Q201.AB.Notes: Chapter 3.7, 4.6

LESSON 1 – 3.7 Implicit Differentiation

INTRODUCTION:

Identify the Shape: $x^2 + y^2 = 1$, $9x^2 + 4y^2 = 40$, $2x^2 - y^2 = 3$, $y^4 + 3y - 4x^3 = 5x + 1$

What does $\frac{dy}{dx}$ represent in reference to the equations above?

Example 1: $y^4 + 3y - 4x^3 = 5x + 1$

A. Find $\frac{dy}{dx}$.

B. Find the slope of the tangent line at the point P(1, -2)

C. Use implicit differentiation to find $\frac{d^2y}{dx^2}$.

2. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ if $x^2 + y^2 = 1$

3. Find $y^/$ if $4xy^3 - x^2y + x^3 - 5x + 6 = 0$

4. Find $\frac{dy}{dx}$ if $y = x^2 \sin y$

5. Find $\frac{dy}{dx}$ if $\tan(xy) + x^2 y + 2 = x$. (This will be covered in Lesson 2)

Find an equation of the line tangent to the curve at the point when $y = 0$.

AP Question (THIS WILL BE USED IN HW AND ON THE PRATICE EXAM)

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$

(b) Find all the points on the curve whose *x*-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the *x*-coordinate of each point on the curve where the tangent line is vertical.

LESSON 1 HW:
AP QUESTION ON PREVIOUS PAGE + SECTION 3.7: # 1, 5, 17, 21, 29, 49

(*Section 3.7*)

#1: Let $x^2y + xy^2 = 6$ Find $\dfrac{dy}{dx}$.

#5: Let $x = \tan(y)$ Find $\dfrac{dy}{dx}$.

#17: Let $x^2 + xy - y^2 = 1$
Find an equation of both the tangent and normal lines to the curve at the point (2, 3).

#21: Let $6x^2 + 3xy + 2y^2 + 17y - 6 = 0$
Find an equation of both the tangent and normal lines to the curve at the point (-1, 0).

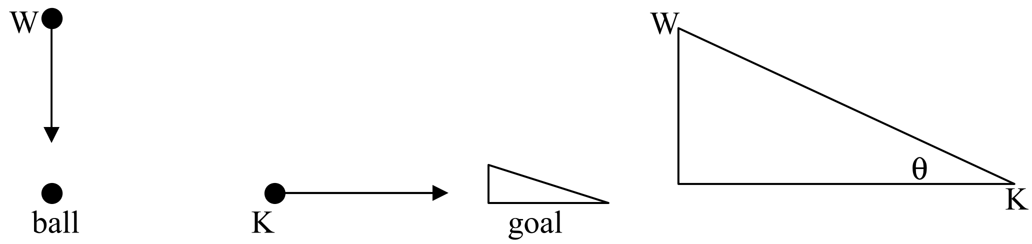
#29: Let $y^2 = x^2 + 2x$ Find $\dfrac{dy}{dx}$ and then $\dfrac{d^2y}{dx^2}$.

#49: Find the two points where the curve $x^2 + xy + y^2 = 7$ crosses the x-axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?

LESSON 2 and 3 – 4.6 Related Rates

Ex: 1 A ladder 20 ft long leans against the wall of a vertical building. If the ladder slides away from the building horizontally at a rate 2 ft/sec, how fast is the ladder sliding down the building when the top of the ladder is 12 ft above the ground?

Ex: 2 A camera is mounted at a point 3000 ft from the base of a rocket launching pad. If the rocket is rising vertically at 880 ft/s when it is 4000 ft above the launching pad, how fast must the camera elevation angle change at that instant to keep the camera aimed at the rocket?



EX 3: Walter approaches the soccer ball, due south, at a rate of 12 ft/s as Kevin approaches the soccer goal, due east, at 10 ft/s. When Walter is 6 ft from the ball and Kevin is 8 ft from the ball …

(a) What is the change in rate of the distance between the two boys?

(b) What is the change in rate of angle θ?

Ex: 4 A water tank has the shape of an inverted right circular cone of altitude 12 ft and base 6 ft. If water is being pumped into the tank at a rate of 1.2 ft^3/min, approximate the rate at which the water level is rising when the water is 3 ft deep.

EX 5: A urinal is 15 ft long and 4 ft across the top. Its ends are isosceles triangles with height 3 ft. Pee runs into the urinal at the rate of 2.5 ft^3/min. How fast is the pee rising when it is 2 ft deep?

LESSON2 NOTES

11. ***Inflating Balloon*** A spherical balloon is inflated with helium at the rate of 100π ft^3/min.

(a) How fast is the balloon's radius increasing at the instant the radius is 5 ft?

(b) How fast is the surface area increasing at that instant?

LESSON 2 NOTES

14. ***Flying a Kite*** Inge flies a kite at a height of 300 ft, the wind carrying the kite horizontally away at a rate of 25 ft/sec. How fast must she let out the string when the kite is 500 ft away from her?

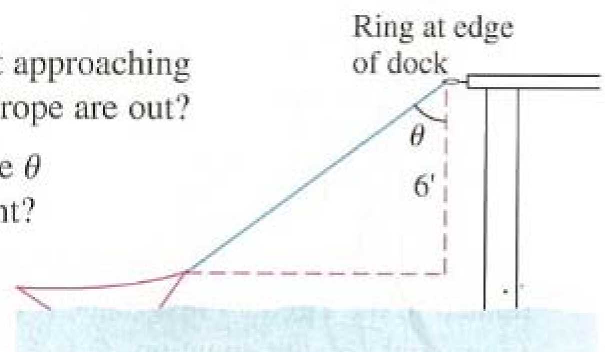
LESSON 2 HW:
SECTION 3.7 #56 SECTION 4.6 #11, 14, 21, 29, 34 AP: 1990 # 4

3.7 #56: The line that is normal to the curve $x^2 + 2xy - 3y^2 = 0$ at (1, 1) intersects the curve at what other point?

21. *Hauling in a Dinghy* A dinghy is pulled toward a dock by a rope from the bow through a ring on the dock 6 ft above the bow as shown in the figure. The rope is hauled in at the rate of 2 ft/sec.

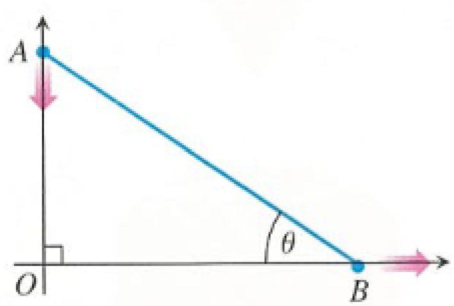
(a) How fast is the boat approaching the dock when 10 ft of rope are out?

(b) At what rate is angle θ changing at that moment?



29. *Moving Shadow* A man 6 ft tall walks at the rate of 5 ft/sec toward a streetlight that is 16 ft above the ground. At what rate is the length of his shadow changing when he is 10 ft from the base of the light?

34. *Walkers* A and B are walking on straight streets that meet at right angles. A approaches the intersection at 2 m/sec and B moves away from the intersection at 1 m/sec as shown in the figure. At what rate is the angle θ changing when A is 10 m from the intersection and B is 20 m from the intersection? Express your answer in degrees per second to the nearest degree.



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4. The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second.

(Note: The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(a) At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?

(b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?

(c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?

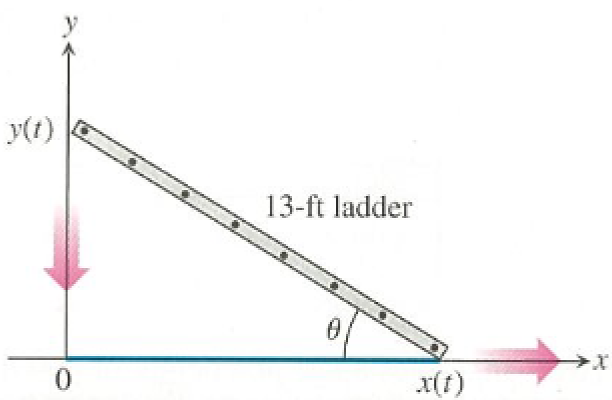
LESSON 3 HW:
SECTION 3.7 #57 SECTION 4.6 #16, 17, 19, 25 AP: 1987 # 5

#57 Find the normals to the curve $xy + 2x - y = 0$ that are parallel to the line $2x + y = 0$

16. ***Growing Sand Pile*** Sand falls from a conveyor belt at the rate of 10 m^3/min onto the top of a conical pile. The height of the pile is always three-eighths of the base diameter. How fast are the **(a)** height and **(b)** radius changing when the pile is 4 m high? Give your answer in cm/min.

17. ***Draining Conical Reservoir*** Water is flowing at the rate of 50 m^3/min from a concrete conical reservoir (vertex down) of base radius 45 m and height 6 m. **(a)** How fast is the water level falling when the water is 5 m deep? **(b)** How fast is the radius of the water's surface changing at that moment? Give your answer in cm/min.

19. ***Sliding Ladder*** A 13-ft ladder is leaning against a house (see figure) when its base starts to slide away. By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.



(a) How fast is the top of the ladder sliding down the wall at that moment?

(b) At what rate is the area of the triangle formed by the ladder, wall, and ground changing at that moment?

(c) At what rate is the angle θ between the ladder and the ground changing at that moment?

25. *Particle Motion* A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x-coordinate (in meters) increases at a constant rate of 10 m/sec. How fast is the angle of inclination θ of theline joining the particle to the origin changing when $x = 3$?

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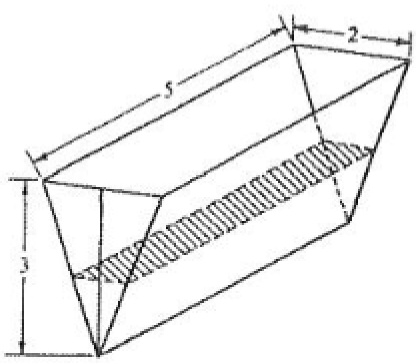
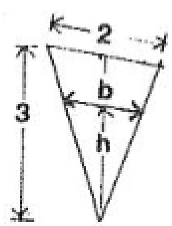
5. The trough shown in the figure above is 5 feet long, and its vertical cross sections are inverted isosceles triangles with base 2 feet and height 3 feet. Water is being siphoned out of the trough at the rate of 2 cubic feet per minute. At any time t, let h be the depth and V be the volume of water in the trough.

(a) Find the volume of water in the trough when it is full.

(b) What is the rate of change in h at the instant when the trough is $\frac{1}{4}$ full by volume?

(c) What is the rate of change in the area of the surface of the water (shaded in the figure) at the instant when the trough is $\frac{1}{4}$ full by volume?





AB: Q201. LESSON 4 – PRACTICE EXAM

Section 3.7:

1. Consider the curve C: $y^2 = x^2 - x - 8$.

A. Find dy/dx

B. Find the points on the curve C when $y = 2$.

C. Find the equations of the respective tangent lines to the curve C at the points found in part B.

D. Show that there are no horizontal tangents to the curve C.

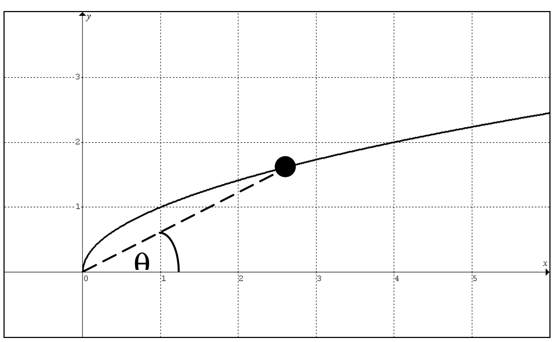
E. Find d^2y/dx^2 (Do not simplified)

F. Find d^2y/dx^2 at the point $(-3,-2)$ (Do simplify)

2. Find dy/dx if $x^3 - xy^3 = 18xy$.

Section 4.6:

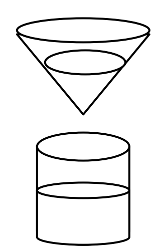
3. A particle moves from left to right along the curve $y = \sqrt{x}$ in such a way that the x-coordinate increases at the rate of 8 m/s. How fast is the angle of inclination θ of the line joining the particle to the origin changing when $x = 4$?



4. Coffee is draining from a conical filter (6 inch base and 6 inch height) into a cylindrical coffeepot (6 inch base) at the rate 10 in^3/min.

A. How fast is the level (height) in the cone falling at the moment when $h = 5$.

B. How fast is the level (height) in the pot rising at the same moment?



5. Notes Packet – AP Question

6. Find dy/dx if $2\cos(xy^2) + y = x^2y$

7. A particle $P(x,y)$ is moving in the coordinate plane in such a way that $dx/dt = -1$ m/sec and $dy/dt = 5$ m/sec. How fast is the particle's distance from the origin changing as it passes through the point (5, 12)?