AB Q302 Practice Review Solutions

#1 $\frac{dy}{dx} = 6(14-y)$

A. $\int \frac{dy}{14-y} = \int 6\,dx$

$-\ln|14-y| = 6x + C$

$\ln|14-y| = -6x + C$

$|14-y| = Ce^{-6x}$

$14-y = C^* e^{-6x}$

$y = 14 - C^* e^{-6x}$

$2 = 14 - C^* \cancel{e^{0}}^{\,1}$

$\therefore C^* = 12$

$$\boxed{y = 14 - 12e^{-6x}}$$

B. $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(6(14-y)\right) = \frac{d}{dx}\left(84-6y\right) = \overset{\text{implicit}}{-6\frac{dy}{dx}} \overset{\text{sub}}{=} -6(84-6y)$

$\left.\frac{d^2y}{dx^2}\right|_{y=4} = -6(84-6(4)) = -360$ ✓

OR

$\left.\frac{d^2y}{dx^2}\right|_{y=4,\ \frac{dy}{dx}=} -6(60) = -360$ ✓

#2 $\frac{dy}{dx} = 6(14+y)$

A. $\int \frac{dy}{14+y} = \int 6\,dx$

$\ln|14+y| = 6x + C$

$|14+y| = Ce^{6x}$

$14+y = C^* e^{6x}$

$y = -14 + C^* e^{6x}$

$2 = -14 + C^* \cancel{e^{0}}^{\,1}$

$C^* = 16$

$$\boxed{y = -14 + 16e^{6x}}$$

B. $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(84+6y\right) = 6\frac{dy}{dx} = 6(84+6y)\Big|_{y=4} = 648$ ✓

#3 $\frac{dy}{dx} = 3x^2y^2$

$\int \frac{dy}{y^2} = \int 3x^2\,dx$

$\int y^{-2}\,dy = \int 3x^2\,dx$

$\frac{y^{-1}}{-1} = x^3 + c$

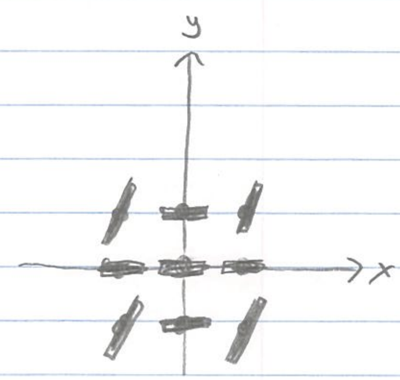
$-\frac{1}{y} = x^3 + c$

$\frac{1}{y} = -x^3 + c$

$y = \frac{1}{-x^3+c}$

$\frac{1}{3} = \frac{1}{0+c} \quad \therefore c = 3$

$$\boxed{y = \frac{1}{-x^3+3}}$$



D: $-x^3+3 \neq 0 \qquad x^3 \neq 3$

$-x^3 \neq -3 \qquad \{x \neq \sqrt[3]{3}\}$

#4 $\frac{dy}{dx} = x^4 - 3y^2 + 6$ CANNOT SEPARATE!

$f(0) = 1$

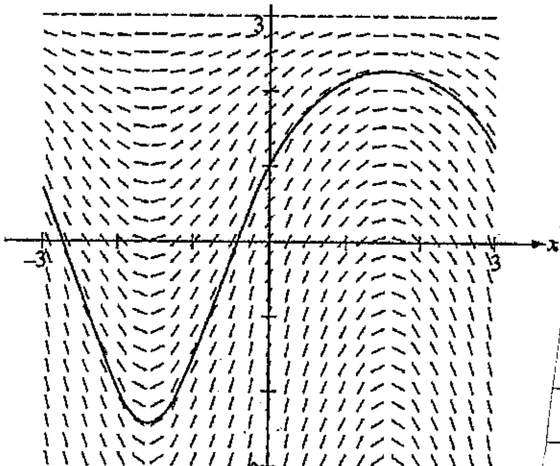
$f'(0,1) = -3(1)^2 + 6 = \boxed{3}$

$f''(x,y) = 4x^3 - 6y\frac{dy}{dx}$

$\left.\frac{d^2y}{dx^2}\right|_{x=0\ y=1,\ \frac{dy}{dx}=3} = 4(0)^3 - 6(1)(3) = \boxed{-18} = f''(0,1)$

$f''(0)$

#5



(b) $\left.\frac{dy}{dx}\right|_{x=0,\,y=1} = (3-1)\cos(0) = \boxed{2}$

$L(x) = f(0) + f'(0,1)(x-0)$

$L(x) = 1 + 2(x)$

$f(0.2) \approx L(0.2) = 1 + 2(0.2) = 1 + 0.4 = \boxed{1.4}$

(c) $\int \frac{dy}{3-y} = \int \cos x\, dx$

$-\ln|3-y| = \sin x + c$

$\ln|3-y| = -\sin x + c$

$|3-y| = ce^{-\sin x}$

$3-y = c^* e^{-\sin x}$

$y = 3 - c^* e^{-\sin x}$

$1 = 3 - c^* e^{0}$ $\quad c^* = 2$

$\boxed{y = 3 - 2e^{-\sin x}}$

$\boxed{\#6}$ $\frac{dy}{dx} = x^2 - \frac{1}{2}y$ $y = h(x)$ $h(0) = 2$

A] $L(x) = h(0) + h'(0,2)(x-0)$

$L(x) = 2 + \left[0^2 - \frac{1}{2}(2)\right]x$ $\left.\frac{dy}{dx}\right|_{x=0,\,y=2} = -1$

$L(x) = 2 - x$

$h(1) \approx L(1) = 2 - 1 = \boxed{1}$

B] $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(x^2 - \frac{1}{2}y\right)$

$= 2x - \frac{1}{2}\frac{dy}{dx}$

$\left.\frac{d^2y}{dx^2}\right|_{x=0,\,y=2,\,\frac{dy}{dx}=-1} \longrightarrow = 2(0) - \frac{1}{2}(-1) = \boxed{\frac{1}{2}} > 0$

$\therefore$ The graph of h is concave <u>up</u> at the point $(0,2)$

$\boxed{\#7}$

$$\frac{dy}{dx} = y^2(2x+2) \qquad y = f(x) \quad f(0) = -1$$

A] $L(x) = f(0) + f'(0,-1)(x-0)$

$L(x) = -1 + \left[(-1)^2(2(0)+2)\right]x \qquad \left.\frac{dy}{dx}\right|_{(0,-1)} = 2$

$L(x) = -1 + 2x$

B] $\int \frac{dy}{y^2} = \int (2x+2)\,dx$ ← separated

$\int y^{-2}\,dy = \int (2x+2)\,dx$

$\frac{y^{-1}}{-1} = x^2 + 2x + C$

$y^{-1} = -x^2 - 2x + C$

$\frac{1}{y} = -x^2 - 2x + C$

$y = \frac{1}{-x^2-2x+C}$

$-1 = \frac{1}{-0^2-2(0)+C}$

$-1 = \frac{1}{C} \quad \therefore C = -1$

$$\boxed{y = \frac{1}{-x^2-2x-1}}$$

OR

$$y = \frac{-1}{x^2+2x+1}$$

AB Q302 Practice Review Solutions

#8 $\frac{dR}{dt} = kR$ $\qquad 24 = C^* \cancel{e^0}^{\,1} \quad \therefore C^* = 24$

$\int \frac{dR}{R} = \int k\,dt$ $\qquad$ (a) $\boxed{R = 24e^{kt}}$

$\ln|R| = kt + C$ $\qquad$ (b) $72 = 24e^{2k}$

$|R| = Ce^{kt}$

$R = C^* e^{kt}$ $\qquad 3 = e^{2k} \rightarrow \ln(3) = 2k \rightarrow \boxed{k = \frac{\ln(3)}{2}}$

(c) $R = 24e^{\frac{\ln(3)}{2}t} \Rightarrow R = 24e^{\ln(3)\cdot\frac{t}{2}}$

$\boxed{R = 24(3)^{\frac{t}{2}}}$ $\qquad R(8) = 24(3)^4 = \boxed{1944 \text{ RATS}}$

#9 $\frac{dT}{dt} = k(20 - T)$

A. $\int \frac{dT}{20 - T} = \int k\,dt$

$-\ln|20 - T| = kt + C$

$\ln|20 - T| = -kt + C$

$|20 - T| = Ce^{-kt}$

$20 - T = C^* e^{-kt}$

$T = 20 - C^* e^{-kt}$

$100 = 20 - C^* \cancel{e^0}^{\,1} \qquad C^* = -80$

$\boxed{T = 20 + 80e^{-kt}}$

B. $60 = 20 + 80e^{-3k}$

$40 = 80e^{-3k}$

$\frac{1}{2} = e^{-3k}$

$\ln(\frac{1}{2}) = -3k$

$\boxed{k = \frac{\ln(1/2)}{-3}}$

C. $T = 20 + 80e^{-\frac{\ln(1/2)}{-3}t}$

$= 20 + 80e^{\ln(1/2)\cdot\frac{t}{3}}$

$\boxed{T = 20 + 80(\frac{1}{2})^{t/3}}$

$T(6) = 20 + 80(\frac{1}{2})^2$

$= 20 + 80(\frac{1}{4}) = 20 + 20$

$= \boxed{40°C}$

D. $30 = 20 + 80(1/2)^{t/3}$

$10 = 80(1/2)^{t/3}$

$\frac{1}{8} = \left(\frac{1}{2}\right)^{t/3}$

$\ln(\frac{1}{8}) = \frac{t}{3}\ln(\frac{1}{2})$

$\frac{t}{3} = \frac{\ln(1/8)}{\ln(1/2)} \rightarrow t = \frac{3\ln(1/8)}{\ln(1/2)}$ min

$t = \frac{3\ln(1/8)}{\ln(1/2)} = \frac{3(-\ln(8))}{-\ln(2)} = \frac{3\ln(8)}{\ln(2)} = \frac{3\ln(2)^3}{\ln(2)} = \frac{9\ln(2)}{\ln(2)} = \boxed{9 \text{ min}}$