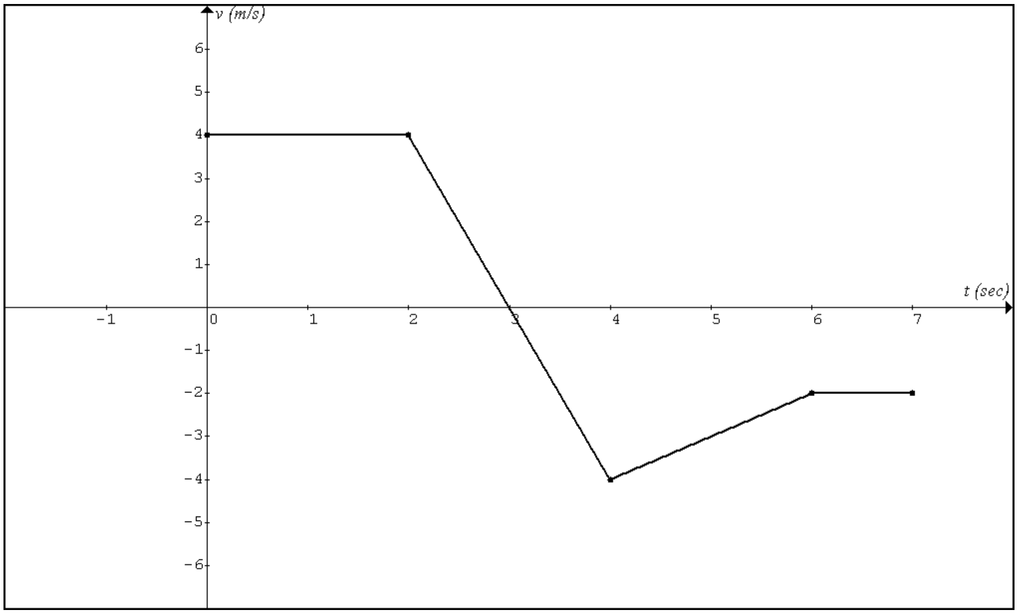
AB Q203: CH4 DERIVATIVE APPLICATIONS
Lesson 1: VELOCITY and ACCELERATION APPLICATIONS

1. The graph shows the velocity $v = v(t)$ of a particle moving along a horizontal coordinate axis. NO TECHNOLOGY:



A. At what time on the interval (0,7) is the particle standing still? Justify.

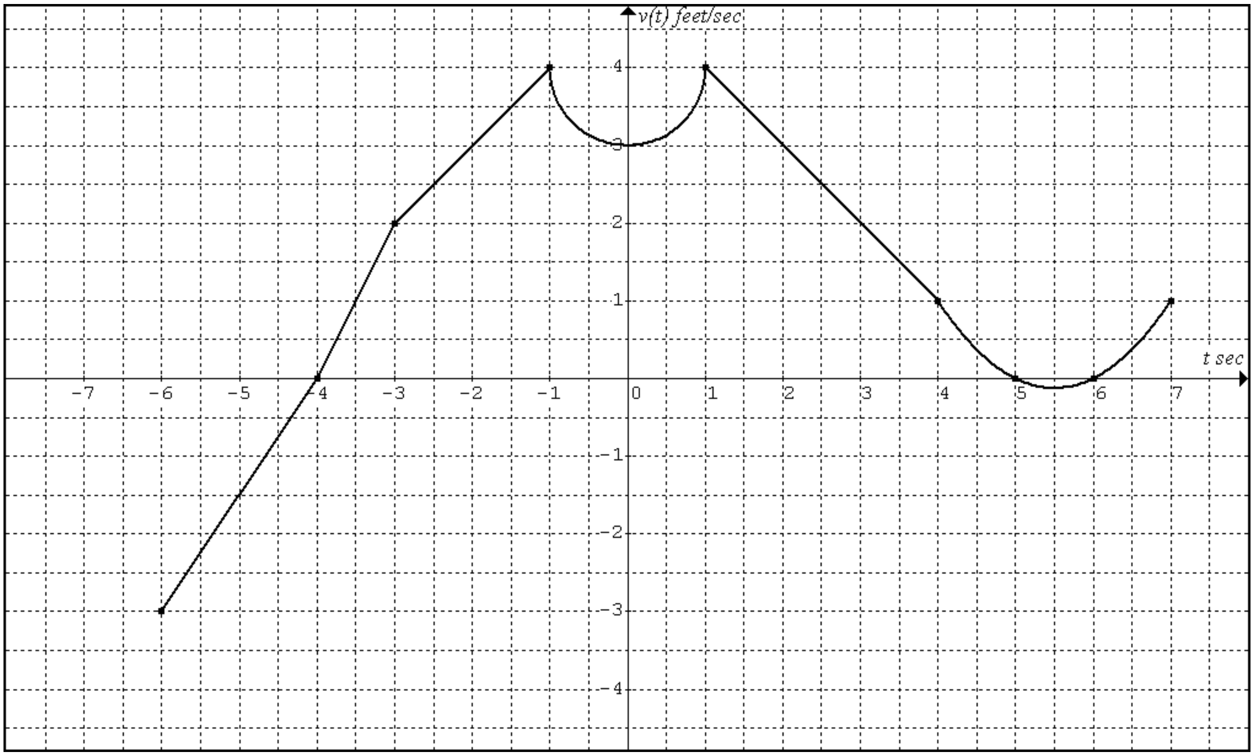
B. On what interval is the particle moving right? Justify.

C. On what interval is the particle moving left? Justify.

D. On what interval is the acceleration positive? Justify.

E. On what interval is the particle getting faster? Justify.

2. The graph shows the velocity $v = v(t)$ of a particle moving along a vertical coordinate axis.
NO TECHNOLOGY:



A. At what time on the interval (-6,7) is the particle standing still? Justify.

B. On what interval is the particle moving up? Justify.

C. On what interval is the particle moving down? Justify.

D. On what interval is the acceleration positive? Justify.

E. On what interval is the particle getting slower? Justify.

F. What are the velocity and acceleration at time $t = 2$?

G. What are the velocity and acceleration at time $t = 0$?

3. The velocity of a particle moving along a horizontal coordinate axis is defined by $v(t) = 8 + 2t - e^{t^2}$ for $-2 < t < 2$.
TECHNOLOGY REQUIRED:

A. At what time on the interval (-2,2) is the particle standing still? Justify.

B. On what interval is the particle moving right? Justify.

C. On what interval is the particle moving left? Justify.

D. On what interval is the acceleration positive? Justify.

E. On what interval is the particle getting slower? Justify.

G. What are the velocity and acceleration at time $t = 0$?

4. The velocity of a particle moving along a horizontal coordinate axis is defined by $v(t) = \cos(t^3)$ for $0 < t < 2$.
TECHNOLOGY REQUIRED:

A. At what time on the interval (0,2) is the particle standing still? Justify.

B. On what interval is the particle moving right? Justify.

C. On what interval is the particle moving left? Justify.

D. On what interval is the acceleration positive? Justify.

E. On what interval is the particle getting faster? Justify.

G. What are the velocity and acceleration at time $t = 1$?

5. The position of a particle moving along a horizontal coordinate axis is defined by $s(t) = \frac{t^3}{3} - \frac{t^2}{2} - 2t + 3$ for $-5 < t < 5$. NO TECHNOLOGY:

A. At what time on the interval (-5,5) is the particle standing still? Justify.

B. On what interval is the particle moving right? Justify.

C. On what interval is the particle moving left? Justify.

D. On what interval is the acceleration positive? Justify.

E. On what interval is the particle getting faster? Justify.

G. What are the position, velocity, and acceleration at time $t = 1$?

6. The position of a particle moving along a horizontal coordinate axis is defined by $s(t) = -\frac{2t^3}{3} + \frac{5t^2}{2} - 3t - 1$ for $0 < t < 2$. NO TECHNOLOGY:

A. At what time on the interval (0,2) is the particle standing still? Justify.

B. On what interval is the particle moving right? Justify.

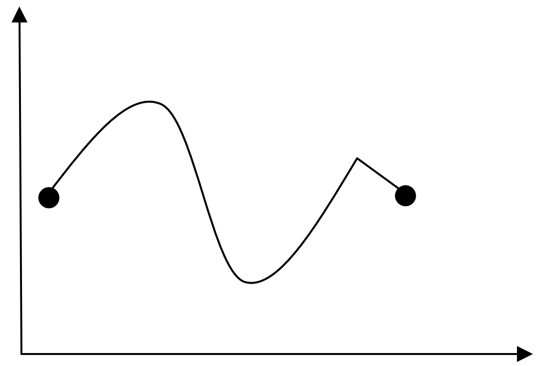
C. On what interval is the particle moving left? Justify.

D. On what interval is the acceleration positive? Justify.

E. On what interval is the particle getting slower? Justify.

G. What are the position, velocity, and acceleration at time $t = 1$?

AB CALCULUS Q203: Lesson 2
CH 4. The First Derivative Test



Assume f is continuous on [a, b]

Relative Extremes on (*a*, *b*) (First Derivative Test)

There is a relative max <u>at</u> because

There is a relative min <u>at</u> because

Relative Extremes on [*a*, *b*] (Endpoints are always relative Extremes)

There are relative extremes at $x = a$ and $x = b$ because there are endpoints at $x = a$ and $x = b$.

Absolute Extremes

is the absolute max because

is the absolute min because

Absolute Extremes (Closed Interval Test)

Find all relative extremes on a closed interval.

Absolute Max: The absolute max is the largest of all the relative extremes.

Absolute Min: The absolute min is the smallest of all the relative extremes.

Intervals of Increasing or Decreasing

f is increasing on because

f is decreasing on because

Section 4.1 Definitions and Theorems

1. Definition: Let a function *f* be defined on an interval *I*, and let x_1 and x_2 denote numbers in *I*.
   (i) *f* is **increasing** on *I* if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$.
   (ii) *f* is **decreasing** on *I* if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$.
   (iii) *f* is **constant** on *I* if $f(x_1) = f(x_2)$ whenever $x_1 \neq x_2$.

2. Definition: Let *c* be a real number in the domain of a function *f*.
   (i) $f(c)$ is a **local (relative) maximum** of *f* if there exist an open interval (a, b) containing *c* such that $f(x) \leq f(c)$ for every *x* in (a, b) in the domain of *f*.
   (ii) $f(c)$ is a **local (relative) minimum** of *f* if there exist an open interval (a, b) containing *c* such that $f(x) \geq f(c)$ for every *x* in (a, b) in the domain of *f*.
   (iii) $f(c)$ is an **absolute (global) maximum** of *f* if $f(x) \leq f(c)$ for every *x* in the domain of *f*.
   (iv) $f(c)$ is an **absolute (global) minimum** of *f* if $f(x) \geq f(c)$ for every *x* in the domain of *f*.

3. Definition: Let *f* be defined on the closed interval $[a, b]$.
   (i) $f(a)$ is a local endpoint maximum if the exist an $\varepsilon > 0$ and interval $[a, a+\varepsilon)$ such that $f(a) \geq f(x)$ for all *x* on $[a, a+\varepsilon)$.
   (ii) $f(a)$ is a local endpoint minimum if the exist an $\varepsilon > 0$ and interval $[a, a+\varepsilon)$ such that $f(a) \leq f(x)$ for all *x* on $[a, a+\varepsilon)$.
   (iii) $f(b)$ is a local endpoint maximum if the exist an $\varepsilon > 0$ and interval $(b-\varepsilon, b]$ such that $f(b) \geq f(x)$ for all *x* on $(b-\varepsilon, b]$.
   (iv) $f(b)$ is a local endpoint minimum if the exist an $\varepsilon > 0$ and interval $(b-\varepsilon, b]$ such that $f(b) \leq f(x)$ for all *x* on $(b-\varepsilon, b]$.

4. Definition: a number *c* in the domain of a function *f* is a critical number of *f* if either $f'(c) = 0$ or $f'(c)$ does not exist.

THEOREM: If a function *f* has a local extremum at a number c is an open interval, then either $f'(c) = 0$ or $f'(c)$ does not exist.

THEOREM: If a function *f* is continuous on a closed interval $[a, b]$ and has a its maximum or minimum value at a number c in the open interval (a, b), then either $f'(c) = 0$ or $f'(c)$ does not exist.

NON – TECHNOLOGY SECTION

1. Let $f(x) = 5 - 7x - 4x^2$.

A. Find the *x*-values where $f(x)$ has relative extremes. Justify using the first derivative test.

B. Find the interval on which $f(x)$ is decreasing. Justify.

2. Let $f(x) = 10x^3(x-1)^2$

A. Find the *x*-values where $f(x)$ has relative extremes. Justify using the first derivative test.

B. Find the interval on which $f(x)$ is increasing. Justify.

3. Let $f(x) = \frac{1}{2}x - \sin x$ on the closed interval $[0, 2\pi]$

A. Find the absolute extremes using the **Closed Interval Test**.

TECHNOLOGY SECTION

1. Let $f'(x) = 8 + 2x - e^{x^2}$ on (-2, 2)

A. Find the *x*-value(s) of any local maximums. Justify using the first derivative test.

B. On what interval is *f* decreasing? Justify.

2. Let $f'(x) = \cos(x^3)$ on (0, 2)

A. Find the *x*-value(s) of any local minimums. Justify using the first derivative test.

B. On what interval is *f* decreasing? Justify.

C. Suppose the function $f'(x) = \cos(x^3)$ was extended to have the domain [0,2].

f has a local _____ at $x = 0$.	A. max, max	B. max, min
f has a local _____ at $x = 2$	C. min, max	D. min, min

Q203. Lesson2.Homework

NO TECHNOLOGY PERMITTED

1. $f(x) = -2x^3 + 6x^2 - 3$

a. Find the interval on which *f* is decreasing. Justify
b. Use the first derivative test to find the *x*-values of any local extremes.
c. What are the local extremes?

2. $f(x) = x + 2\cos(x)$ on $[0, 2\pi]$

a. Find the interval on which *f* is increasing. Justify
b. Use the first derivative test to find the *x*-values of any local extremes on $(0, 2\pi)$.
c. Will $f(0)$ be a local max or min? Will $f(2\pi)$ be a local min or max?
d. Find the absolute extremes using the closed interval test.

3. $f(x) = x\sqrt{x^2 - 9}$ SKIP THIS PROBLEM

a. Find the interval on which *f* is decreasing. Justify
b. Find the *x*-values of any local extremes.

TECHNOLOGY REQUIRED

4. $f'(x) = x - \cos(\pi x) - \sin x$ on $(-1,1)$

a. Graph $f'(x)$
b. Find the zeros of $f'(x)$.
c. Find the interval on which *f* is increasing. Justify.
d. Indicate the *x*-value(s) where *f* has a local max. Justify.

5. $f'(x) = 5e^{-x^2/2} - 1$ on $(-3,3)$

a. Graph $f'(x)$
b. Find the zeros of $f'(x)$.
c. Find the interval on which *f* is increasing. Justify.
d. Indicate the *x*-value(s) where *f* has a local max. Justify.

6. $f'(x) = \dfrac{x}{20}\cos(x^2)$ on $[-2,2]$

a. Graph $f'(x)$
b. Find the zeros of $f'(x)$.
c. Find the interval on which *f* is increasing. Justify.
d. Indicate the *x*-value(s) where *f* has a local min. Justify.

AP QUESTION: 1990-AB5 (NO CALCULATOR) (CLOSED INTERVAL TEST)

NO CALCULATOR

1990 – AB5

5. Let f be the function defined by $f(x) = \sin^2 x - \sin x$ for $0 \le x \le \frac{3\pi}{2}$.

(a) Find the x-intercepts of the graph of f.

(b) Find the intervals on which f is increasing.

(c) Find the absolute maximum value and the absolute minimum value of f. Justify your answer.

Use the Closed Interval Test to do part (c).

Q203: CH4 DERIVATIVE APPLICATIONS
Lesson 3: The Second Derivative - Applications

The First Derivative Test is for finding ____.

The Second Derivative Test is for finding ____.

The Concavity test is for finding ____.

The Point of Inflection test is for finding ____.

CONCAVITY:

CONCAVITY TEST:

POINTS OF INFLECTION:

POINTS OF INFLECTION TEST:

SECOND DERIVATIVE TEST:

NO TECHNOLOGY

Use the 2^{nd} derivative test to find the *x*-value where the function has a relative maximum and or minimum.

Example 1: $f(x) = x^3 - x^2 - 5x - 5$

Example 2: $f(x) = 12 + 12x^2 - x^4$

Example 3: $y = 3 + \sin x \quad x \in (0, 2\pi)$

Example 1 Continued: $f(x) = x^3 - x^2 - 5x - 5$

A. Use the concavity test to determine on what interval the function is concave up.

B. Use the point of inflection test to identify the *x*-values of any point of inflection.

Example 2 Continued: $f(x) = 12 + 12x^2 - x^4$

A. Use the concavity test to determine on what interval the function is concave down.

B. Use the point of inflection test to identify the *x*-values of any point of inflection.

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1992 AP 1

Let f be the function defined by $f(x) = 3x^5 - 5x^3 + 2$.

(a) On what intervals is f increasing? Justify.

(b) On what intervals is the graph of f concave upward? Justify.

(c) Write the equation of each horizontal tangent line to the graph of f.

TECHNOLOGY REQUIRED

Consider the derivative $f^{/}(x)$ of the function $y = f(x)$ on the interval (1,4)

$f^{/}(x) = x^3 \cos(2x+1)$ on (1,4)

A. On what interval if *f* decreasing? Justify your answer.

B. For what values of *x* does *f* has a local maximum? Justify your answer.

C. On what interval is *f* concave down? Justify your answer.

CHAPTER 4.3 DEFINITIONS AND TESTS

Definition: Let *f* be differentiable on an open interval I. The graph of *f* is

(i) **concave upward** on I if $f^{/}$ is increasing on I.

(ii) **concave downward** on I if $f^{/}$ is decreasing on I.

Definition: A point (c, $f(c)$) on the graph of *f* is a **point of inflection** if the following two conditions are satisfied:

(i) *f* is continuous at c.

(ii) There is an open interval (a,b) containing c such that the graph is concave upward on (a, c) and concave downward on (c, b), or vise versa.

Concavity Test:

If $f^{//}(x) > 0$ on I, then *f* is **concave upward** on I.

If $f^{//}(x) < 0$ on I, then *f* is **concave downward** on I.

Point of Inflection Test:

If *f* is continuous at $x = c$ and $f^{//}(x)$ changes sign at $x = c$, then *f* has a point of inflection at $x = c$.

Second Derivative Test:

If $f^{/}(c) = 0$ and $f^{//}(c) < 0$, then *f* has a local maximum at c.

If $f^{/}(c) = 0$ and $f^{//}(c) > 0$, then *f* has a local minimum at c.

AB.Q203.L3: HOMEWORK

TECHNOLOGY SECTION

1. A function f is defined on (0.250, 1.250) and the *derivative* of the function is given by $f'(x) = x\sin\left(e^{2x} - x\right)$

Round Answer to three decimal places

A. Find the interval(s) on which f is increasing. Justify your answer.

B. For what value(s) of x does f have a local minimum? Justify your answer.

C. On what interval is f concave upward? Justify your answer.

D. How many points of inflection are there on the graph of $f(x)$? Justify your answer.

E. Suppose we extended f to include the endpoints at $a = 0.250$ and $b = 1.250$. Use an endpoint analysis to determine whether each endpoint is a local maximum or local minimum.

There is a local _______ at $x = 0.250$

There is a local _______ at $x = 1.250$

F. If $f(0.522) = 4$, write and equation of the line tangent to $f(x)$ at $x = 0.522$.

G. Use the tangent line in part (F) to estimate $f(0.3)$

NON TECHNOLOGY SECTION

2. Let $y = 4x^3 + 21x^2 + 36x - 20$

A. On what interval is the graph of *y* concave downward? Justify your answer.

B. The graph of *y* has how many points of inflection? Justify your answer.

3. Let $f(x) = 3x - x^3 + 5$

Use the second derivative test to determine <u>the</u> local extremes of $f(x)$.

4. Let $f(x) = xe^x$

Use the second derivative test to determine the local extremes of $f(x)$.

5. Let $f'(x) = (x-1)^2(x-2)$ be the derivative of the function of $y = f(x)$.

A. Find the *x*-value(s) where the function *f* has a local minimum. Justify your answer.

B. Find the *x*-value(s) where the function *f* has a point of inflection. Justify your answer.

GRAPH ANALYSIS SECTION

6. Consider the function $y = f(x)$ shown below.

A. Estimate the x value(s) where $f'(x) = 0$.

B. Estimate the x value(s) where $f'(x) > 0$.

C. Estimate the x value(s) where $f'(x) < 0$.

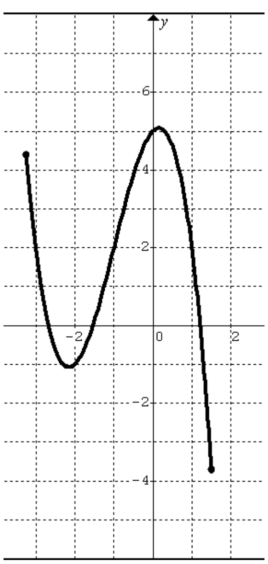
D. Estimate the x value(s) where $f''(x) = 0$.

E. Estimate the x value(s) where $f''(x) > 0$.

F. Estimate the x value(s) where $f''(x) < 0$.

G. Estimate the x value(s) where $f'(x)$ is decreasing.

7. Consider the **derivative** graph function $y = f'(x)$ shown below.



A. Estimate the x-value(s) where f is increasing.

B. Estimate the x-value(s) where f is decreasing.

C. Estimate the x-value(s) where f is concave up.

D. Estimate the x-value(s) where f is concave down.

AB: Q203 – LESSON 4: PRACTICE EXAMINATION

TECHNOLOGY SECTION: *Round answers to three decimal places.*

1. The velocity of a particle moving along a horizontal is given as $v(t) = 8\cos(t) + \ln\left(\sin(t) + t^2\right)$ on $0.1 < t \le 8$

A. On what time interval is the particle moving to the right? Justify.

B. What are the velocity and acceleration at time $t = 5$?

C. Is the particle speeding up or slowing down at $t = 3.5$? Justify.

2. The derivative of *f* is given by $f'(x) = e^{x^2} - 5x^3 + x$ on $0 \le t < 3$

A. On what interval is *f* decreasing? Justify.

B. At what *x*-value(s) does *f* have a relative maximum? Justify.

C. On what interval is *f* concave upward? Justify.

NO TECHNOLOGY SECTION

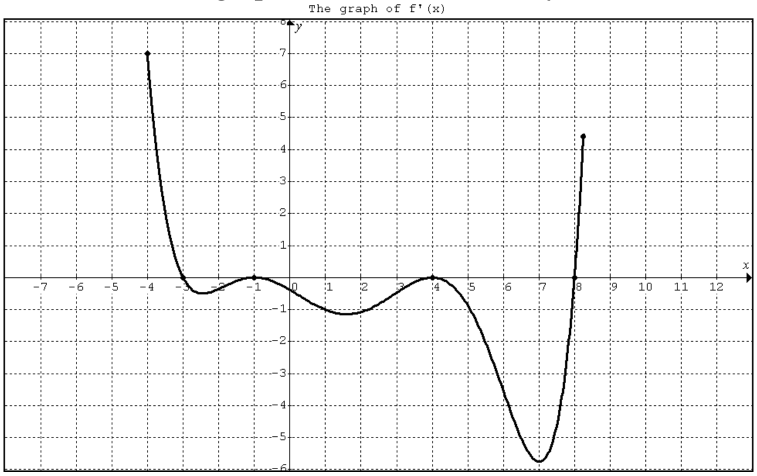
1. Let *f* be defined by $f(x) = \ln(2 + \sin x)$ for $\pi \le x \le 2\pi$.
Find the absolute maximum value and the absolute minimum value of *f* using the closed interval test.

2. Let $f''(x) = (x-1)(x+2)^2 e^{x^2}$.
   A. When is the graph of $f(x)$ concave upward? Justify.
   B. How many points of inflection are on *f*? Justify.

3. A particle moves along a horizontal line. It's position at time t is given as
$s(t) = \frac{2}{3}t^3 - \frac{5}{2}t^2 - 3t$.
On what time interval is the particle slowing down? Justify.

4. Consider the graph of the *derivative* of f below.

The graph of f'(x)



A. For what x – value(s) does f have a local minimum? Justify.

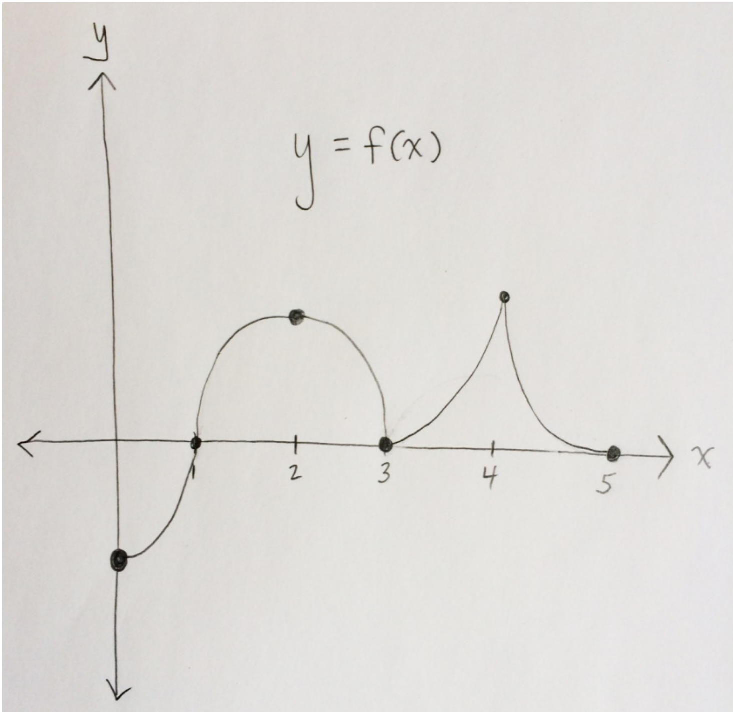
B. On what interval is f increasing? Justify.

C. On what interval is f concave upward? Justify.

D. How many points of inflection are on f?

GRAPH THEORY

5. Below is Steven's graph of $y = f(x)$.



THE CHART REPRESENTS STEVEN'S GRAPH

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$	4	$4 < x < 5$	5
$f(x)$									**+**		
$f'(x)$			**DNE**								
$f''(x)$											

FILL IN EACH BLANK IN THE CHART ABOVE WITH ONE OF THE FOLLOWING:

+ for positive

– for negative

0 for zero

DNE for Does not Exist