NAME:	
DATE:	Solutions

CALCULATOR SECTION
30 minutes

AP CALCULUS AB – Q303 PRACTICE EXAM : CH5A (FTC2)

1[10]. Suppose $f'(x) = \sin(x^2)$ and $f(2) = 3$. Find $f(1)$. Show reasoning.

$$\underbrace{\int_1^2 f'(x)\,dx}_{0.495} = f(2) - f(1)$$

$$0.495 = 3 - f(1)$$

$$f(1) = 3 - 0.495 = \boxed{2.505}$$

2.[25] A water tank at Camp Newton holds 1200 gallons of water at time t = 0. During the time interval $0 \le t \le 18$ hours, water is pumped into the tank at the rate

$W(t) = 95\sqrt{t}\sin^2\left(\frac{t}{6}\right)$ gallons per hour.

During the same time interval, water is removed from the tank at the rate

$R(t) = 275\sin^2\left(\frac{t}{3}\right)$ gallons per hour.

A. Is the amount of water in the tank increasing or decreasing at the time $t = 15$? Justify.

B. Write an equation for $A(t)$, in terms of an integral expression, the total amount of water in the tank at time t on $0 \le t \le 18$.

C. To the nearest whole number, how many gallons of water are in the tank at time $t = 18$?

D. For $t > 18$, no water is pumped into the tank, but water continues to be removed at the rate $R(t)$ until the tank becomes empty. Write, but do not solve, an equation involving an integral expression that can be used to find t, the time it takes for the tank to empty.

A(t) is the amount of water at time t

A'(t) = rate at which the amount of water is changing at time t

$A'(t) = W(t) - R(t)$ — $W(t)$ = rate <u>into</u> tank, $R(t)$ = rate <u>out</u> of tank

A] $A'(15) = W(15) - R(15) = -121.09$ gallons/hr ∴ the water is decreasing at a rate 121.09 gal/hr

B] $A(t) = A(0) + \int_0^t [W(u) - R(u)]\,du$

$= 1200 + \int_0^t [W(u) - R(u)]\,du$

C] $A(18) = 1200 + \int_0^{18} (W(t) - R(t))\,dt = 1310$ gallons

D] $1310 - \int_{18}^t R(u)\,du = 0$ or $\int_{18}^t R(u)\,du = 1310$

Multiple Choice [5 points each]

81. If $G(x)$ is an antiderivative for $f(x)$ and $G(2)=-7$, then $G(4)=$

(A) $f'(4)$

(B) $-7+f'(4)$

(C) $\int_2^4 f(t)\,dt$

(D) $\int_2^4 (-7+f(t))\,dt$

(E) $\boxed{-7+\int_2^4 f(t)\,dt}$

Could be x instead of t.

$$\int_2^4 f(x)\,dx = G(4)-G(2)$$

$$= G(4)+7$$

$$\therefore G(4) = -7+\int_2^4 f(x)\,dx$$

87. An object traveling in a straight line has position $x(t)$ at time t. If the initial position is $x(0)=2$ and the velocity of the object is $v(t)=\sqrt[3]{1+t^2}$, what is the position of the object at time $t=3$?

(A) 0.431 (B) 2.154 (C) 4.512 (D) 6.512 (E) 17.408

$$x(3) = x(0) + \int_0^3 v(t)\,dt = 2+\int_0^3 v(t)\,dt = 6.512$$

91. What is the average value of $y=\dfrac{\cos x}{x^2+x+2}$ on the closed interval [−1, 3]?

(A) −0.085 (B) 0.090 (C) 0.183 (D) 0.244 (E) 0.732

$$\text{Av. value of } y = \frac{\int_{-1}^{3} \frac{\cos x}{x^2+x+2}}{3-(-1)} = \frac{1}{4}\int_{-1}^{3} \frac{\cos x}{x^2+x+2}\,dx = 0.183$$

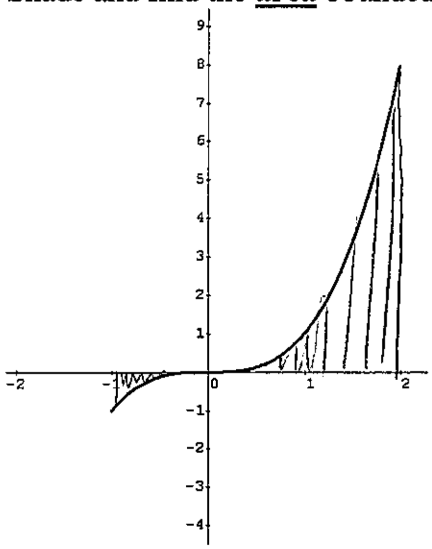
$=$

NAME: Solutions
DATE:

NON CALCULATOR SECTION
30 minutes

AP CALCULUS AB – Q303 PRACTICE EXAM : CH5A (FTC2)

1[20]. Below is a graph of $y = x^3$ from $x = -1$ to $x = 2$.
Shade and find the **area** bounded by graph of y and the x-axis. Show work.



$$A = -\int_{-1}^{0} x^3 dx + \int_{0}^{2} x^3 dx$$

dont forget the anti-derivative!

$$= -\left[\frac{x^4}{4}\right]_{-1}^{0} + \left[\frac{x^4}{4}\right]_{0}^{2}$$

$$= -\left[0 - \frac{1}{4}\right] + \left[4 - 0\right]$$

$$= \frac{1}{4} + 4 = 4\frac{1}{4} \text{ or } \boxed{\frac{17}{4}}$$

2[5].

Using the substitution $u = x^2 - 3$, $\int_{-1}^{4} x(x^2-3)^5\, dx$ is equal to which of the following?

(A) $2\int_{-2}^{13} u^5\, du$

(B) $\int_{-2}^{13} u^5\, du$

(C) $\frac{1}{2}\int_{-2}^{13} u^5\, du$

(D) $\int_{-1}^{4} u^5\, du$

(E) $\frac{1}{2}\int_{-1}^{4} u^5\, du$

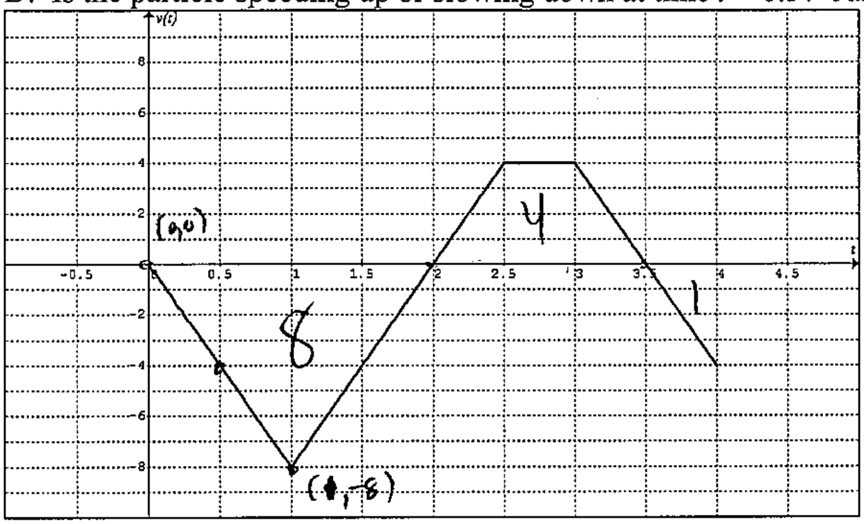
$u = x^2 - 3 \quad du = 2x\,dx \quad dx = \frac{du}{2x}$

$x = -1 : u = 1 - 3 = -2$

$x = 4 : u = 16 - 3 = 13$

$$\frac{1}{2}\int_{-2}^{13} u^5\, du$$

3[25]. A particle moves along the x-axis so that its velocity $v(t)$ at time $t \geq 0$ is given by the graph below.
The position of the particle at time t is $x(t)$ and its position at time $t = 0$ is $x(0) = 5$.
Be sure to show the appropriate set up for each part.
A. Find the total distance traveled by the particle from time $t = 0$ to $t = 4$.
B. Find the position of the particle at time $t = 4$.
C. Find the velocity and acceleration of the particle at time $t = 0.5$.
D. Is the particle speeding up or slowing down at time $t = 0.5$? Justify.



A] Total distance $= \int_0^4 |v(t)|\,dt = \frac{1}{2}(2\cdot 8) + \frac{1.5+0.5}{2}\cdot 4 + \frac{1}{2}(0.5\cdot 4)$
$= 8 + 4 + 1 = \boxed{13}$

B] $x(4) = x(0) + \int_0^4 v(t)\,dt = 5 + \int_0^4 v(t)\,dt = 5 - 8 + 4 - 1$
$= \boxed{0}$ ← position origin

C] $v(0.5) = -4$ (point on $y = v(t)$ graph)
$a(0.5) = -8$ (slope of $y = v(t)$ graph)

D] The particle is speeding up at $t = 0.5$ because the velocity and acc. are the same sign at $t = 0.5$ (as seen in part C)