AB: Q204: Practice Exam Solutions

1. $A(x) = \underbrace{2x}_{\text{base}}\overbrace{(8\cos(0.3x))}^{\text{height}}$ D: $0 < x < 5.236$

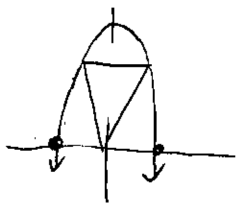
$A'(x) = 0$ at $x = 2.868$

A is max at $x = 2.868$ b/c
A' goes from positive to negative at $x = 2.868$

$\boxed{\text{Answer: } A(2.868) = 29.925 \text{ units}^2}$

2. MAX $A = \frac{1}{2}(\text{base})\text{height} = \frac{1}{2}(2x)(27 - x^2) = x(27 - x^2) = 27x - x^3$

D: $0 < x < \sqrt{27}$



$A(x) = 27x - x^3$ $A'(x) = 27 - 3x^2 = 0$

$x^2 = 9 \rightarrow \cancel{x = -3}$ or $x = 3$

out of domain

$A''(x) = -6x$ $A''(3) = -6(3) = -18 < 0$

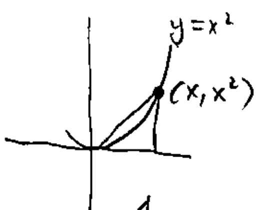
$\therefore$ A is max at $x = 3$

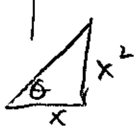
$\boxed{\text{Answer: } A(3) = 27(3) - (3)^3 = 27(2) = 54 \text{ units}^2}$

3. Given $\frac{dx}{dt} = 10$ m/sec

Find $\frac{d\theta}{dt}$ when $x = 3$

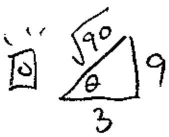
Rel $\tan\theta = \frac{x^2}{x}$





$\tan\theta = x$

$\sec^2\theta \frac{d\theta}{dt} = \frac{dx}{dt}$



$\cos\theta = \frac{3}{\sqrt{90}}$

$\sec\theta = \frac{\sqrt{90}}{3}$

$\left(\frac{90}{9}\right)\frac{d\theta}{dt} = 10$

$\boxed{\frac{d\theta}{dt} = 1 \text{ radian/sec}}$

4. Given $\frac{dV}{dt} = -50 \text{ m}^3/\text{min}$

a) Find $\frac{dh}{dt}$ when $h = 5$m

Rel UP $V = \frac{1}{3}\pi r^2 h$

$\frac{45}{6} = \frac{r}{h}$

$45h = 6r$

$r = \frac{15}{2}h$

$V = \frac{1}{3}\pi\left(\frac{15}{2}h\right)^2 h$

$* \; V = \frac{225}{12}\pi h^3 \; *$

$\frac{dV}{dt} = \frac{225}{4}\pi h^2 \frac{dh}{dt}$

$-50 = \frac{225}{4}\pi(25)\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-8}{225\pi} \text{ m/min}}$

b) Find $\frac{dr}{dt}$ when $h = 5$

R $r = \frac{15}{2}h$

$\frac{dr}{dt} = \frac{15}{2}\frac{dh}{dt}$

$\frac{dr}{dt} = \frac{15}{2} \cdot \frac{-8}{225\pi} = \boxed{\frac{-4}{15\pi} \text{ m/min}}$

5. $f(x) = xe^{2x} + 2x + 4$

$f'(x) = xe^{2x} \cdot 2 + e^{2x}(1) + 2 = 2xe^{2x} + e^{2x} + 2$

$f(0) = 0(e)^0 + 2(0) + 4 = 4$

$f'(0) = 2(0)e^0 + e^0 + 2 = 3$

$L(x) = 4 + 3(x - 0)$

$f(-0.3) \approx L(-0.3) = 4 + 3(-0.3) = \boxed{3.1}$

6. $L(x) = f(1) + f'(1)(x-1)$

$L(x) = 3.156 + (1)(x-1)$

$L(x) = 3.156 + (x-1)$

$f'(1) = \ln(\cos^2(0)) + e^{\sin(0)}$

$= \ln(1) + e^0$

$= 0 + 1$

$= 1$

$f(1.216) \approx L(1.216) = 3.156 + (0.216) = \boxed{3.372}$

7. Ave rate Δ $[-6, \frac{13}{6}]$ $= \dfrac{f(13/6) - f(-6)}{\frac{13}{6} - (-6)} = \dfrac{14 - 14}{\frac{13}{6} + 6} = 0$

Inst. rate Δ $= f'(x) = \begin{cases} 2(x+2) & ; x < 1 \leftarrow [-6, 1) \\ 6 & ; x \geq 1 \leftarrow [1, 13/6] \end{cases}$

$f'(x) = 0$ $\quad 2x + 4 = 0$ or $\cancel{6 = 0}$ $\rightarrow$ $2x + 4 = 0$

$\boxed{x = -2} \in (-6, 1) \in (-6, 13/6)$

8. Ave rate Δ $[-2, 1]$ $= \dfrac{f(1) - f(-2)}{1 - (-2)} = \dfrac{2.334}{3} = 0.778$

Inst. rate Δ $= f'(x) = \cancel{0.785}$ 0.778 at $x = -1.310$ or $x = 0.344$

which both belong to the Interval $(-2, 1)$ ✓

$y1 = \tan^{-1}(e^{x+2}) + \sin(x^2)$

Home: $(y1(1) - y1(-2))/(3)$

Home: $d(y1(x), x)$ ← Copy * you may notice cosh def... this is a special way to write the function.

$y2 = $ paste

$y3 = 0.785$ 〉 Find intersection [or] $y2 = $ paste $- 0.778$ 〉 Find zeros

9. $$\lim_{x\to 0^+} \frac{\ln(x^2+2x)}{\ln(x)} = \lim_{x\to 0^+} \frac{\frac{1}{x^2+2x}(2x+2)}{\frac{1}{x}}$$

$$= \lim_{x\to 0^+} \frac{2x+2}{x^2+2x} \cdot \frac{x}{1} = \lim_{x\to 0^+} \frac{2x+2}{x+2} = \frac{2}{2} = \boxed{1}$$

10. $$\lim_{x\to 0^+} \frac{\tan(x)}{2x} = \lim_{x\to 0^+} \frac{\sec^2(x)}{2} = \boxed{\frac{1}{2}}$$

11. $$\lim_{x\to 0^+} (\sin x)^x \qquad \text{let } y = (\sin x)^x$$

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} x \cdot \ln(\sin x) = \lim_{x\to 0^+} \frac{\ln(\sin x)}{\frac{1}{x}}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{\sin x} \cdot \cos x}{-\frac{1}{x^2}} = \lim_{x\to 0^+} \frac{-\cos x}{\sin x} \cdot x^2$$

$$= \lim_{x\to 0^+} \frac{-\cos x(2x) + x^2(\sin x)}{\cos x} = 0$$

$$\lim_{x\to 0^+} (\sin x)^x = \lim_{x\to 0^+} y = \lim_{x\to 0^+} e^{\ln y} = e^{\lim_{x\to 0^+} \ln y} = e^0 = \boxed{1}$$

12. $$\lim_{x\to 0^+} (\sin x)^{\tan x} \qquad \text{let } y = (\sin x)^{\tan x}$$

$$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \tan x \cdot \ln(\sin x) = \lim_{x\to 0^+} \frac{\ln(\sin x)}{\cot x}$$

$$= \lim_{x\to 0^+} \frac{\frac{1}{\sin x} \cdot \cos x}{-\csc^2 x} = \lim_{x\to 0^+} \frac{\cos x}{\sin x} \cdot \frac{-\sin^2 x}{1} = \lim_{x\to 0^+} \cos x(-\sin x) = 0$$

$$\lim_{x\to 0^+} (\sin x)^{\tan x} = \lim_{x\to 0^+} y = \lim_{x\to 0^+} e^{\ln y} = e^{\lim_{x\to 0^+} \ln y} = e^0 = \boxed{1}$$