

AB: Q302 – CHAPTER 6: PRACTICE EXAM

#1 (No Calculator) Let $\frac{dy}{dx} = 6(14 - y)$ where $y(0) = 2$.

A. Solve the differential equation by separating the variables

B. Find $\frac{d^2y}{dx^2}$ in terms of y and use it to find $\frac{d^2y}{dx^2}$ at $y = 4$.

#2 (No Calculator) Let $\frac{dy}{dx} = 6(14 + y)$ where $y(0) = 2$.

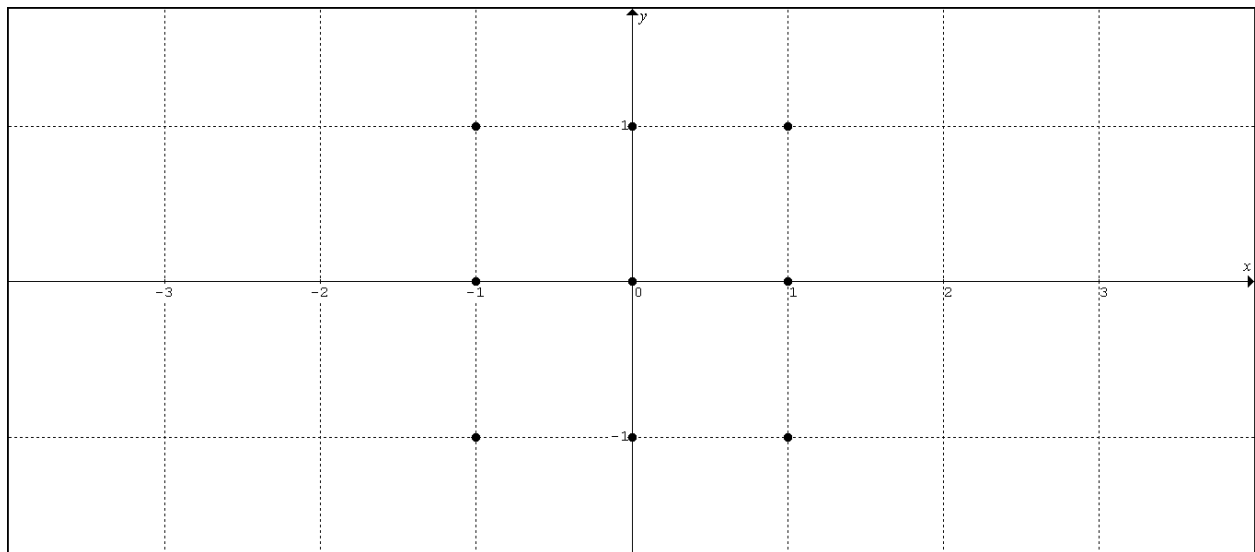
A. Solve the differential equation by separating the variables

B. Find $\frac{d^2y}{dx^2}$ in terms of y and use it to find $\frac{d^2y}{dx^2}$ at $y = 4$.

#3 (No Calculator) Let $\frac{dy}{dx} = 3x^2y^2$ where $y(0) = \frac{1}{3}$.

A. Solve the differential equation by separating the variables.

B. Draw a slope for the indicated point in the graph below.



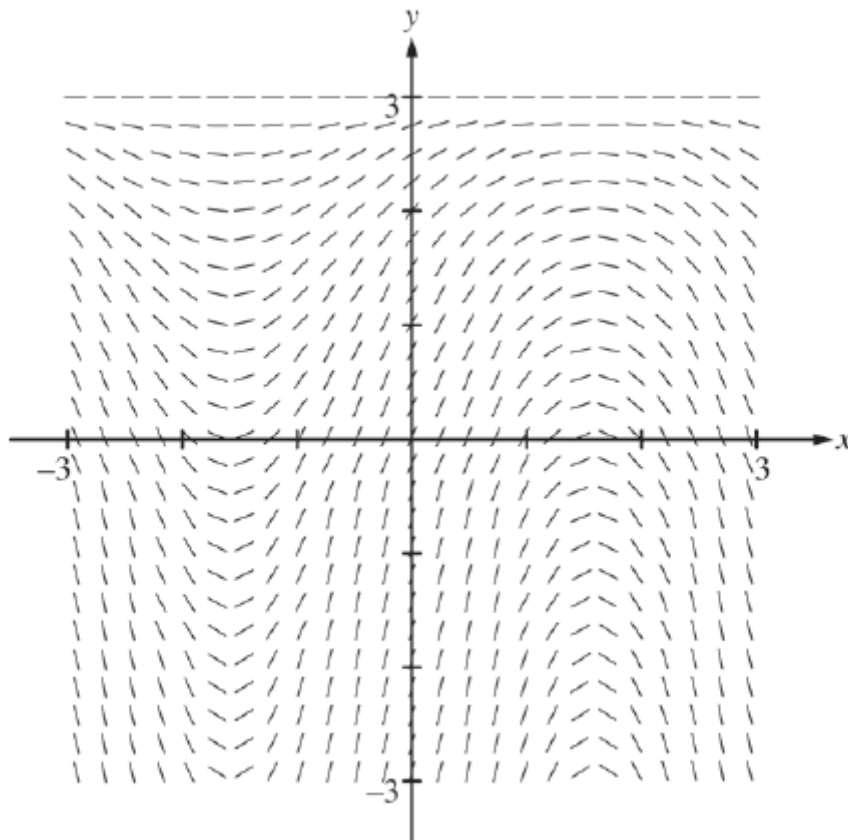
#4 (No Calculator)

Let $\frac{dy}{dx} = x^4 - 3y^2 + 6$ where $y = f(x)$ is the particular solution to the differential equation with the initial condition $f(0) = 1$. Find $f'(0)$ and $f''(0)$.

#5 (No Calculator) Consider the differential equation $\frac{dy}{dx} = (3 - y)\cos(x)$.

Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(0) = 1$. The function f is defined for all real numbers.

(a) A portion of the slope field of the differential equation is given below. Sketch the solution curve through the point $(0, 1)$.



(b) Write an equation for the line tangent to the solution curve in part (a) at the point $(0, 1)$. Use the equation to approximate $f(0.2)$.

(c) Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = 1$.

#6. Consider the differential equation $\frac{dy}{dx} = x^2 - \frac{1}{2}y$

Let $y = h(x)$ be the particular solution to the differential equation with $h(0) = 2$.

- A. Use a linearization centered at $x = 0$ to approximate $h(1)$.
- B. Is the graph of $y = h(x)$ concave upward or concave downward at $x = 0$? Justify.

#7. Consider the differential equation $\frac{dy}{dx} = y^2(2x + 2)$

Let $y = f(x)$ be the particular solution to the differential equation with $f(0) = -1$.

- A. Use a linearization centered at $x = 0$ to approximate $f\left(\frac{1}{2}\right)$.
- B. Find $y = f(x)$, the particular solution to the differential equation with initial condition $f(0) = -1$.

#8. (No Calculator) Let $R(t)$ represent the number of rats in a farm house at time t days. The rate at which the rat population grows is directly proportional to the number of rats present where the constant of proportionality is k .

- A. If there were 24 rats initially, find the function $R(t)$ in terms of t and k .
- B. If there are 72 rats in 2 days, find k .
- C. How many rats would we expect to find after 8 days?

#9. (No Calculator) Let $T(t)$ represent the temperature (degrees Celsius) of tea in a cup at time t minutes, with $t \geq 0$. The temperature $T(t)$ is changing at a rate directly proportional to the difference between the room temperature (20°C) and the temperature of the tea, i.e.

$$\frac{dT}{dt} = k(20 - T) \text{ where the constant of proportionality is } k.$$

- A. If the tea is poured with an initial temperature of 100 degrees Celsius, find $T(t)$ in terms of t and k .
- B. If the tea cools to 60 degrees Celsius in 3 minutes, find k .
- C. What will be the temperature of the tea after 6 minutes?
- D. At what time will the temperature reach 30 degrees Celsius?