AB:Q204. PRACTICE EXAM

OPTIMIZATION

1. CALCULATOR - REQUIRED

Inscribing a Rectangle A rectangle is inscribed under one arch of $y = 8 \cos(0.3x)$ with its base on the x-axis and its upper two vertices on the curve symmetric about the y-axis. What is the largest area the rectangle can have?

2. NO – CALCULATOR

Area of Triangle An isosceles triangle has its vertex at the origin and its base parallel to the x-axis with the vertices above the axis on the curve $y = 27 - x^2$. Find the largest area the triangle can have.

RELATED RATES

3. NO – CALCULATOR

Particle Motion A particle moves along the parabola $y = x^2$ in the first quadrant in such a way that its x-coordinate (in meters) increases at a constant rate of 10 m/sec. How fast is the angle of inclination θ of theline joining the particle to the origin changing when $x = 3$?

4. NO - CALCULATOR

Draining Conical Reservoir Water is flowing at the rate of 50 m^3/min from a concrete conical reservoir (vertex down) of base radius 45 m and height 6 m. **(a)** How fast is the water level falling when the water is 5 m deep? **(b)** How fast is the radius of the water's surface changing at that moment? ~~Give your answer in cm/min.~~

LINEARIZATION

5. NO – CALCULATOR

Let $f(x) = xe^{2x} + 2x + 4$.

Find a linearization of *f* at $x = 0$, and use it to approximate *f* at $x = -0.3$.

6. NO – CALCULATOR

Let *f* be a function with $f(1) = 3.156$ and $f'(x) = \ln(\cos^2(x-1)) + e^{\sin(x-1)}$.

Find a linearization of *f* at $x = 1$, and use it to approximate *f* at $x = 1.216$.

MEAN VALUE THEOREM

7. NO – CALCULATOR

Let $f(x) = \begin{cases} (x+2)^2 - 2 & x < 1 \\ 6x + 1 & x \geq 1 \end{cases}$ on the interval $\left[-6, \frac{13}{6}\right]$

Assuming that *f* satisfies the hypothesis of the mean value theorem (which it does)...

Find the value(s) of *c* that satisfies the conclusion of the Mean Value Theorem.

(Show Work and no decimal answers)

8. CALCULATOR – REQUIRED

Let $f(x) = \tan^{-1}\left(e^{x+2}\right) + \sin\left(x^2\right)$ on the interval $[-2, 1]$

Assuming that *f* satisfies the hypothesis of the mean value theorem (which it does)...

Find the value(s) of *c* that satisfies the conclusion of the Mean Value Theorem.

(Round to three decimal places)

L'HÔPITAL'S RULE

Find each limit, provided it exists. Show work. NO CALCULATOR

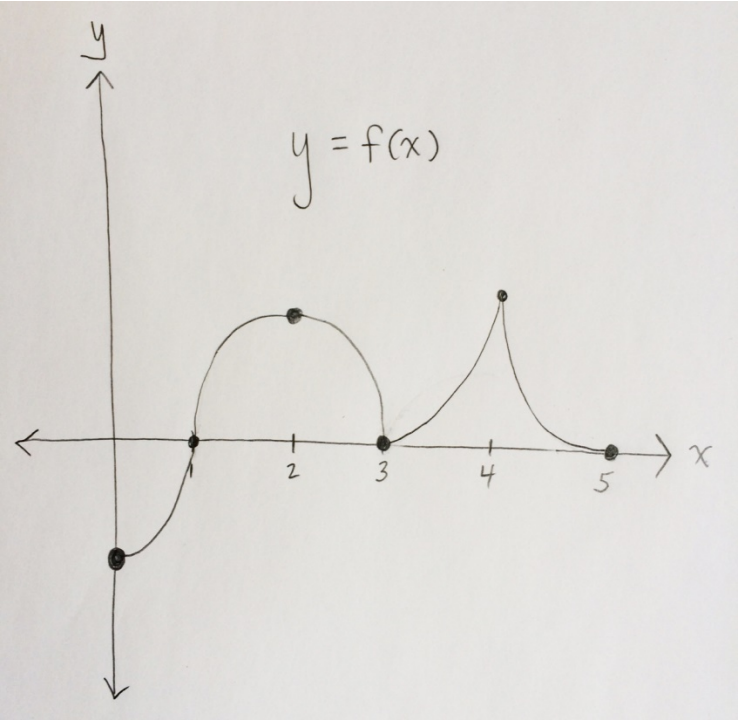
9. $\lim_{x \to 0^+} \frac{\ln\left(x^2 + 2x\right)}{\ln x}$

10. $\lim_{x \to 0^+} \frac{\tan(x)}{2x}$

11. $\lim_{x \to 0^+} (\sin x)^x$

12. $\lim_{x \to 0^+} (\sin x)^{\tan x}$

13. Below is Steven's graph of $y = f(x)$.



THE CHART REPRESENTS STEVEN'S GRAPH

x	0	$0 < x < 1$	1	$1 < x < 2$	2	$2 < x < 3$	3	$3 < x < 4$	4	$4 < x < 5$	5
$f(x)$									+		
$f'(x)$			DNE								
$f''(x)$											

FILL IN EACH BLANK IN THE CHART ABOVE WITH ONE OF THE FOLLOWING:

+ for positive

– for negative

0 for zero

DNE for Does not Exist