Q304

~~Q401~~ CH 5 PRACTICE EXAM – CALCULATOR SECTION

1. (a) $\int_0^{12} P(t)\,dt = \Delta t\,[P(2) + P(6) + P(10)] = 4\,[46 + 57 + 62] = 660 \text{ ft}^3$

   (b) $\int_0^{12} R(t)\,dt = 305 \text{ ft}^3$

   (c) Amount $= 1000 + 660 - 305 = 1355 \text{ ft}^3$

   (d) $A(t) = 1000 + \int_0^t [P(u) - R(u)]\,du$

   (e) $A'(t) = P(t) - R(t)$ $\quad A'(4) = P(4) - R(4) = 53 - 0.556 \approx 52$

   $A'(4)$ is the rate at which the pool water volume is changing at $t = 4$ hours.

2. A. Av $W(t) = \frac{1}{15}\int_0^{15} W(t)\,dt \approx \frac{1}{15}[\text{Trap}] = \frac{1}{15}\left[\frac{3}{2}[20 + 2(31) + 2(28) + 2(24) + 2(22) + 21]\right]$

   $= 25.1\ ^\circ C$

   B. $W'(12)$ is the rate of change in temperature (in $^\circ C$ per day) at time $t = 12$ days.

   C. $W'(12) \approx \dfrac{W(15) - W(9)}{15 - 9} = \dfrac{21 - 24}{6} = \dfrac{-1}{2}\ ^\circ C/\text{day}$ (average on a small neighborhood method)

3. FTC-2 (in disguise) $\int_0^3 g'(x)\,dx = g(3) - g(0)$

   $2.019 = 7 - g(0)$ $\quad \therefore \boxed{g(0) = 4.981}$

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4. $g'(x) = f(x)$ The graph we see is the derivative!

   (a) $g(3) = \int_0^3 f(x)\,dx = \frac{\pi(2)^2}{4} - \frac{1}{2}(1) = \boxed{\pi - \frac{1}{2}}$

   $*\ g'(x) = f(x) = 0$ at $x = -2, 2, 4$

   (b) g has a relative max at $x = 2$ b/c $g'(x) = f(x)$ goes from positive to negative at $x = 2$. Also from * above $g'(2) = f(2) = 0$

   (c) $y - g(3) = g'(3)(x - 3) \longrightarrow \boxed{y - (\pi - \tfrac{1}{2}) = -1(x - 3)}$

   $g'(3) = f(3) = -1$

   (d) g has a point of inflection at $x = 0, 3$ b/c $g''(x) = f'(x)$ changes sign at these x-values

   (e) $x = 4$ is not a candidate $\quad x = -2, 2, 5$ are candidates

   endpt $g(-2) = \int_0^{-2} f(x)\,dx = -\int_{-2}^{0} f(x)\,dx = -(\pi) = -\pi$ (could have also eliminated $x = -2$ with explanation)

   $g(2) = \int_0^2 f(x)\,dx = \pi$

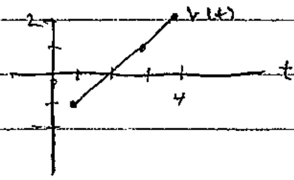
   $g(5) = \int_0^5 f(x)\,dx = \pi - \frac{1}{2}(2) + \frac{1}{2} = \pi - \frac{1}{2}$

   The absolute max is $\underline{\underline{\pi}}$

   (f) $B'(x) = g'(x) + \frac{1}{2} = f(x) + \frac{1}{2} = 0$ when $f(x) = -\frac{1}{2}$ which occurs at $x = 2.5$ and $x = 3.5$. B has a relative minimum at $x = 3.5$ b/c B' goes from negative to positive at $x = 3.5$

5. $x(1)=5 \qquad x'(t)=v(t)=t-2$

A]



You can appeal to geometry:

$$TD = -\int_1^2 v(t)\,dt + \int_2^4 v(t)\,dt = +\frac{1}{2} + \frac{1}{2}(4) = \frac{5}{2}$$

$$-(-1/2) + (2)$$

B] $$x(4) = x(1) + \int_1^4 x'(t)\,dt = 5 + \underbrace{\int_1^4 v(t)\,dt}_{\text{displacement}} = 5 - \frac{1}{2} + 2 = \frac{13}{2}$$

6. $$\left[e^{(3x^2)^3} - \cos^5(3x^2)\right] 6x$$

chain Rule

7. $u = x^2 - 3$

$x=-1 : u=-2$

$x=4 : u=13$

$$\int_{-1}^{4} x(x^2-3)^5\,dx = \frac{1}{2}\int_{-2}^{13} u^5\,du \quad [C]$$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

76 $\quad 78 - 15 = 63$

8. $$\int_{-2}^{1} (x^2-5x)\,dx = \left[\frac{x^3}{3} - \frac{5x^2}{2}\right]_{-2}^{1} = \left(\frac{1}{3} - \frac{5}{2}\right) - \left(\frac{-8}{3} - \frac{5(4)}{2}\right)$$

$$= \frac{2}{6} - \frac{15}{6} + \frac{16}{6} + \frac{60}{6} = \frac{63}{6} = \boxed{\frac{21}{2}}$$

9. $$\text{AV } f(x) = \frac{\int_{-1}^{2} f(x)\,dx}{2-(-1)} = \frac{1}{3}\int_{-1}^{2} 2x^4\,dx = \frac{2}{3}\left[\frac{x^5}{5}\right]_{-1}^{2} = \frac{2}{3}\left(\frac{32}{5} + \frac{1}{5}\right) = \frac{2}{3}\cdot\frac{33}{5}$$

$$= \frac{22}{5}$$

$f(z) = 2z^4 = \frac{22}{5} \qquad z^4 = \frac{11}{5}$

$$\boxed{z = \sqrt[4]{\frac{11}{5}}}$$

$z = -\sqrt[4]{11/5}$ out of domain