AB CALCULUS: Q204. [Derivative Applications] **Lesson 1** – Chapter 4.4 Notes (Optimization)

NO – TECHNOLOGY

[BASIC – MAX WHAT]

1. A garden is to be laid out in a rectangular area and protected by a chicken wire fence. What is the largest possible area of the garden if only 100 running feet of chicken wire is available for the fence?

NO – TECNOLOGY

[BASIC – MAX WHERE]

2. An open box is to be made from a 16-inch by 30-inch piece of cardboard by cutting out squares of equal size from four corners and bending up the sides. What size should the squares be to obtain a box with the largest volume?

TECHNOLOGY REQUIRED

[INSCRIBED]
3. A rectangle is to be inscribed under one arch of the cosine curve. What is the largest area the rectangle can have?

4. NO TECHNOLGY

[INSCRIBED - RELATIONSHIP]
Find the radius and height of the right cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inches and height 10 inches.

HOMEWORK

NO TECHNOLOGY

1. What is the smallest perimeter possible for a rectangle whose area is 16 cm^2, and what are its dimensions?

2. A rectangular plot of farmland will be bounded on one side by a river and on the other three sides by a single-strand electric fence. With 800 m of wire at your disposal, what is the largest area you can enclose, and what are its dimensions?

TECHNOLOGY REQUIRED

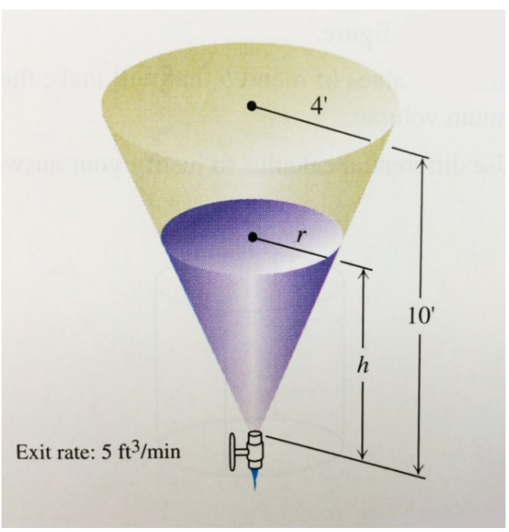
3. A rectangle is to be inscribed under the arch of the curve $y = 4\cos(0.5x)$ from $x = -\pi$ to $x = \pi$. What is the largest area of this rectangle can have?

4. A rectangle has its base on the *x*-axis and is inscribed under the curve $y = \dfrac{4.6 - x^2}{1 + x^2}$.

What is the largest area of this rectangle can have?

NO TECHNOLOGY (REVIEW)

5. Water drains from the conical tank shown in the figure at a rate of 5 ft^3/min.
How fast is the water level changing when h = 6 ft?



6. Let *f* be the function given by $f(x) = x^3 - 7x + 6$.

A. Write an equation of the tangent to the graph of *f* at $x = -1$.
B. Use the tangent line (linearization) to approximate $f(-1.02)$
C. Find to the average rate of change if *f* on the interval [1, 3].
D. Find the number *c* on the interval (1, 3), such that the instantaneous rate of change in *f* at $x = c$ is equal to the average rate of change found in part C.

7. Let *f* be a function with $f(0) = 10.2$ and $f'(x) = 2\sin\left(\frac{\pi}{2} - x\right)$.
Use a linearization of *f* at $x = 0$ and use it to approximate $f(-0.3)$

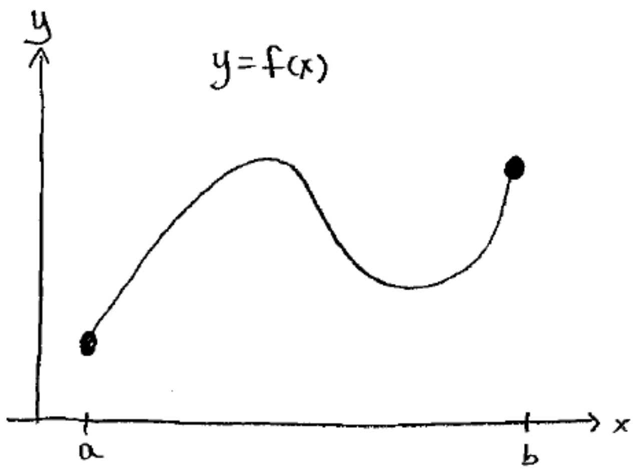
8. Consider the continuous and differentiable function $y = f(x)$ with selected values shown in the chart below.

x	-4	-3	-2	-1	0	1	2
f(*x*)	-139	72	41	2	-3	-4	17

What is the minimum number of times that $f'(x) = 0$ on the domain [-4, 2]. Explain your reasoning.

AB.Q204.LESSON2.NOTES: Theorems that Guarantee
MEAN VALUE THEOREM

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exist at least one point $x = c$ in (a, b) such that:

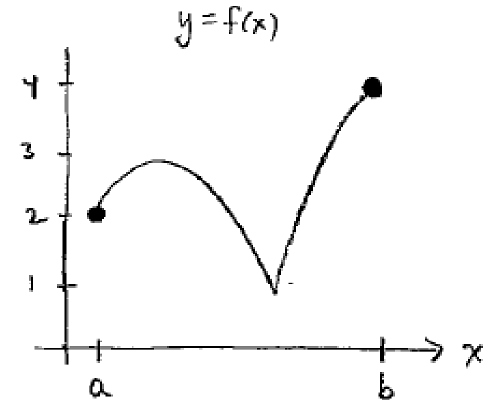


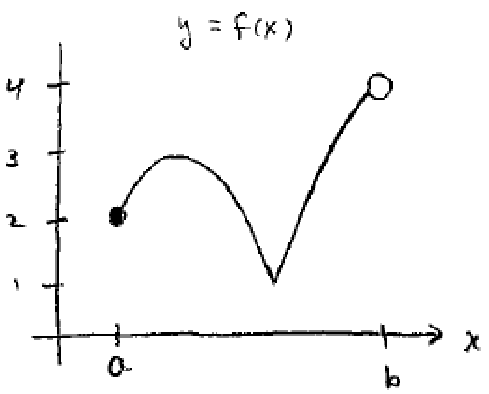
EXAMPLES:

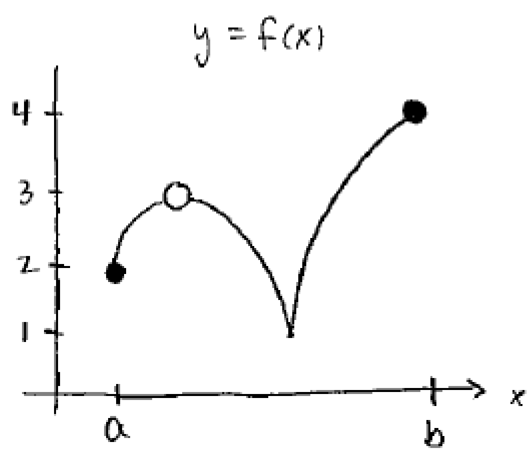
1. If it applies, find the value(s) of c that satisfy the Mean Value Theorem.

EXTREME VALUE THEOREM

If *f* is continuous on the closed interval [*a*, *b*], then *f* has both an absolute maximum and absolute minimum on the interval.



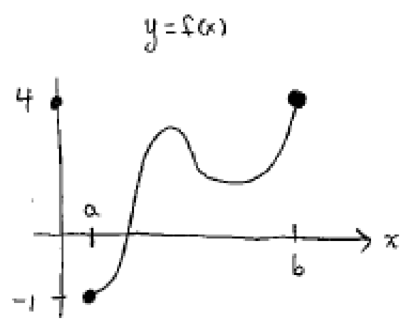


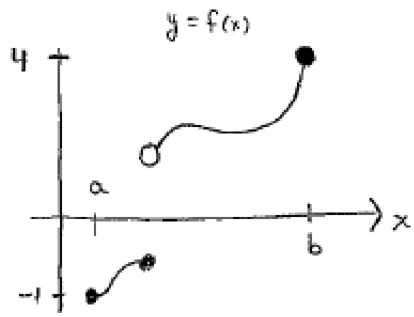


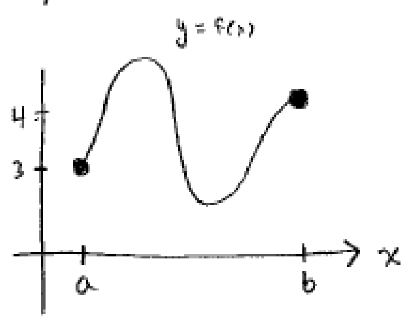
INTERMEDIATE VALUE THEOREM

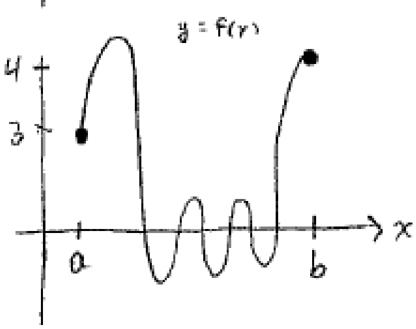
If f is continuous on the closed interval $[a, b]$, then f takes on every value between $f(a)$ and $f(b)$.

If f' is continuous on the closed interval $[a, b]$, then f' takes on every value between $f'(a)$ and $f'(b)$.









AB.Q204.DERIVATIVE APPLICATIONS – LESSON 2 (PRACTICE AND REVIEW)
(NO CALCULATOR – UNLESS NOTED OTHERWISE)

1. Determine whether *f* satisfies the hypothesis of the Mean Value Theorem on [a, b], and, if so, find all numbers *c* in (a, b) that satisfy the conclusion of the Mean Value Theorem.

A. $f(x) = x^2 + 2x - 1$ on [0,1]

B. $f(x) = \ln(x-1)$ on [2, 4]

C. TECHNOLOGY REQUIRED: $f(x) = x\cos(x^2)$ on [1.5, 3] (round answer to three places)

2. Let $f(x) = \sqrt{x^2 + 9}$. Use a linearization of *f* at $x = -4$ and use it to approximate $f(-3.9)$.

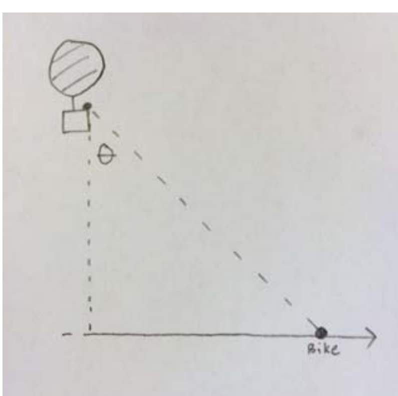
3. Let $f(x) = \ln(x+1)$. Use a linearization of *f* at $x = 0$ and use it to approximate $f(0.2)$.

4. Let *f* be a function with $f(0) = 2$ and $f'(x) = e^x \cos(x)$.
Use a linearization of *f* at $x = 0$ and use it to approximate $f(0.5)$

5. A man inside a hot-air balloon basket, directly above a level, straight path, is filming a bicyclist traveling along the path. The balloon is rising vertically at a constant rate of 1 m/sec. The cyclist is moving away from the point directly below the balloon at a constant rate of 5 m/sec. Consider the point in time when the camera is 24 m above the ground and the bicycle is 7 m from the point directly below the balloon?

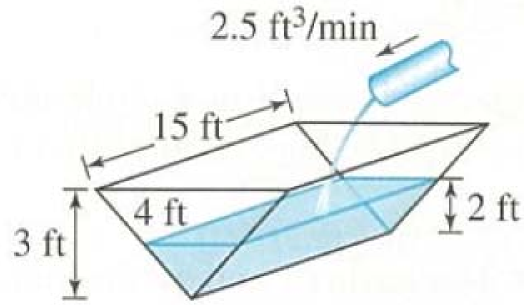
A. How fast is the distance between the bicycle and the man's camera changing?

B. How fast is the angle of the camera changing?



6.

Filling a Trough A trough is 15 ft long and 4 ft across the top as shown in the figure. Its ends are isosceles triangles with height 3 ft. Water runs into the trough at the rate of $2.5 \text{ ft}^3/\text{min}$. How fast is the water level rising when it is 2 ft deep?



7.

Particle Motion A point moves smoothly along the curve $y = x^{3/2}$ in the first quadrant in such a way that its distance from the origin increases at the constant rate of 11 units per second. Find dx/dt when $x = 3$.

8.

Largest Rectangle A rectangle has its base on the x-axis and its upper two vertices on the parabola $y = 12 - x^2$. What is the largest area the rectangle can have, and what are its dimensions?

AB.Q204.LESSON3.NOTES: LÔPITAL'S RULE (CH 8.2)

Indeterminate Forms and L'Hôpital's Rule Reference Sheet

Indeterminate Form	Limit Form $\lim_{x\to c} \dfrac{f(x)}{g(x)}$
$\dfrac{0}{0}$	$\lim_{x\to c} f(x)=0$ and $\lim_{x\to c} g(x)=0$
$\dfrac{\infty}{\infty}$	$\lim_{x\to c} f(x)=\infty$ or $-\infty$ and $\lim_{x\to c} g(x)=\infty$ or $-\infty$

L'Hôpital's Rule: Suppose that f and g are differentiable on an open interval (a, b) containing c, except possibly at c itself. If $f(x)/g(x)$ has the indeterminate form $0/0$ or ∞/∞ at $x=c$ and if $g'(x)\neq 0$ for $x\neq c$, then $\lim_{x\to c} \dfrac{f(x)}{g(x)} = \lim_{x\to c} \dfrac{f'(x)}{g'(x)}$ provided the limit exist or equals 0.

Indeterminate Form	Limit Form $\lim_{x\to c}[f(x)g(x)]$
$0\cdot\infty$	$\lim_{x\to c} f(x)=0$ and $\lim_{x\to c} g(x)=\infty$ or $-\infty$

Guidelines:

1. Write $f(x)g(x)$ as $\dfrac{f(x)}{1/g(x)}$ or $\dfrac{g(x)}{1/f(x)}$
2. Apply L'Hôpital's Rule for $\dfrac{0}{0}$ or $\dfrac{\infty}{\infty}$

Indeterminate Form	Limit Form $\lim_{x\to c} f(x)^{g(x)}$
0^0	$\lim_{x\to c} f(x)=0$ and $\lim_{x\to c} g(x)=0$
∞^0	$\lim_{x\to c} f(x)=\infty$ or $-\infty$ and $\lim_{x\to c} g(x)=0$
1^∞	$\lim_{x\to c} f(x)=1$ and $\lim_{x\to c} g(x)=\infty$ or $-\infty$

Guidelines:

1. Let $y=f(x)^{g(x)}$
2. Take the natural log of both sides
   $\ln y = \ln f(x)^{g(x)} = g(x)\ln f(x)$
3. Investigate $\lim_{x\to c} \ln y = \lim_{x\to c}[g(x)\ln f(x)]$

Conclude:

a) If $\lim_{x\to c} \ln y = L$, then $\lim_{x\to c} y = e^L$
b) If $\lim_{x\to c} \ln y = \infty$, then $\lim_{x\to c} y = \infty$
c) If $\lim_{x\to c} \ln y = -\infty$, then $\lim_{x\to c} y = 0$

AB: Q204 Lesson 3 Notes: (Please Bring Additional Paper to Take Notes**)**

Q204 - AB PRACTICE L3 - L'HOPITAL'S RULE

5. $\lim\limits_{x \to 0} \dfrac{1-\cos x}{x^2}$

49. $\lim\limits_{x \to 1} \dfrac{x^3-1}{4x^3-x-3}$

13. $\lim\limits_{x \to \pi} \dfrac{\csc x}{1+\cot x}$

17. $\lim\limits_{x \to 0^+} x \cdot \ln x$

21. $\lim\limits_{x \to 0} (e^x + x)^{1/x}$

25. $\lim\limits_{x \to 0^+} \left(1 + \frac{1}{x}\right)^x$

31. $\lim\limits_{x \to \infty} (1+x)^{1/x}$

33. $\lim\limits_{\theta \to 0} \dfrac{\sin(\theta^2)}{\theta}$

35. $\lim\limits_{x \to \infty} \dfrac{\log_2 x}{\log_3 (x+3)}$

37. $\lim\limits_{y \to \pi/2} \left(\frac{\pi}{2} - y\right) \tan y$

41. $\lim\limits_{x \to \infty} \dfrac{3x-5}{2x^2-x+2}$

43. $\lim\limits_{x \to \infty} (1+2x)^{1/(2\ln x)}$

45. $\lim\limits_{x \to 0^+} (1+x)^{1/x}$

47. $\lim\limits_{x \to 1^+} x^{1/(1-x)}$

AB:Q204. LESSON 4 – PRACTICE EXAM

OPTIMIZATION

1. CALCULATOR - REQUIRED

Inscribing a Rectangle A rectangle is inscribed under one arch of $y = 8\cos(0.3x)$ with its base on the x-axis and its upper two vertices on the curve symmetric about the y-axis. What is the largest area the rectangle can have?

2. NO – CALCULATOR

Area of Triangle An isosceles triangle has its vertex at the origin and its base parallel to the x-axis with the vertices above the axis on the curve $y = 27 - x^2$. Find the largest area the triangle can have.

RELATED RATES

3. NO – CALCULATOR

Particle Motion A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x-coordinate (in meters) increases at a constant rate of 10 m/sec. How fast is the angle of inclination θ of theline joining the particle to the origin changing when $x = 3$?

4. NO - CALCULATOR

Draining Conical Reservoir Water is flowing at the rate of 50 m^3/min from a concrete conical reservoir (vertex down) of base radius 45 m and height 6 m. **(a)** How fast is the water level falling when the water is 5 m deep? **(b)** How fast is the radius of the water's surface changing at that moment? ~~Give your answer in cm/min.~~

LINEARIZATION

5. NO – CALCULATOR

Let $f(x) = xe^{2x} + 2x + 4$.

Find a linearization of *f* at $x = 0$, and use it to approximate *f* at $x = -0.3$.

6. NO – CALCULATOR

Let *f* be a function with $f(1) = 3.156$ and $f^{/}(x) = \ln(\cos^2(x-1)) + e^{\sin(x-1)}$.

Find a linearization of *f* at $x = 1$, and use it to approximate *f* at $x = 1.216$.

MEAN VALUE THEOREM

7. NO – CALCULATOR

Let $f(x) = \begin{cases} (x+2)^2 - 2 & x < 1 \\ 6x + 1 & x \geq 1 \end{cases}$ on the interval $\left[-6, \frac{13}{6}\right]$

Assuming that *f* satisfies the hypothesis of the mean value theorem (which it does)...

Find the value(s) of *c* that satisfies the conclusion of the <u>Mean Value Theorem.</u>

(Show Work and no decimal answers)

8. CALCULATOR – REQUIRED

Let $f(x) = \tan^{-1}\left(e^{x+2}\right) + \sin\left(x^2\right)$ on the interval $[-2, 1]$

Assuming that *f* satisfies the hypothesis of the mean value theorem (which it does)...

Find the value(s) of *c* that satisfies the conclusion of the <u>Mean Value Theorem.</u>

(Round to three decimal places)

L'HÔPITAL'S RULE

Find each limit, provided it exists. Show work. NO CALCULATOR

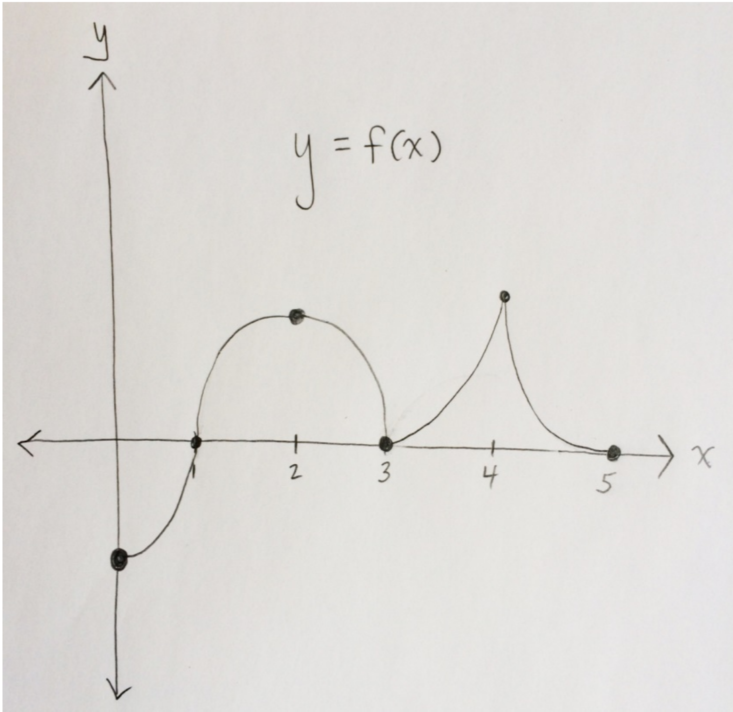
9. $\lim_{x \to 0^+} \frac{\ln\left(x^2 + 2x\right)}{\ln x}$

10. $\lim_{x \to 0^+} \frac{\tan(x)}{2x}$

11. $\lim_{x \to 0^+} (\sin x)^x$

12. $\lim_{x \to 0^+} (\sin x)^{\tan x}$

13. Below is Steven's graph of $y = f(x)$.



THE CHART REPRESENTS STEVEN'S GRAPH

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3	$3<x<4$	4	$4<x<5$	5
$f(x)$									**+**		
$f'(x)$			**DNE**								
$f''(x)$	■										■

FILL IN EACH BLANK IN THE CHART ABOVE WITH ONE OF THE FOLLOWING:

+ for positive

– for negative

0 for zero

DNE for Does not Exist