**AB.Q103.NOTES: Chapter 2.4, 3.1, 3.2 LESSON 1**

**DEFINITION DEVELOPMENT**

Average rate of change of *f* on [a,b]
Slope of the secant line though curve *f* on [a, b]

Standard:

Alternate:

Instantaneous rate of change of *f* at $x = a$
Slope of the line tangent to the curve of *f* at $x = a$
Derivative of *f* at $x = a$

Standard:

Alternate:

1. Consider the function $y = f(x)$ where $f(x) = 3x^2 - 2$

a. Find the average rate of change in $f$ on the interval [-1, 2]

b. Using the standard definition of a derivative at $x = a$ …
Find the instantaneous rate of change in $f$ at $x = 1$. (*Precede your answer with Lagrange notation).*

c. Write an equation of the tangent line to the graph of $f(x)$ at $x = 1$.

2. Consider the function $y = f(x)$ where $f(x) = \sqrt{x+1}$

a. Find the average rate of change in *f* on the interval [0, 8]

b. Using the alternate definition of a derivative at $x = a$ …
Find the instantaneous rate of change in *f* at $x = 3$. (*Precede your answer with Lagrange notation).*

c. Write an equation of the tangent line to the graph of $f(x)$ at $x = 3$.

3. Consider the function $y = f(x)$ where $f(x) = \begin{cases} 5 - 2x; & x \geq 1 \\ 6 - 4x + x^2; & x < 1 \end{cases}$

a. Find the average rate of change in *f* on the interval [0, 2]

b. Using the standard definition of a derivative at $x = a$ …
Find the instantaneous rate of change in *f* at $x = 1$. (*Precede your answer with Lagrange notation).*

c. Write an equation of the tangent line to the graph of $f(x)$ at $x = 1$.

4. Consider the function $y = g(x)$ where $g(x) = \begin{cases} 5 - x; & x \geq 0 \\ 2x + 5; & x < 0 \end{cases}$

a. Find the average rate of change in $g$ on the interval [-2, 5]

b. Using the standard definition of a derivative at $x = a$ …
Find the instantaneous rate of change in $g$ at $x = 0$. (*Precede your answer with Lagrange notation).*

c. Write an equation of the tangent line to the graph of $g(x)$ at $x = 0$.

*Details and Summary*

What is the derivative of $f$ at $x = a$?

DEF (Sandard): The derivative of the function $f$ at the point $x = a$ is

$f^{/}(a) = \lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$, provided it exists.

DEF (Alternate): The derivative of the function $f$ at the point $x = a$ is

$f^{/}(a) = \lim_{x\to a} \dfrac{f(x) - f(a)}{x-a}$, provided it exists.

Is the function differentiable at $x = a$?

DEF (Std): A function $f$ is differentiable at $x = a$ if $\lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$ exists.

If the limit does not exists then we say that the function is not differentiable at $x = a$.

DEF (alt): A function $f$ is differentiable at $x = a$ if $\lim_{x\to a} \dfrac{f(x) - f(a)}{x-a}$ exists.

If the limit does not exists then we say that the function is not differentiable at $x = a$.

**AB.Q103.LESSON 1 – HW**:

1. If $f(2)=3$ and $f^{/}(2)=5$, find an equation of (a) the tangent line, and (b) the normal line to the graph of $y=f(x)$ at the point where $x=2$.

2. Consider the function $y=f(x)$ where $f(x)=x^2-4x$
a. Find the average rate of change in *f* on the interval [-2, 4]
b. Find the instantaneous rate of change in $f(x)$ at $x=1$.
Classify $f(x)$ as either smooth, a corner, a cusp, a vertical line tangent, or discontinuous at $x=1$.
c. Write an equation of the tangent line to the graph of $f(x)$ at $x=1$.

3. Consider the function $y=f(x)$ where $f(x)=\begin{cases}-x; & x<0 \\ x^2-x; & x\geq 0\end{cases}$
a. Find the average rate of change in *f* on the interval [-2, 4]
b. Find the instantaneous rate of change in $f(x)$ at $x=0$.
Classify $f(x)$ as either smooth, a corner, a cusp, a vertical line tangent, or discontinuous at $x=0$.
c. Write an equation of the tangent line to the graph of $f(x)$ at $x=0$.

4. Consider the function $y=f(x)$ where $f(x)=\begin{cases}-x^2; & x\geq -2 \\ 2x; & x<-2\end{cases}$
a. Find the average rate of change in *f* on the interval [-4, 0]
b. Find the instantaneous rate of change in $f(x)$ at $x=-2$.
Classify $f(x)$ as either smooth, a corner, a cusp, a vertical line tangent, or discontinuous at $x=-2$.
c. Write an equation of the tangent line to the graph of $f(x)$ at $x=-2$.

5. Consider the function $g(x)=\frac{1}{x}$.
a. Find $g^{/}(2)$ using the standard definition of the derivative at $x=a$.
b. Find $g^{/}(2)$ using the alternate definition of the derivative at $x=a$.

6. Let $p(x)=\begin{cases}\frac{x^2-1}{x+1}; & x\neq -1 \\ \sec(\pi x)-\ln(x^2); & x=-1\end{cases}$. Prove that *p* is or is not <u>continuous</u> **at** $x=-1$.

**AB.Q103.NOTES: Chapter 2.4, 3.1, 3.2 LESSON 2**

1. Let $p(x)=\begin{cases}\dfrac{x^2-1}{x+1}; & x\neq -1\\ \sec(\pi x)-\ln(x^2); & x=-1\end{cases}$. Prove that *p* is or is not <u>continuous</u> **at** $x=-1$.

2. Let $f(x)=\begin{cases}5-2x; & x\geq 1\\ x^2+1; & x<1\end{cases}$. Prove that *f* is or is not <u>continuous</u> **at** $x=1$.

3. THEOREM:

## 4. OVERVIEW

5. $f(x)=\begin{cases} -x; & x<0 \\ x^2-x; & x\geq 0 \end{cases}$

A. Prove that *f* is or is not continuous **at** $x=0$.

B. Prove that *f* is or is not differentiable **at** $x=0$.

6. $f(x)=\begin{cases}3-x, & x\geq -1\\ x^2+3, & x<-1\end{cases}$

A. Prove that $f$ is or is not <u>continuous</u> **at** $x=-1$.

B. Prove that $f$ is or is not <u>differentiable</u> **at** $x=-1$.

7. $p(x)=\begin{cases} 2x, & x \geq 1 \\ 2x+3, & x < 1 \end{cases}$

A. Prove that $p$ is or is not continuous **at** $x=1$.

B. Prove that $p$ is or is not differentiable **at** $x=1$.

**AB.Q103.LESSON 2 – HW**:

1. Consider the function $y = f(x)$ where $f(x) = \begin{cases} x+1; & x > 1 \\ -x^2 + 3x; & x \le 1 \end{cases}$

a. Find the average rate of change in *f* on the interval [-1, 2]

b. Using the standard definition of a derivative at $x = a$ …
Find the instantaneous rate of change in *f* at $x = 1$. (*Precede your answer with Lagrange notation).*

c. Write an equation of the tangent line to the graph of $f(x)$ at $x = 1$.

2. Consider the function $b(x) = \begin{cases} x; & x > 1 \\ -2 + x; & x \le 1 \end{cases}$

a. Prove that *b* is or is not continuous at $x = 1$.

b. Using the standard definition of a derivative at $x = a$, prove that *b* is not differentiable $x = 1$.

c. Explain how you could have used part (a) to prove that *b* is not differentiable $x = 1$.

3. Use the table below to estimate a) $f'(1.57)$ and b) $f'(3)$

| *t* | 0.00 | 0.56 | 0.92 | 1.19 | 1.30 | 1.39 | 1.57 | 1.74 | 1.98 | 2.18 | 2.41 | 2.64 | 3.24 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| *f*(*t*) | 1577 | 1512 | 1448 | 1384 | 1319 | 1255 | 1191 | 1126 | 1062 | 998 | 933 | 869 | 805 |

4. Using the definition for a derivative at $x = a$,
prove that $f(x) = \begin{cases} x; & x \le 1 \\ 1/x; & x > 1 \end{cases}$ is or is not <u>differentiable</u> **at** $x = 1$.
Classify $f(x)$ as either smooth, a corner, a cusp, a vertical line tangent, or discontinuous at $x = 1$.

5. Prove that $g(x) = \begin{cases} 2 + \sqrt{x}; & x > 0 \\ 1 + e^{-x}; & x \le 0 \end{cases}$ is or is not <u>continuous</u> **at** $x = 0$.

**AB.Q103.NOTES: Chapter 2.4, 3.1, 3.2 LESSON 3**

Definition: The derivative of the function $f$ with respect to the variable $x$ is the <u>function</u> $f'(x)$ whose value at $x$ is $f'(x) = \lim_{h\to 0} \dfrac{f(x+h) - f(x)}{h}$, provided it exists.

NO ALTERNATE DEFINITION FOR $f'(x)$

1. Consider the function $f(x) = 3x^2 - 2$.

A. Find $f'(x)$.

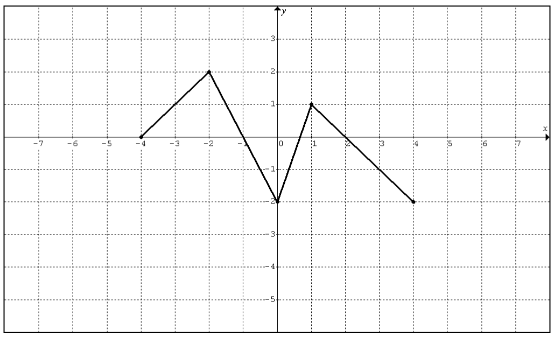
B. Write an equation of the tangent line to the graph of $f(x)$ at $x = -1$ and $x = 5$.

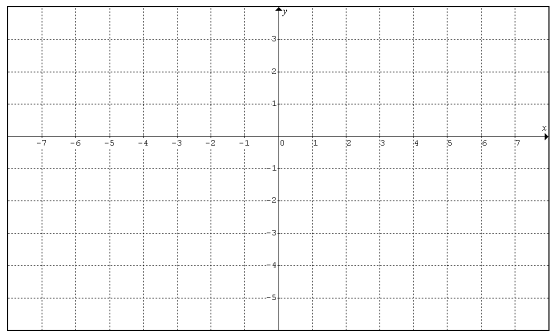
C. Write an equation of the normal to the graph of $f(x)$ at $x = -1$ and $x = 5$.

D. Find the points on the graph of $f$ where the slope of the tangent line is parallel to $y = 4x + 5$.

2. **Graph** $f'(x)$ **from** $f(x)$

The graph of $y = g(x)$ shown here is made of line segments joined end to end. Graph the function's derivative.

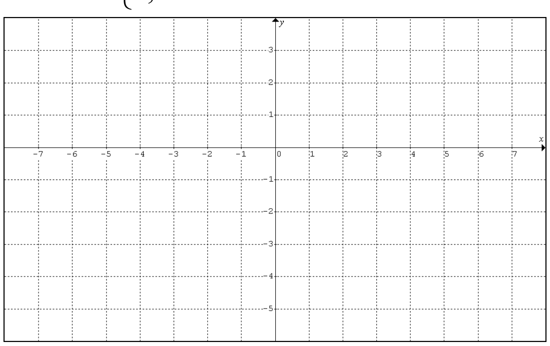




3. **Graph** $f(x)$ **from** $f'(x)$

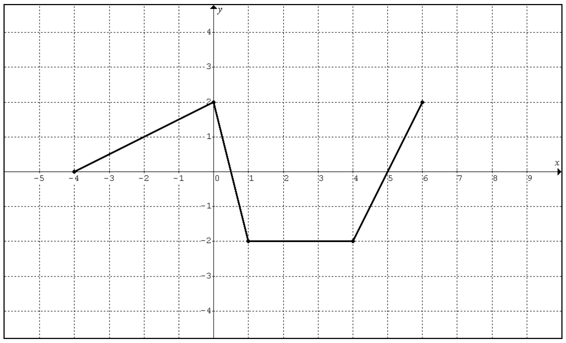
Sketch a possible graph of a continuous function *f* that has domain [-3, 3], where $f(-1) = -2$ and the equation of $y = f'(x)$ is shown below.

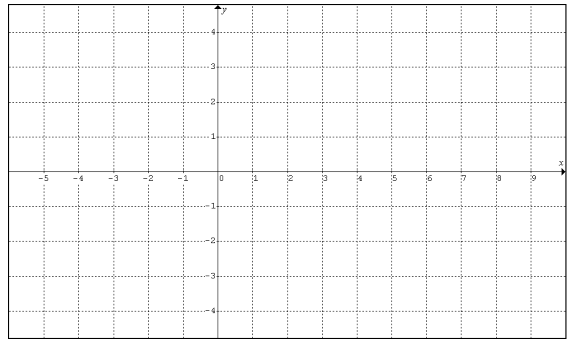
$$f'(x) = \begin{cases} -1; & x < -1 \\ 0; & -1 < x < 2 \\ 3; & x > 2 \end{cases}$$



**AB.Q103.LESSON 3 – HW**:

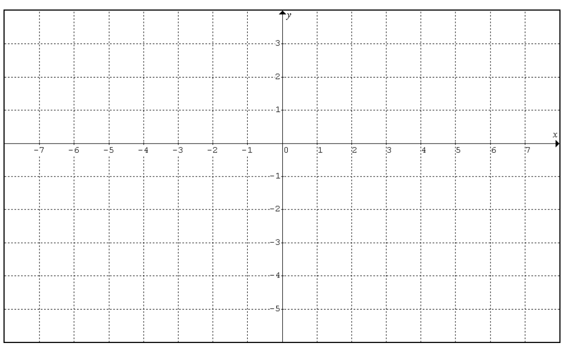
1. The graph of the function $y = f(x)$ shown here is made of line segments joined end to end. Graph $y = f'(x)$ and state its domain.





2. Sketch the graph of a continuous function with domain [-2,2], $f(0) = -1$, and

$$f'(x) = \begin{cases} 1; & x < -1 \\ -2; & x > -1 \end{cases}.$$



3. Using the information from problem 2, write an equation of the line tangent to $f$ at $x = 0$.

4. Find $f'(x)$ for $f(x) = 5x^2 - 2x + 1$ using the appropriate definition.

5. Find the value of $x$ for which the tangent to $f(x) = 5x^2 - 2x + 1$ is horizontal.

6. Find $f'(x)$ for $f(x) = \dfrac{1}{x+1}$ using the appropriate definition.

7. Find $f'(x)$ for $f(x) = \sqrt{2x+1}$ using the appropriate definition.

8. Consider the function $y = f(x)$ where $f(x) = \begin{cases} x+1; & x > 1 \\ -x^2 + 3x; & x \leq 1 \end{cases}$.

Find $f'(x)$.

9. Using the definition for a derivative at $x = a$,

prove that $m(x) = \begin{cases} \sqrt{x}; & x \geq 1 \\ \dfrac{x}{2} + \dfrac{1}{2}; & x < 1 \end{cases}$ is or is not <u>differentiable</u> **at** $x = 1$.

Classify $m(x)$ as either smooth, a corner, a cusp, a vertical line tangent, or discontinuous at $x = 1$.