AB CALCULUS

Q101

ALGEBRAIC and TRIGONOMETRIC ESSENTIALS FOR CALCULUS

NO CALCULATORS

AB. Q101. LESSON 1. NOTES
Definition of Absolute Value and Writing Piecewise Functions

Define $\sqrt{x^2}$: Define $|x|$:

1. Write the function $f(x) = |3x-2|$ without using the absolute-value symbol.

2. Write the function $g(x) = 5 - |x+1|$ without using the absolute-value symbol.

3. Write the function $y = \frac{|x+2|}{x+2}$ without using the absolute-value symbol. GRAPH IT.

4. Write the function $h(x) = 4 - \sqrt{x^2} + x|x|$ without using the absolute-value symbol.

5. Write the function $f(x) = |2-x| + 1$ without using the absolute-value symbol. GRAPH IT.

6. Write the function $f(x) = 3 - 2|x-1|$ without using the absolute-value symbol.

DOMAIN / SET AND INTERVAL NOTATION

STATE THE DOMAIN FOR EACH FUNCTION (USING BOTH **INTERVAL** AND **SET** NOTATIONS)

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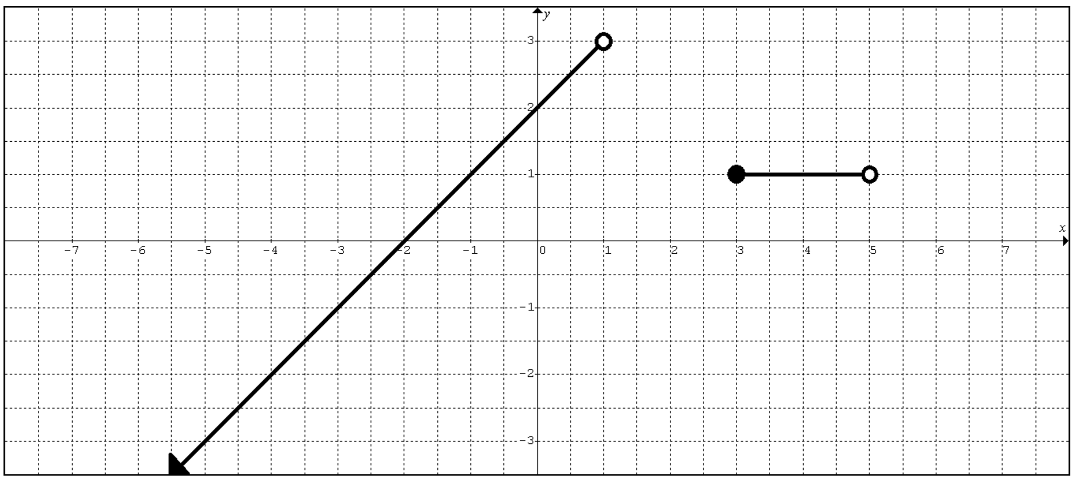

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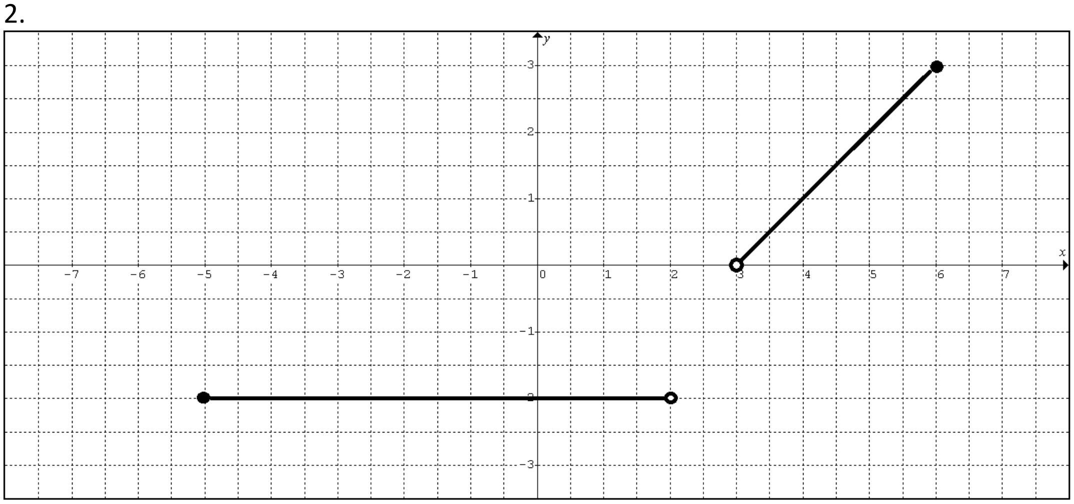

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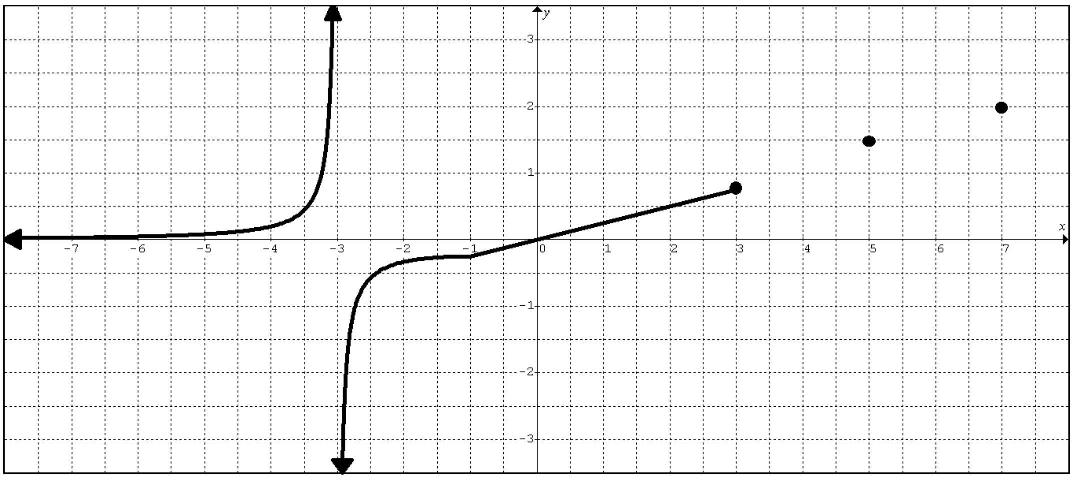

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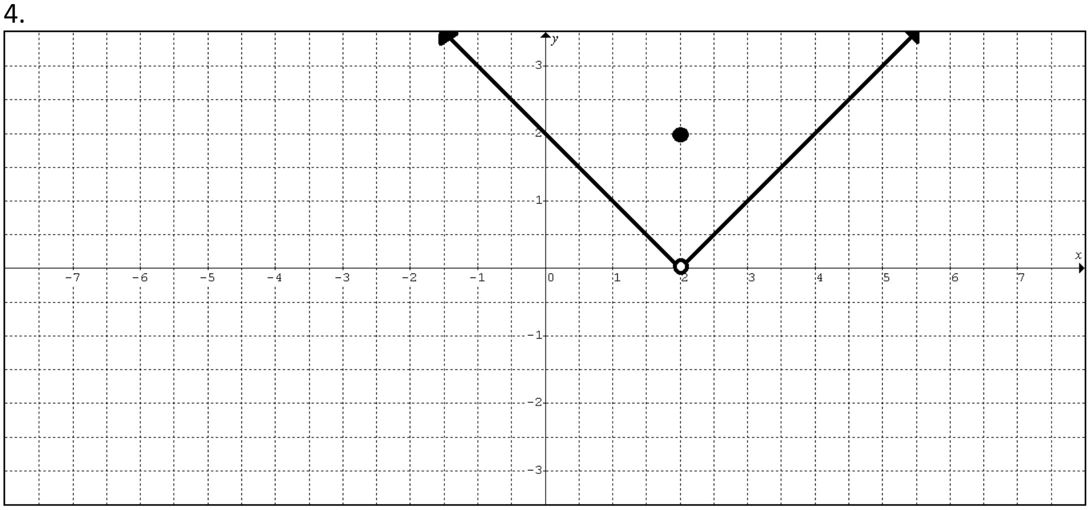

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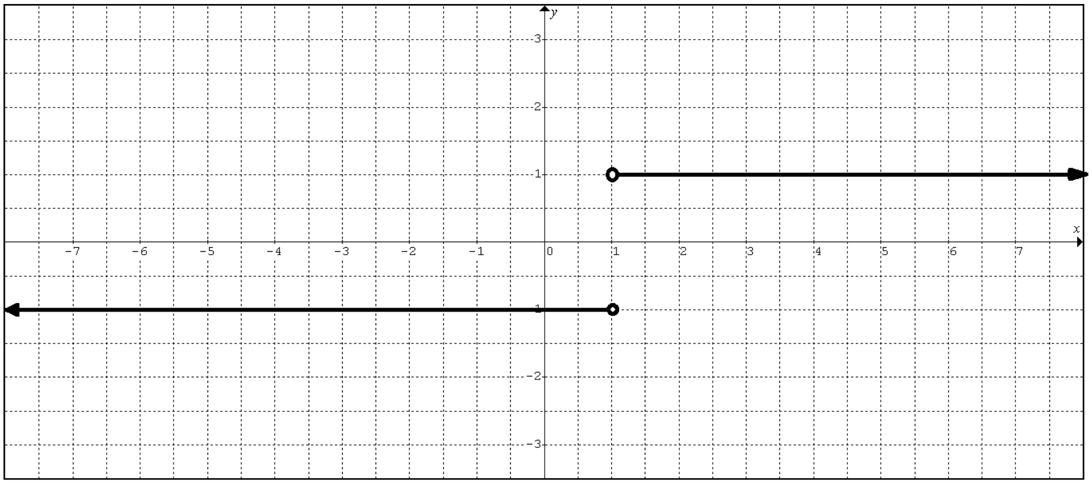

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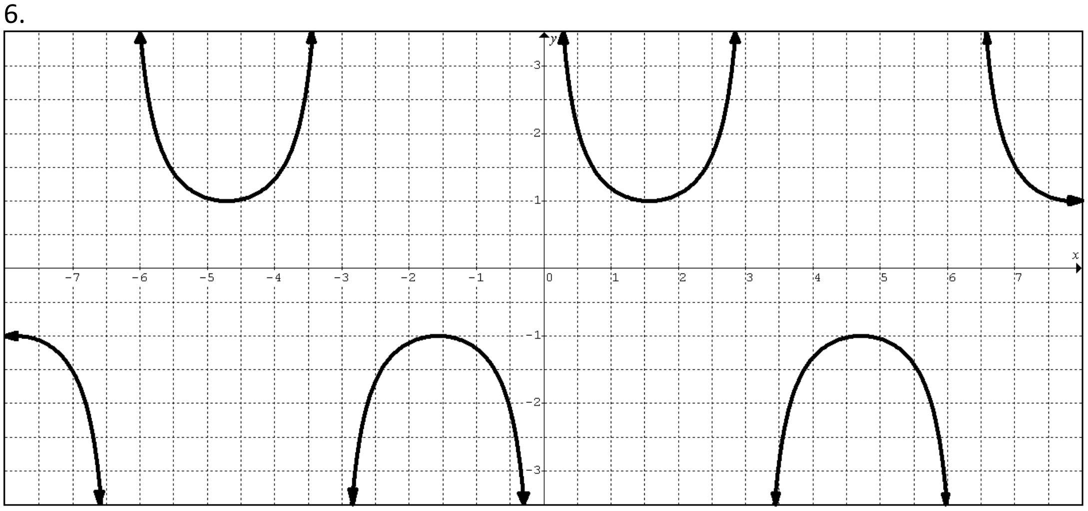

STATE THE DOMAIN FOR EACH FUNCTION (USING **SET** NOTATION)

1. $y = \sqrt{x-2}$

2. $y = \dfrac{3}{\sqrt{x}}$

3. $y = \dfrac{|x+5|}{x+5}$

4. $y = \sqrt{5-x}$

5. $y = \sqrt{x^2 - 4}$

6. $y = \dfrac{2x}{(x-3)(3x+5)}$

7. $y = \dfrac{10}{16-(x+2)^2}$

8. $y = \dfrac{\sqrt{4+x}}{1-x}$

9. $y = \ln(x)$

10. $y = 5\ln(x-2)$

11. $y = \sin(x)$

12. $y = \tan(x)$

13. $y = e^x$

AB. Q101. LESSON 1 HOMEWORK

Write each function without the absolute value symbol using a piecewise representation.

1. $f(x) = 2 + |5 - 2x|$

2. $g(x) = 5 + \sqrt{(7 + x)^2}$

Report the domain using SET and INTERVAL notations.

3. $y = \dfrac{x+1}{(x-3)(x+2)}$

4. $y = \dfrac{\sqrt{7-x}}{\sqrt{x+1}}$

5. $y = \begin{cases} 5; & x < 0 \\ \dfrac{\sqrt{x+3}}{x-9}; & elsewhere \end{cases}$

6. $y = \begin{cases} \ln(x+4); & x < 0 \\ \dfrac{1}{x-4}; & x \geq 0 \end{cases}$

7. $y = \sec x$

Mixed Review (Algebraic Essentials)

8. Solve the inequality: $-5 \leq \dfrac{14-3x}{2} < 1$

9. Solve the inequality: $x^2 - 10 > 3x$

10. Solve the inequality: $|x - 3| < \dfrac{1}{2}$

11. Solve the inequality: $|2x - 7| > 3$

12. Write an equation of a line (in point-slope form) that passes through the points (1,7) and (-3, 2).

13. Simplify the expression $\dfrac{f(2+h)-f(2)}{h}$ for the function $f(x)=x^2+6x-4$.

14. Graph the function $f(x)=\begin{cases} 2x+3; & x<0 \\ x^2; & 0\le x<2 \\ 1; & x\ge 2 \end{cases}$.

15. Graph the function $g(x)=\begin{cases} \dfrac{|x-2|}{x-2}; & x\ne 2 \\ 0; & x=2 \end{cases}$

16. Graph the function $r(x)=\begin{cases} x+2; & x\ne -1 \\ 5; & x=-1 \end{cases}$

17. Graph the function $h(x)=\begin{cases} x^2; & x\le 0 \\ x; & x>0 \end{cases}$

18. Graph the function $f(x)=x+\sqrt{x^2}+2$

19. Graph $y=f(g(x))$ **AND** $y=g(f(x))$ for the functions $f(x)=16-x^2$ and $g(x)=\sqrt{x}$.
Also state the domain of each.

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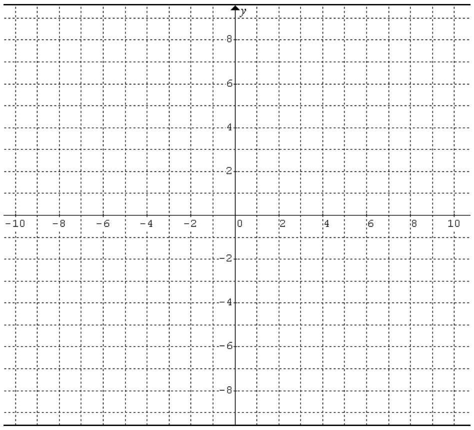

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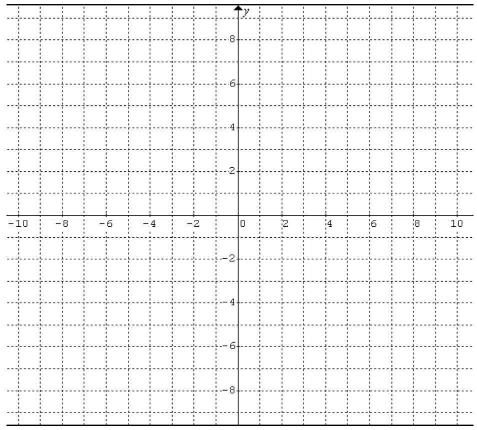

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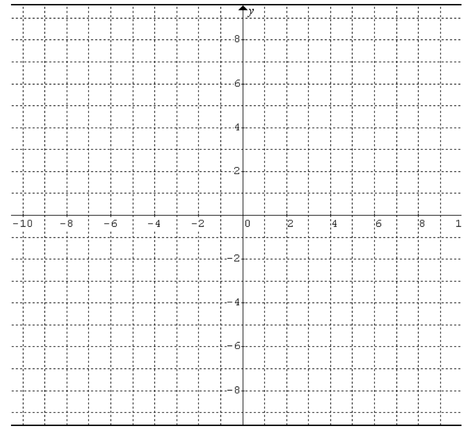

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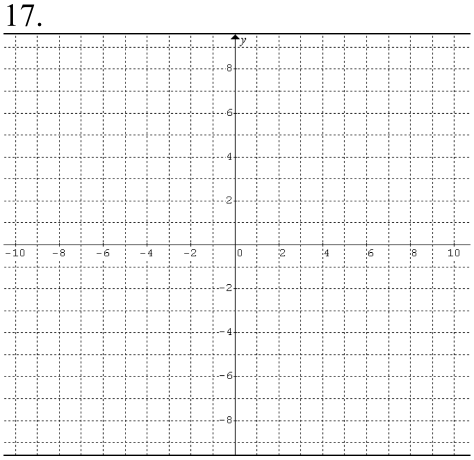

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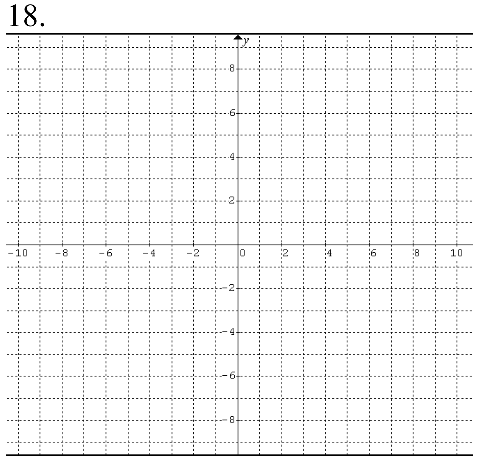

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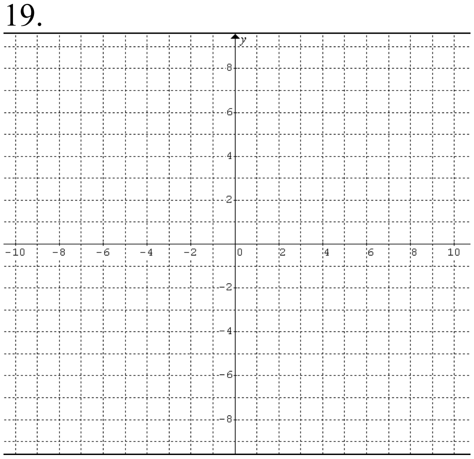

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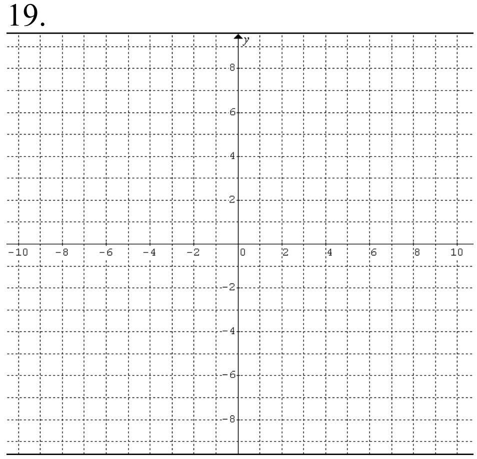


AB. Q101. LESSON 2. NOTES: Powerful Graphing

Consider a rational function $f(x)$ in factored form.

Graphing $f(x)$:

- Find the holes and write the coordinates of each.
- Find the vertical asymptotes and write the equation of each
  - ✓ (determine how the graph approaches the asymptote, i.e. from the same or opposite direction)
- Find the zeros of the function and write the coordinates for each
  - ✓ (determine how the graph touches the *x*-axis, i.e. bounce, plateau, cut through)
- Find the *y*-intercept and write the coordinates.
- Determine the horizontal asymptotes and write the equation for each

Guidelines Details

❖ **Holes (or removable) discontinuities** occur for *x* values that make a canceled factor go to zero.

❖ **Vertical Asymptote discontinuities** occur for *x* values that make a simplified rational function undefined.
After simplifying, set the denominator equal to zero to find these vertical asymptotes.
- ✓ If the asymptote repeats an even number of times, then the graph will approach the vertical asymptote from the same direction.
- ✓ If the asymptote repeats an odd number of times (or does not repeat), then the graph will approach the vertical asymptote from opposite directions.

DEF: A line $x = a$ is called a vertical asymptote of the graph of a function *f* if $f(x) \to +\infty$ or $f(x) \to -\infty$ as *x* approaches *a* from the left or right.

❖ A function's **Zeros** occur for the *x* values that make a simplified rational function equal zero.
After simplifying, set the numerator equal to zero to find these zeros.
- ✓ If the zero repeats an even number of times, then the graph will bounce off the *x*-axis.
- ✓ If the zero repeats an odd number of times, then the graph will plateau on the *x*-axis.
- ✓ If the zero does not repeat, then the graph will simply cut through the *x*-axis.

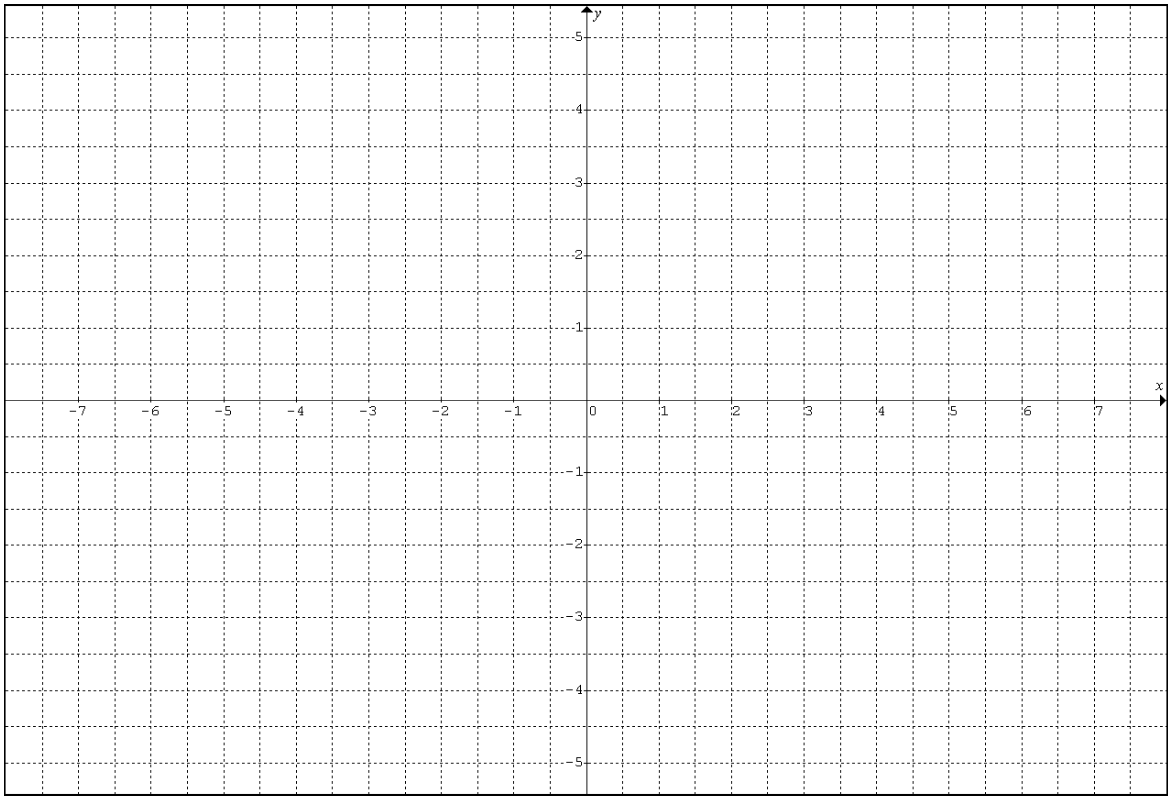
❖ A function's **Y-intercept** occurs when the *x*-value equals zero.
Plug zero in for x and solve for y to find the value of the y-intercept.

❖ **Horizontal Asymptote:**

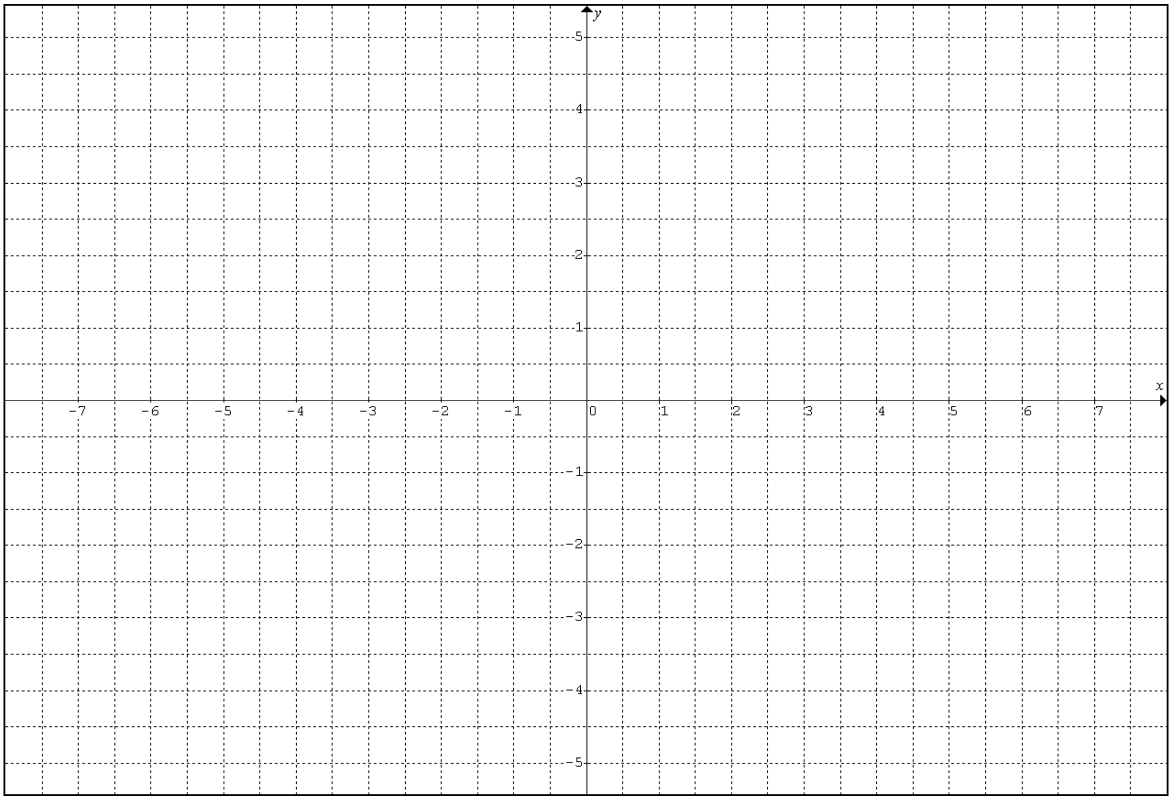
Choose the highest power term in both the numerator and denominator.

- If the power in the numerator is higher than the power in the denominator, then there will not be a horizontal asymptote.
- If the power in the numerator is less than the power in the denominator, then there will be a horizontal asymptote at *y* = 0. (end behavior not necessarily local)
- If the power in the numerator is equal to the power in the denominator, then there will be a horizontal asymptote at $y = \frac{a}{b}$ where *a* is the leading coefficient of the numerator and *b* the leading coefficient of the denominator.

DEF: A line $y = L$ is called a horizontal asymptote of the graph of a function *f* if $\lim_{x \to +\infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$



EXAMPLE 1



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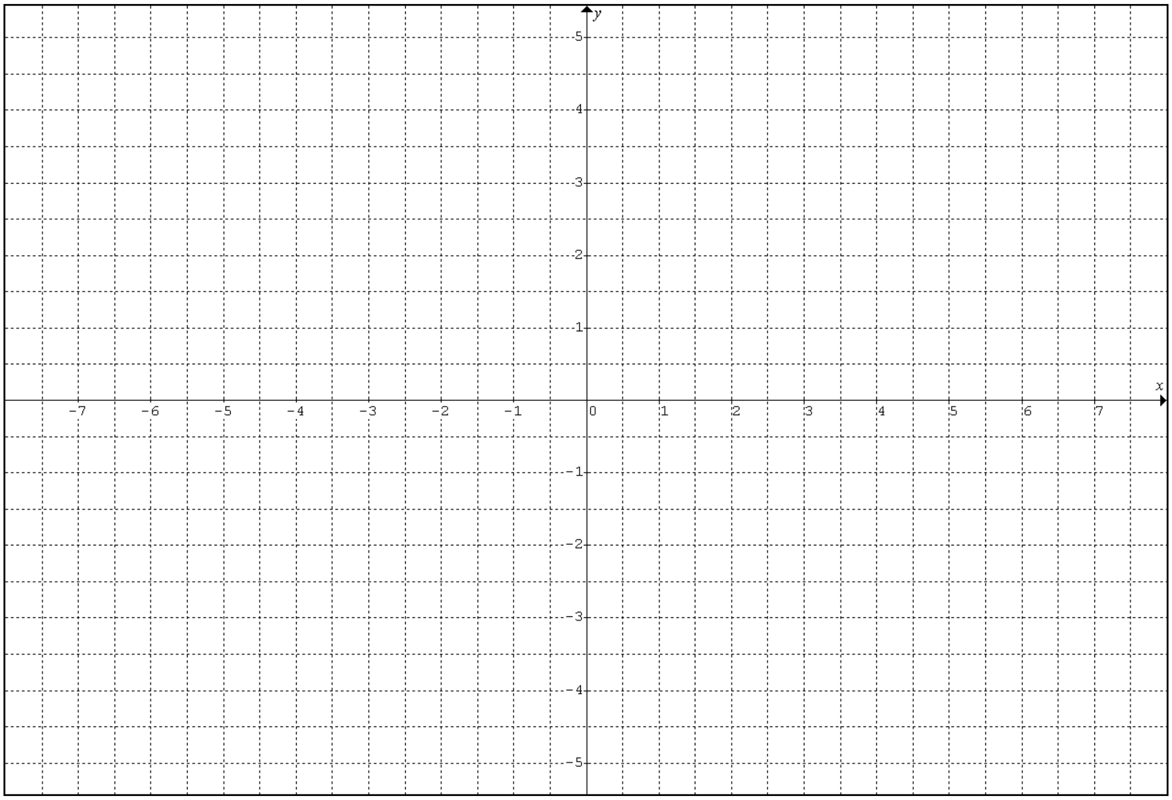

EXAMPLE 3

Q101 Lesson 2 HW

- Carefully **graph** the function. Show work.
- Show the **appropriate limit notation** leading to the horizontal asymptote
- Show the **appropriate limit notation** leading to the vertical asymptote(s).
- Carefully highlight and indicate the **coordinates** of any key points.
- Carefully **label** any asymptotes and axes.

1. $f(x)=\dfrac{x^2+x-6}{x^2-9}=\dfrac{(x-2)(x+3)}{(x-3)(x+3)}$

2. $f(x)=\dfrac{x^2-1}{x^3-3x^2+4}=\dfrac{(x+1)(x-1)}{(x+1)(x-2)^2}$

3. $f(x)=\dfrac{6x^2-x-2}{10x^2+9x+2}=\dfrac{(2x+1)(3x-2)}{(2x+1)(5x+2)}$

4. $f(x)=\dfrac{3x-1}{27x^3-27x^2+9x-1}=\dfrac{3x-1}{(3x-1)^3}$

5. $f(x)=\dfrac{x^2-5x+6}{x^3+x^2-8x-12}=\dfrac{(x-3)(x-2)}{(x-3)(x+2)^2}$

6. $f(x)=\dfrac{14(x^2-81)}{9(x+3)^2(x-4)(x-7)}$

7. $f(x)=\dfrac{2x^2-3x-2}{x-2}=\dfrac{(2x+1)(x-2)}{(x-2)}$

8. $f(x)=\dfrac{5x^2+20x+20}{x^2+4x+4}=\dfrac{5(x+2)^2}{(x+2)^2}$

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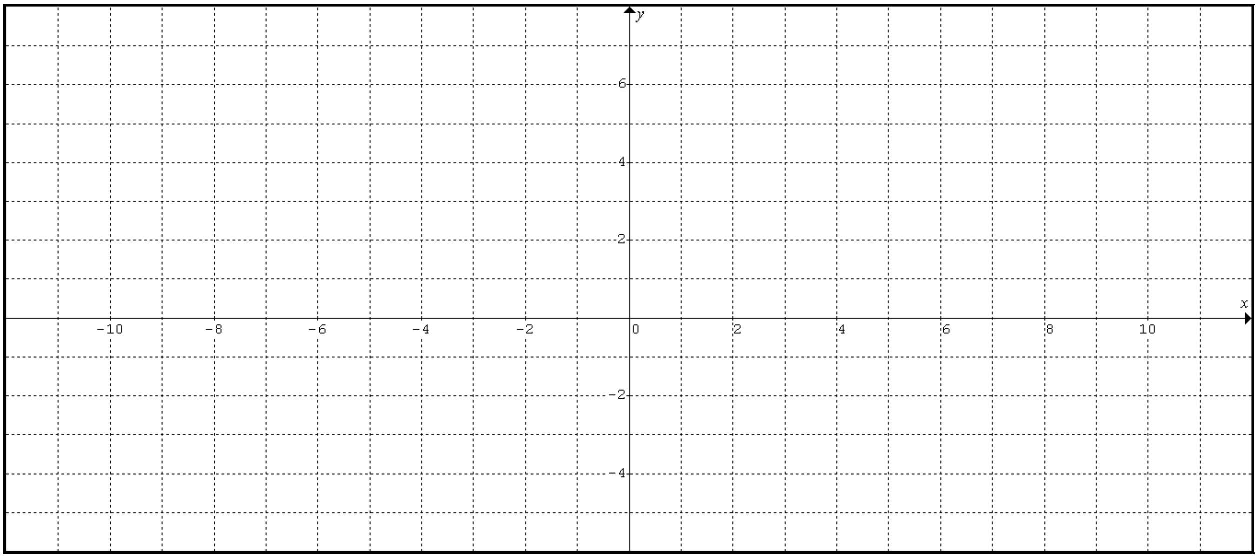

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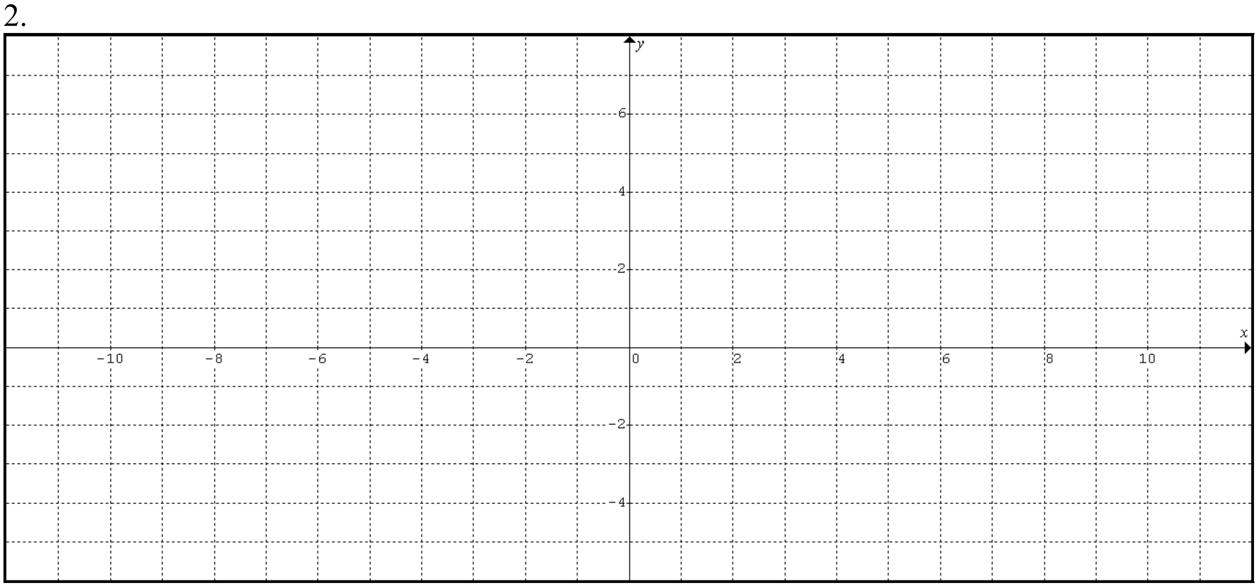

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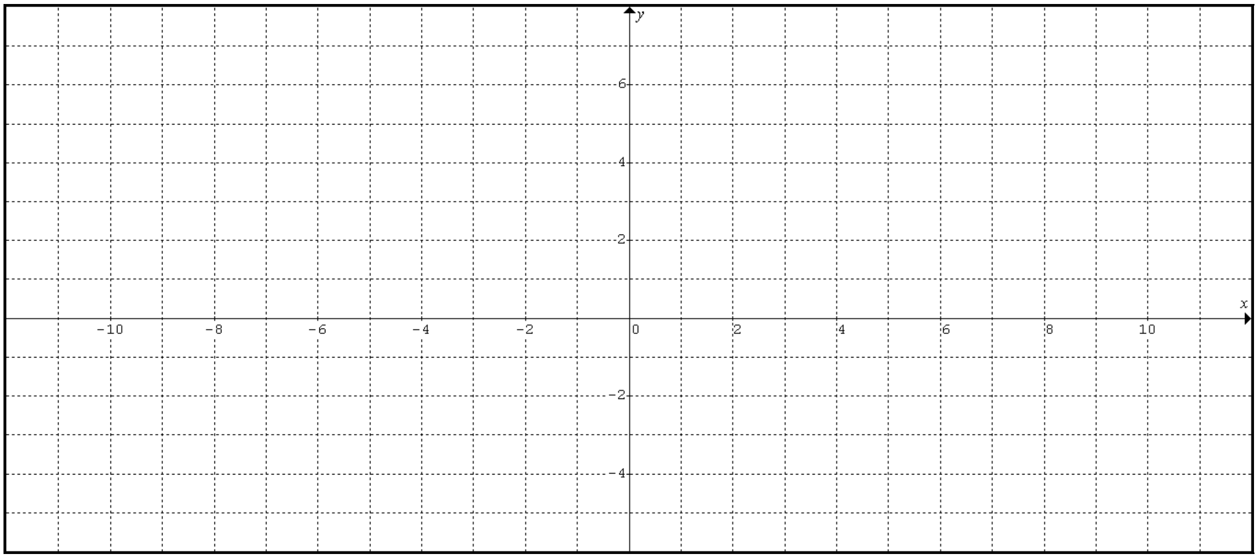

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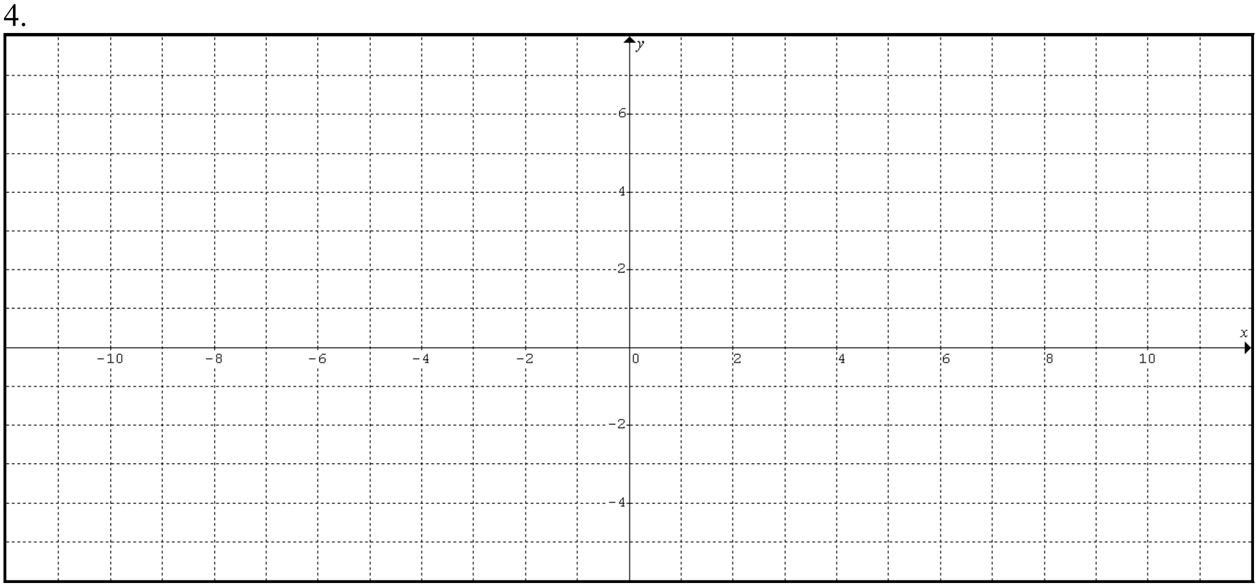

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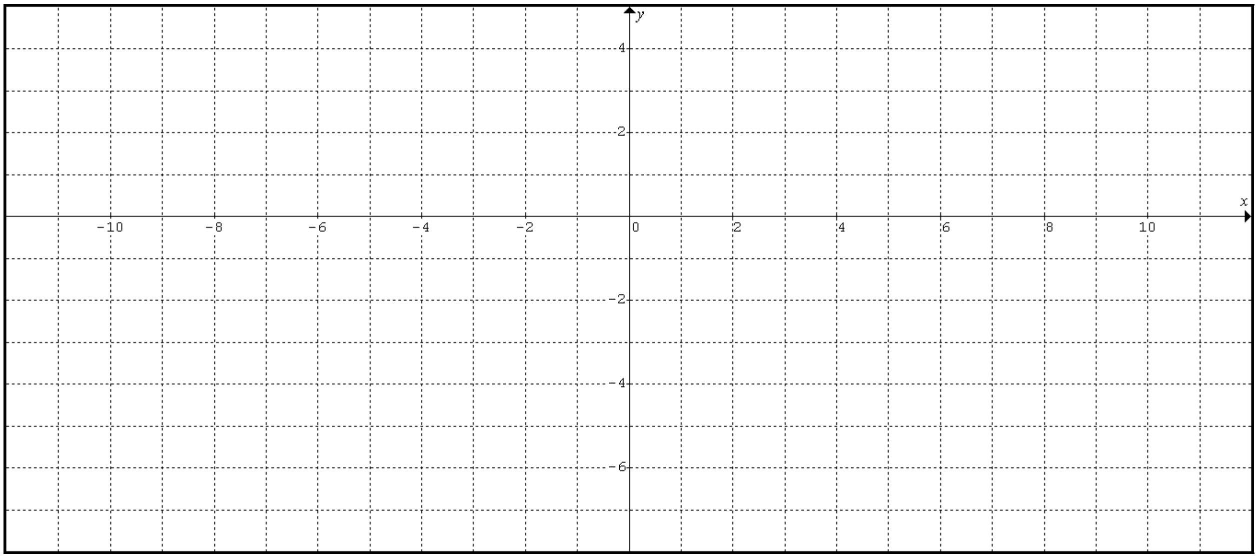

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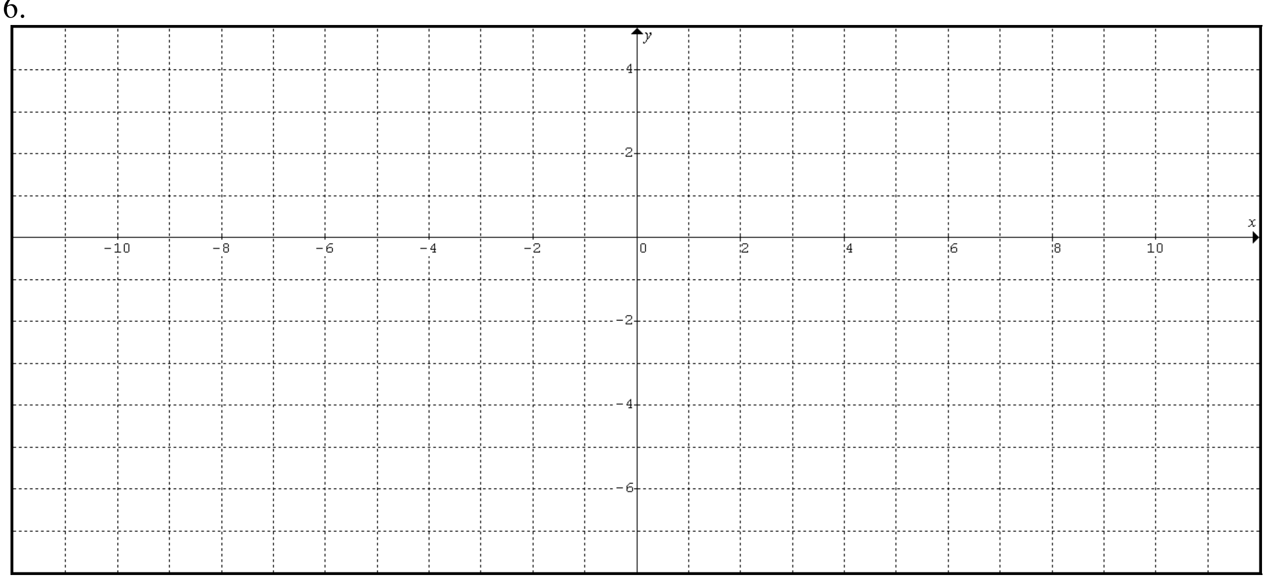

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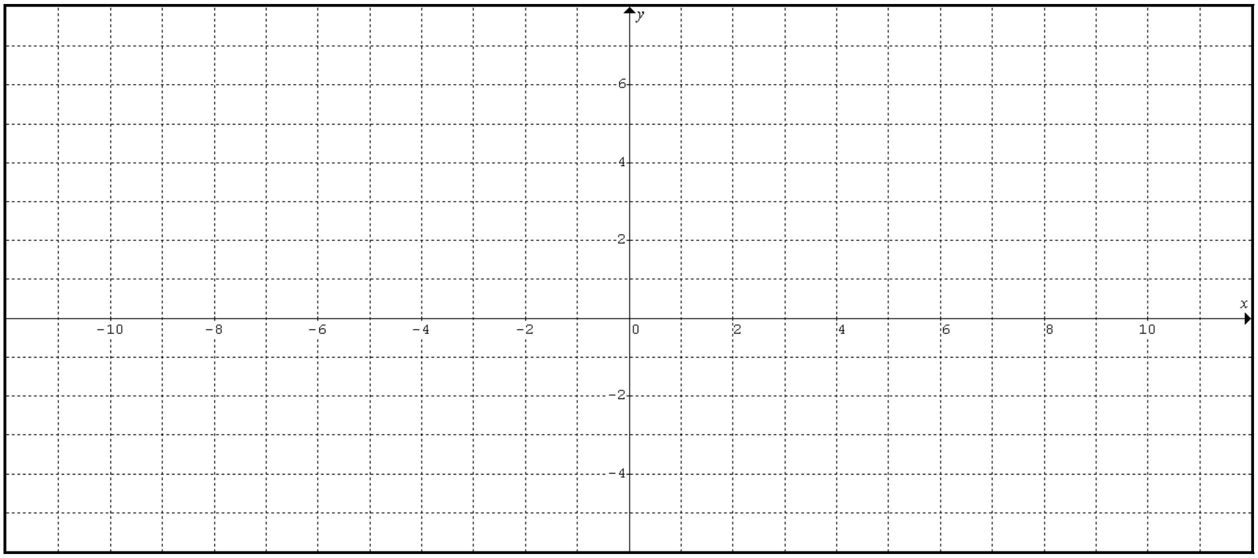

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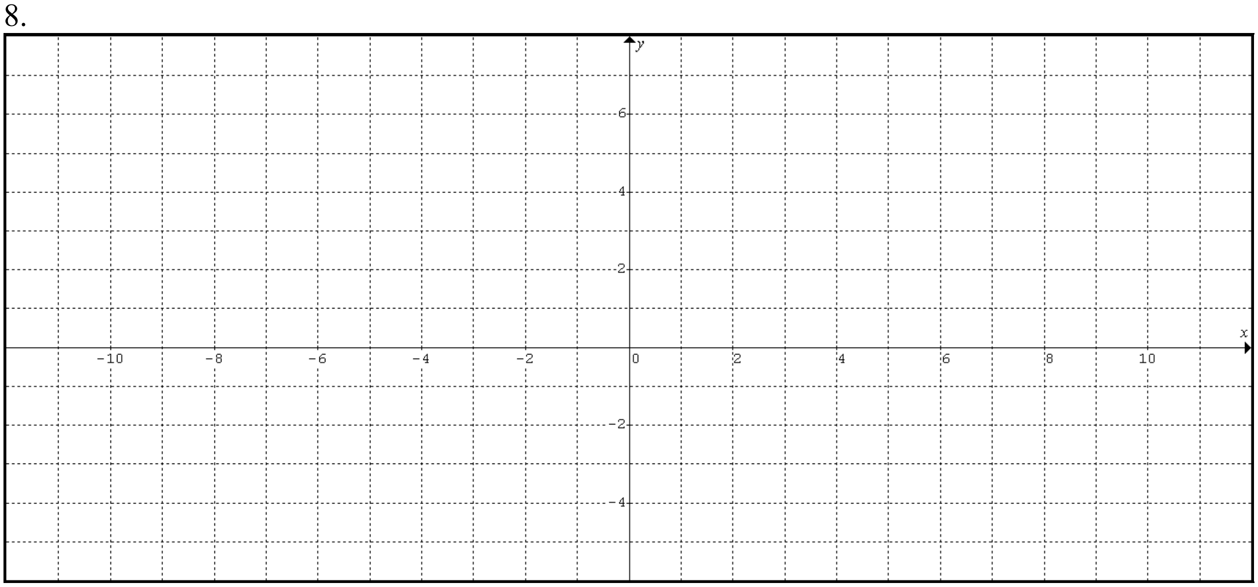

Mixed Review (Trigonometric Essentials)

Evaluate the following:

1. $\sin\left(\frac{5\pi}{6}\right)$

2. $\cos\left(\frac{11\pi}{6}\right)$

3. $\tan\left(\frac{4\pi}{3}\right)$

4. $\sec\left(\frac{2\pi}{3}\right)$

5. $\csc\left(\frac{7\pi}{6}\right)$

Solve each equation on the domain $\left[0, 2\pi\right]$ (answer in radians):

6. $\sin(x) = \frac{1}{2}$

7. $\tan(x) = -\frac{1}{\sqrt{3}}$

8. $\sin(x) + \cos(x) = 0$

9. $\cos^2(x) + \cos(x) = 0$

10. $\sin(x) - 2\cos(x)\sin(x) = 0$