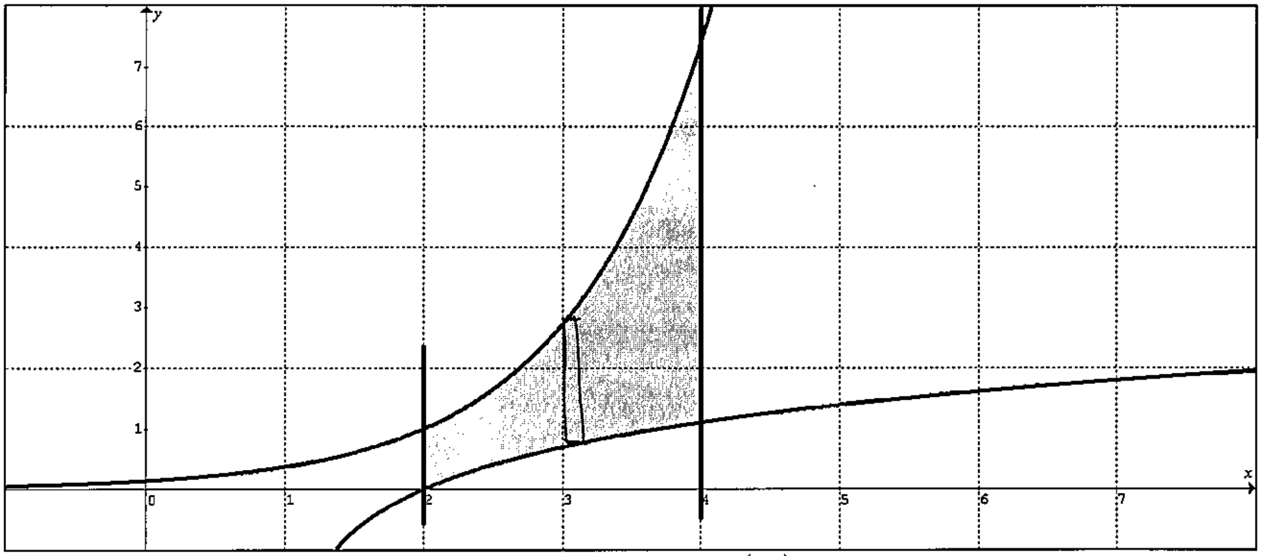
AB: Q502 REVIEW

(The exam will be very similar in format)

$\boxed{\text{Solutions}}$



1: Consider the region R bounded by the graphs of $y = e^{(x-2)}$, $y = \ln(x-1)$, $x = 2$, and $x = 4$ as shown by the shaded region above.

A. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the x axis.

$$V = \pi \int_2^4 \left[\left(e^{(x-2)}\right)^2 - \left(\ln(x-1)\right)^2 \right] dx$$

B. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the y axis.

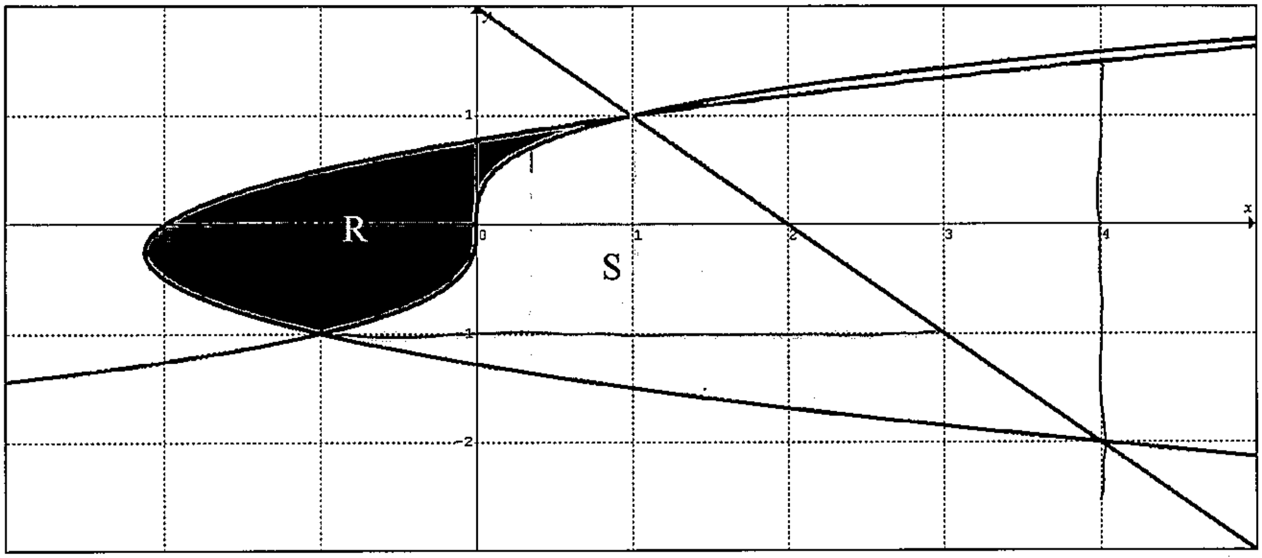
$$V = 2\pi \int_2^4 (x)\left(e^{x-2} - \ln(x-1)\right) dx$$

C. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $x = 16$.

$$V = 2\pi \int_2^4 (16 - x)\left(e^{x-2} - \ln(x-1)\right) dx$$

D. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $y = 16$.

$$V = \pi \int_2^4 \left[\left(16 - \ln(x-1)\right)^2 - \left(16 - e^{x-2}\right)^2 \right] dx$$



2: Consider the region **R** bounded by the graphs of $x = 2y^2 + y - 2$ and $y = \sqrt[3]{x}$ as shown by the dark shaded region above. Consider, also, the region **S** bounded by the graphs of $x = 2y^2 + y - 2$, $y = \sqrt[3]{x}$, and $x + y = 2$ as shown by the light shaded region above.

$$x = 2y^2 + y - 2 \qquad x = y^3 \qquad x = 2 - y$$

A. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $x = 4$.

$$V = \pi \int_{y=-1}^{y=1} \left[\left(4 - (2y^2 + y - 2)\right)^2 - \left(4 - y^3\right)^2 \right] dy$$

B. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $y = 4$.

$$V = 2\pi \int_{-1}^{1} (4 - y)\left(y^3 - (2y^2 + y - 2)\right) dy$$

C. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region S is revolved about the line $x = 4$.

$$V = \pi \int_{-2}^{-1} \left[\left(4 - (2y^2 + y - 2)\right)^2 - \left(4 - (2 - y)\right)^2 \right] dy + \pi \int_{-1}^{1} \left[\left(4 - y^3\right)^2 - \left(4 - (2 - y)\right)^2 \right] dy$$

D. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region S is revolved about the line $y = 4$.

$$V = 2\pi \int_{-2}^{-1} (4 - y)\left(2 - y - (2y^2 + y - 2)\right) dy + 2\pi \int_{-1}^{1} (4 - y)\left(2 - y - y^3\right) dy$$

3: If $\dfrac{dP}{dt} = 0.001P(25-P)$ and $P(0) = 15\ldots$ $\quad K = 0.001 \quad L = 25$

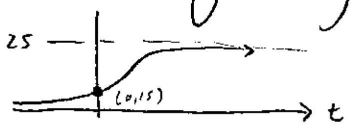
A. Find the function $P(t)$. Provide a quick sketch.

B. Find the value of P when $\dfrac{dP}{dt}$ is at its greatest.

$$P = \frac{25}{1+c^{*}e^{-0.025t}} \longrightarrow 15 = \frac{25}{1+c^{*}} \longrightarrow 1+c^{*} = \frac{25}{15} \longrightarrow c^{*} = \frac{5}{3}-1 = \frac{2}{3}$$

A. $\boxed{P = \dfrac{25}{1+\frac{2}{3}e^{-0.025t}}}$

B. P is growing fastest $P = \dfrac{25}{2} = \boxed{12.5}$



4: If $\dfrac{dP}{dt} = 40P - 5P^2$ and $P(0) = 3\ldots$

A. Find the function $P(t)$. $\longrightarrow \dfrac{dP}{dt} = 5P(8-P) \quad K=5 \quad L=8$

B. Find the value of $\dfrac{dP}{dt}$ when $\dfrac{dP}{dt}$ is at its greatest.

$$P = \frac{8}{1+c^{*}e^{-40t}} \longrightarrow 3 = \frac{8}{1+c^{*}} \longrightarrow 1+c^{*} = \frac{8}{3} \longrightarrow c^{*} = \frac{5}{3}$$

A. $\boxed{P = \dfrac{8}{1+\frac{5}{3}e^{-40t}}}$

B. P is growing fastest when $P = 4$ and $\dfrac{dP}{dt} = 40(4) - 5(16) = \boxed{80}$

5: If $\dfrac{40}{Z}\dfrac{dZ}{dt} = 5 - \dfrac{Z}{60}$ and $Z(0) = 25\ \ldots \longrightarrow \dfrac{dZ}{dt} = \dfrac{Z}{40}\left(5 - \dfrac{Z}{60}\right) \rightarrow \dfrac{dZ}{dt} = \dfrac{Z}{2400}(300 - Z)$

A. Find the function $Z(t)$.

B. Find $\lim\limits_{t\to\infty} Z(t)$. $\quad K = \dfrac{1}{2400} \quad L = 300$

$$Z = \frac{300}{1+c^{*}e^{-\frac{1}{8}t}} \rightarrow 25 = \frac{300}{1+c^{*}} \longrightarrow 1+c^{*} = 12 \longrightarrow c^{*} = 11$$

$\boxed{Z = \dfrac{300}{1+11e^{-\frac{1}{8}t}}}$

B. $\lim\limits_{t\to\infty} Z(t) = \underline{\underline{300}}$

6. Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$

Let $y = h(x)$ be the particular solution to the differential equation with $h(0) = 2$.

A. Use a linearization centered at $x = 0$ to approximate $h(1)$.

B. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $h(1)$.

A] $L(x) = h(0) + h'(0,2)(x-0)$

$L(x) = 2 + \left[0^2 - \frac{1}{2}(2)\right](x)$

$L(x) = 2 - x$

$h(1) \approx L(1) = 2 - 1 = \boxed{1}$

B] $(\overset{x_0}{0}, \overset{y_0}{2})$ $\hat{y}_1 = 2 + \left[0^2 - \frac{1}{2}(2)\right]\underbrace{0.5}_{\Delta x}$ $\Delta x = 0.5$

$= 2 - [1]0.5$

$= \frac{3}{2}$

$(0.5, \frac{3}{2})$ $\hat{y}_2 = \frac{3}{2} + \left[\left(\frac{1}{2}\right)^2 - \frac{1}{2}\left(\frac{3}{2}\right)\right]\underbrace{0.5}_{\Delta x}$

$= \frac{3}{2} + \left[\frac{1}{4} - \frac{3}{4}\right]\frac{1}{2}$

$= \frac{3}{2} + \left[-\frac{1}{2}\right]\frac{1}{2}$

$= \frac{3}{2} - \frac{1}{4}$

$= \frac{6}{4} - \frac{1}{4}$

$= \boxed{\frac{5}{4}}$

$\boxed{h(1) \approx E(1) = \frac{5}{4}}$

7. Consider the differential equation $\frac{dy}{dx} = y^2(2x+2)$

Let $y = f(x)$ be the particular solution to the differential equation with $f(0) = -1$.

A. Use Euler's method, starting at $x = 0$ with two steps of equal size, to approximate $f\left(\frac{1}{2}\right)$.

$\Delta x = \frac{1}{4}$

B. Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

A]. $\begin{matrix} x_0 & y_0 \\ (0, & -1) \end{matrix}$

$$\hat{y}_1 = -1 + \left[(-1)^2(2(0)+2)\right]\frac{1}{4}$$
$$= -1 + [2]\frac{1}{4}$$
$$= -1 + \frac{1}{2} = -\frac{1}{2}$$

$\left(\frac{1}{4}, -\frac{1}{2}\right)$

$$\hat{y}_2 = -\frac{1}{2} + \left[\left(\frac{-1}{2}\right)^2\left(2\frac{1}{4}+2\right)\right]\frac{1}{4}$$
$$= -\frac{1}{2} + \left[\frac{1}{4}\left(\frac{5}{2}\right)\right]\frac{1}{4}$$
$$= -\frac{1}{2} + \left[\frac{5}{32}\right] = -\frac{16}{32} + \frac{5}{32} = \frac{-11}{32}$$

$$f\left(\frac{1}{2}\right) \approx E\left(\frac{1}{2}\right) = \boxed{\frac{-11}{32}}$$

B]

$$\int \frac{dy}{y^2} = \int (2x+2)\,dx$$
$$\int y^{-2}\,dy = \int (2x+2)\,dx$$
$$\frac{y^{-1}}{-1} = x^2 + 2x + c$$

$$\frac{-1}{y} = -x^2 - 2x + c$$
$$y = \frac{1}{-x^2 - 2x + c}$$
$$-1 = \frac{1}{c} \quad \therefore c = -1$$
$$y = \frac{1}{-x^2-2x-1} \quad \text{or} \quad \frac{-1}{x^2+2x+1}$$