?