# AB CALCULUS

# Q102

## LIMITS and CONTINUITY FOR CALCULUS

## NO CALCULATORS

**AB CALCULUS Q102: Limits – Lesson 1**

1. $\lim_{x \to 2} 5x - 3$

2. $\lim_{x \to 1} \frac{x^2 + x - 2}{x - 1}$

3. $\lim_{x \to 9} \frac{x - 9}{\sqrt{x} - 3}$

4. Let $f(x) = \begin{cases} 3 - x; & x < 1 \\ 4; & x = 1 \\ x^2 + 1; & x > 1 \end{cases}$ Find $\lim_{x \to 1} f(x)$

5. Let $f(x) = \frac{|x|}{x}$ Find $\lim_{x \to 0} f(x)$

6. Let $f(x) = x^2 \sin \frac{1}{x^2}$ Find $\lim_{x \to 0} f(x)$

7. Let $\lim_{x \to 5} f(x) = 0$ and $\lim_{x \to 5} g(x) = 2$

Show Properties

a. $\lim_{x \to 5} [f(x) + g(x)]$

b. $\lim_{x \to 5} [f(x) \cdot g(x)]$

c. $\lim_{x \to 5} e^{g(x)}$

d. $\lim_{x \to 5} \sqrt[3]{f(x) + 27}$

8. $\lim_{x \to 0} \frac{\sin x}{x}$

9. Let $f(x) = \frac{1}{x}$. Find $\lim_{x \to 0} \frac{1}{x}$

10. $f(x) = \frac{1}{(x-2)^2}$. Find $\lim_{x \to 2} \frac{1}{(x-2)^2}$

Summary of Analytic techniques to find the limit of a function as $x$ approaches a real number:

1.

2.

3.

4.

5.

6.

Definition of Vertical Asymptote of a function $f$:

TWO-ONE SIDED LIMIT THEOREM:

**AB.Q102.LESSON 1 – HW**:

Textbook Section 2.1:

11, 18, 19, 22, 49, 35, 51, 62, 37, 43

Textbook Section 2.2:

13, 14, 27, 53

*These have been typed out on the next page.*

Section 2.1 . Find the limit or state the limit does not exist.

11. $\lim_{y\to -3} \dfrac{y^2+4y+3}{y^2-3}$

18. $\lim_{x\to 0} \dfrac{(4+x)^2-16}{x}$

19. $\lim_{x\to 1} \dfrac{x-1}{x^2-1}$

22. $\lim_{x\to 0} \dfrac{\frac{1}{2+x}-\frac{1}{2}}{x}$

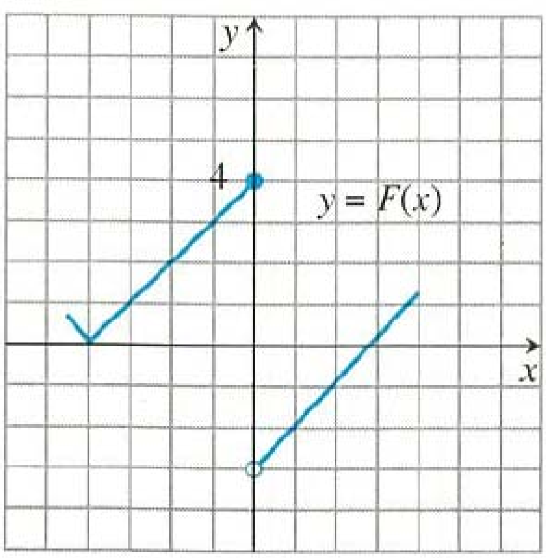
49. Assume $\lim_{x\to 4} f(x)=0$ and $\lim_{x\to 4} g(x)=3$.

(a) $\lim_{x\to 4}\left(g(x)+3\right)$ (b) $\lim_{x\to 4} xf(x)$ (c) $\lim_{x\to 4} g^2(x)$ (d) $\lim_{x\to 4} \dfrac{g(x)}{f(x)-1}$

51. $\lim_{x\to 2} f(x)$ where $f(x)=\begin{cases} 3-x, & x<2 \\ \dfrac{x}{2}+1, & x>2 \end{cases}$.

62. $\lim_{x\to 0} x^2\cos\dfrac{1}{x^2}$

43.




(a) $\lim_{x\to 0^-} F(x)$

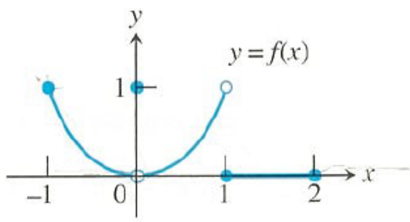
(b) $\lim_{x\to 0^+} F(x)$

(c) $\lim_{x\to 0} F(x)$

(d) $F(0)$

37. True or False

**37.**




**(a)** $\lim_{x \to -1^+} f(x) = 1$

**(b)** $\lim_{x \to 0^-} f(x) = 0$

**(c)** $\lim_{x \to 0^-} f(x) = 1$

**(d)** $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

**(e)** $\lim_{x \to 0} f(x)$ exists

**(f)** $\lim_{x \to 0} f(x) = 0$

**(g)** $\lim_{x \to 0} f(x) = 1$

**(h)** $\lim_{x \to 1} f(x) = 1$

**(i)** $\lim_{x \to 1} f(x) = 0$

**(j)** $\lim_{x \to 2^-} f(x) = 2$

Section 2.2. Find the limit or state the limit does not exist.

13. $\lim_{x \to 2^+} \frac{1}{x-2}$ 14. $\lim_{x \to 2^-} \frac{x}{x-2}$

53. For $f(x) = \begin{cases} 1/x, & x < 0 \\ -1, & x \geq 0 \end{cases}$

(a) $\lim_{x \to -\infty} f(x)$ (b) $\lim_{x \to \infty} f(x)$ (c) $\lim_{x \to 0^-} f(x)$ (d) $\lim_{x \to 0^+} f(x)$

27. For $f(x) = \frac{1}{x^2 - 4}$ ,

Write limit statements for any vertical asymptotes of the graph of $f(x)$.

Write limit statements for any horizontal asymptotes of the graph of $f(x)$.

**AB CALCULUS Q102: Limits Continued – Lesson 2 (Part 1)**

1. Find $\lim_{x \to -\infty} \frac{2x^2 - 5}{3x^2 + x + 2}$

2. Find $\lim_{x \to \infty} \frac{2x^3 - 5}{3x^2 + x + 2}$

3. Find $\lim_{x \to \infty} \frac{\sqrt{9x^2 + 2}}{4x + 3}$

4. Find $\lim_{x \to \infty} \sqrt[3]{\frac{3x + 2}{5x - 1}}$

Definition of a Horizontal Asymptote:

**AB CALCULUS Q102: CONTINUITY – Lesson 2 (Part 2)**

Definition of a function *f* continuous at $x = a$ :

EXAMPLE:

EXAMPLE:

EXAMPLE:

**AB CALCULUS Q102: IVT – Lesson 2 (Part 3)**

**Intermediate Value Theorem** (the no-duh theorm)

If $f$ is continuous on a closed interval [a, b], then $f$ takes on all values between $f(a)$ to $f(b)$.

Example: Let $y = f(x)$ be continuous on [-4, 2] with corresponding values as shown in the table below:

| $x$ | -4 | -3 | -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | -139 | 72 | 41 | 2 | -3 | -4 | 17 |

A. How many times will $y = f(x)$ obtain the value of -120? Justify using the intermediate value theorem.

B. How many zeros will $y = f(x)$ obtain on the interval [-4, 2]? Justify using the intermediate value theorem.

**AB.Q102.LESSON 2 – HW**:

**CONTINUITY at $x = a$ : Read Section 2.3**

1. Prove that $f(x) = x^2 - 2x + 3$ is or is not continuous at $x = 1$.

2. Prove that $g(x) = \begin{cases} 2 + \sqrt{x}; & x > 0 \\ 1 + e^{-x}; & x \le 0 \end{cases}$ is or is not continuous at $x = 0$.

3. Prove that $r(x) = \begin{cases} x + 2; & x \ne -1 \\ 5; & x = -1 \end{cases}$ is or is not continuous at $x = -1$.

4. Prove that $f(x) = \begin{cases} x^2 + 2; & x \ge 0 \\ 1/x; & x < 0 \end{cases}$ is or is not continuous at $x = 0$.

5. A. Prove that $g(x) = \dfrac{x^2 - 9}{x - 3}$ is not continuous at $x = 3$.
   B. Extend *g*(*x*), making it a piecewise function that is continuous at $x = 3$.

6. A. Prove that $d(x) = \dfrac{x - 4}{\sqrt{x} - 2}$ is not continuous at $x = 4$.
   B. Extend *d*(*x*), making it a piecewise function that is continuous at $x = 4$.

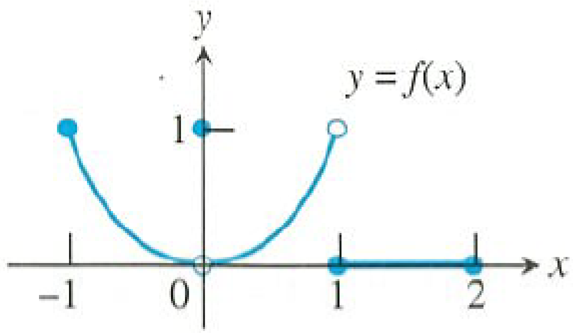
7. Prove that $h(x) = \begin{cases} x \sin \dfrac{1}{x} & x \ne 0 \\ 0 & x = 0 \end{cases}$ is or is not continuous **at** $x = 0$.

Hint: See the Sandwich Theorem of Limits.

LESSON 2 HW CONTINUED …
Section 2.3 #23, 41 – 44, 47

23. Find all domain values of $x$ such that $f(x)$ is not continuous. (proof not required). Classify the discontinuity as a jump, removable, or infinite discontinuity.

23.




**41-44: Sketch a possible graph for a function *f* that has the given properties.**

41. $f(3)$ exists, but $\lim_{x \to 3} f(x)$ does not exist.
42. $f(-2)$ exists, $\lim_{x \to -2^+} f(x) = f(-2)$, but $\lim_{x \to -2^-} f(x)$ does not exist .
43. $f(4)$ exists, $\lim_{x \to 4} f(x)$ exists, but f is not continuous at $x = 4$ .
44. $f(x)$ is continuous for all $x$ except $x = 1$ . $f$ has a removable discontinuity at $x = 1$ .

**47.** Find a value of the constant "a" such that $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$ is continuous at $x = 3$ .

**Section 2.2 #6, 9, 22**

6. For $f(x) = \dfrac{2x-1}{|x|-3}$

Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ and use this to describe the end behavior of the graph of $f$

9. Find $\lim_{x \to \infty} \dfrac{1 - \cos x}{x^2}$ using the sandwich theorem.

22. $f(x) = \left(\dfrac{2}{x} + 1\right)\left(\dfrac{5x^2 - 1}{x^2}\right)$

Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ and use this to describe the end behavior of the graph of $f$