# Review Solutions

① $\int x^2 \ln x \, dx = \frac{x^3}{3}\ln x - \int \frac{x^2}{3}\,dx = \frac{x^3}{3}\ln x - \frac{x^3}{9} + C$

$u = \ln x \quad dv = x^2$

$du = \frac{dx}{x} \quad v = \frac{x^3}{3}$

NOTE: $u = x^2 \quad dv = \ln x$ not wise b/c $\int \ln x \, dx$ not straight forward.

② $\int x^2 e^{-2x}\,dx = -\frac{1}{2}x^2 e^{-2x} - \frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x} + c$

| | | |
|---|---|---|
| $x^2$ | $+$ | $e^{-2x}$ |
| $2x$ | $-$ | $-\frac{1}{2}e^{-2x}$ |
| $2$ | $+$ | $\frac{1}{4}e^{-2x}$ |
| $0$ | | $-\frac{1}{8}e^{-2x}$ |

$u = 2x+1 \quad du = 2dx \rightarrow dx = \frac{du}{2}$

③ $\int \frac{11x+2}{2x^2-5x-3}\,dx = \int \frac{dx}{2x+1} + \int \frac{5}{x-3}\,dx = \frac{1}{2}\ln|2x+1| + 5\ln|x-3| + C$

$\frac{11x+2}{2x^2-5x-3} = \frac{A}{2x+1} + \frac{B}{x-3} = \frac{1}{2x+1} + \frac{5}{x-3}$

$A(x-3) + B(2x+1) = 11x + 2$

$(A+2B)x - 3A + B = 11x + 2$

$A + 2B = 11 \qquad B = 5 \quad A = 1$

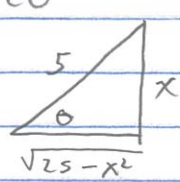
$-3A + B = 2$

④ $\int \frac{dx}{x^2\sqrt{25-x^2}} = \int \frac{5\cos\theta \, d\theta}{25\sin^2\theta\sqrt{25-25\sin^2\theta}} = \int \frac{\cos\theta}{25\sin^2\theta\cos\theta}\,d\theta$

$x = 5\sin\theta$

$dx = 5\cos\theta\,d\theta$

$= \int \frac{1}{25}\csc^2\theta\,d\theta = \frac{-1}{25}\cot\theta + C$




$= \frac{-1}{25}\frac{\sqrt{25-x^2}}{x} + C$

PARTIAL $\displaystyle\int \frac{dx}{1-x^2} = \int \frac{1/2\,dx}{1-x} + \int \frac{1/2\,dx}{1+x} = -\frac{1}{2}\ln|1-x| + \frac{1}{2}\ln|1+x| + C$ ✓

TRIG SUB $\displaystyle\int \frac{dx}{1-x^2} = \int \frac{\cos\theta\,d\theta}{1-\sin^2\theta} = \int \frac{d\theta}{\cos\theta} = \int \sec\theta\,d\theta$

$x = \sin\theta$

$dx = \cos\theta\,d\theta$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|\frac{1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}}\right| + C \checkmark$$

$$= \ln\left|\frac{1+x}{\sqrt{1-x^2}}\right| + C = \ln|1+x| - \frac{1}{2}\ln|1-x^2| + C$$

$$= \ln|1+x| - \frac{1}{2}\ln|1-x| - \frac{1}{2}\ln|1+x| + C$$

$$= \frac{1}{2}\ln|1+x| - \frac{1}{2}\ln|1-x| + C \quad ☺$$

SAME!