Solutions

AB: Q203

BC: Q201 – EXAMINATION REVIEW (Lessons 1 – 3)

TECHNOLOGY SECTION: *Round answers to three decimal places.*

1. The velocity of a particle moving along a horizontal is given as $v(t) = 8\cos(t) + \ln(\sin(t) + t^2)$ on ~~$0 < t \le 8$~~. $0.1 < t \le 8$

A. On what time interval is the particle moving to the right? Justify.

$v(t) = 0$ at $t = 1.746$ and $t = 4.343$

It is ok if you used a parenthesis

moving right on $(\cancel{0}\ 0.1, 1.746) \cup (4.343, 8]$ b/c $v(t) > 0$ on this interval

B. What are the velocity and acceleration at time $t = 5$? Round answers to three decimal places.

$v(5) = 5.449$

$a(5) = 8.099$

C. Is the particle speeding up or slowing down at $t = 3.5$? Justify.

$v(3.5) = -5.015$

$a(3.5) = 3.316$

The particle is slowing down at $t = 3.5$ b/c the velocity and acceleration have different signs at $t = 3.5$

2. The derivative of f is given by $f'(x) = e^{x^2} - 5x^3 + x$ on $0 \le t < 3$

A. On what interval is f decreasing? Justify. $f'(x) = 0$ at $x = 0.824$ and $x = 1.836$

f is decreasing on $[0.824, 1.836]$ b/c $f'(x) < 0$ on $(0.824, 1.836)$

B. At what x-value(s) does f have a relative maximum? Justify. $f'(x) = 0$ at $x = 0.824$

f has a relative max at $x = 0.824$ b/c $f'(x)$ goes from positive to negative at $x = 0.824$

C. On what interval is f concave upward? Justify.

f is concave upward on $(0, 0.344) \cup (1.559, 3)$ b/c $f''(x) > 0$ on this interval.

NO TECHNOLOGY SECTION

1. Let f be defined by $f(x) = \ln(2 + \sin x)$ for $\pi \le x \le 2\pi$.
Find the absolute maximum value and the absolute minimum value of f using the closed interval test.

out of domain

$$f'(x) = \frac{1}{2+\sin x} \cdot \cos x = 0 \qquad \cos x = 0 \qquad x = \cancel{\tfrac{\pi}{2}},\ \frac{3\pi}{2}$$

endpoints at $x = \pi, 2\pi$

$f\left(\frac{3\pi}{2}\right) = \ln(1) = 0$

$f(\pi) = \ln(2)$

$f(2\pi) = \ln(2)$

The abs. max is $\ln 2$

The abs min is 0

2. A. When is the graph of $f(x)$ concave upward if $f''(x) = (x-1)(x+2)^2 e^{x^2}$. Justify.
B. How many points of inflection are on f? Justify.

$f''(x) = 0 \qquad x = 1, -2$

x: -2, 1

$f''(x)$: $-$ $\quad$ $-$ $\quad$ $+$

A. f is concave up on $(1, \infty)$ b/c $f''(x) > 0$ on this interval

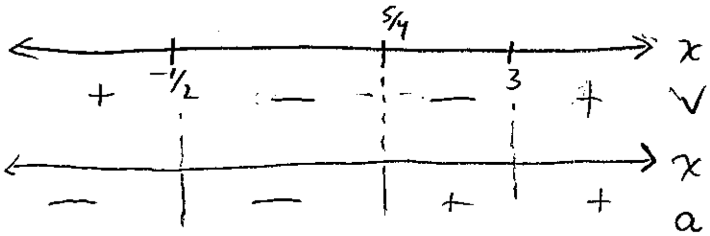
B. There is one point of inflection at $x = 1$ b/c $f''(x)$ changes sign at this x-value

3. A particle moves along a horizontal line. It's position at time t is given as $s(t) = \frac{2}{3}t^3 - \frac{5}{2}t^2 - 3t$.

On what time interval is the particle slowing down? Justify.

$v(t) = 2t^2 - 5t - 3 = 0 \qquad (2t+1)(t-3) = 0 \qquad t = -\frac{1}{2},\ t = 3$

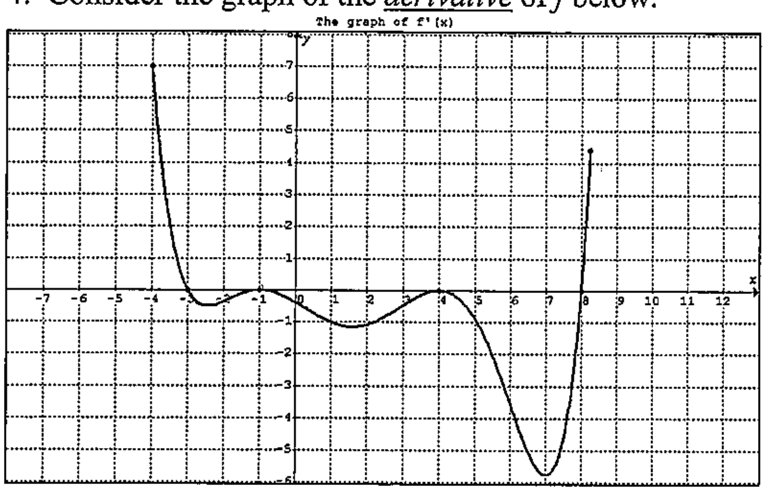
$a(t) = 4t - 5 = 0 \qquad t = \frac{5}{4}$



The particle is slowing down on $(-\infty, -\frac{1}{2}) \cup (\frac{5}{4}, 3)$ b/c $v(t)$ and $a(t)$ have opposite signs on this interval

- on $(-\infty, -\frac{1}{2})$ $v(t) > 0$ and $a(t) < 0$
- on $(\frac{5}{4}, 3)$ $v(t) < 0$ and $a(t) > 0$

4. Consider the graph of the *derivative* of f below.



A. For what x – values does f have a local minimum? Justify.

f has a local min at $x = 8$ b/c $f'(x)$ goes from negative to positive at $x = 8$. $f'(x) = 0$ at $x = 8$. Also f has a min at $x = -4$ b/c f is increasing away from the left endpoint

B. On what interval is f increasing? Justify.

f is increasing on $[-4, -3] \cup [8, 8.25]$ b/c $f'(x) > 0$ on $(-4, -3) \cup (8, 8.25)$

ish

C. On what interval is f concave upward? Justify.

f is concave up on $(-2.5, -1) \cup (1.5, 4) \cup (7, 8.25)$ b/c $f'(x)$ is increasing on this interval.

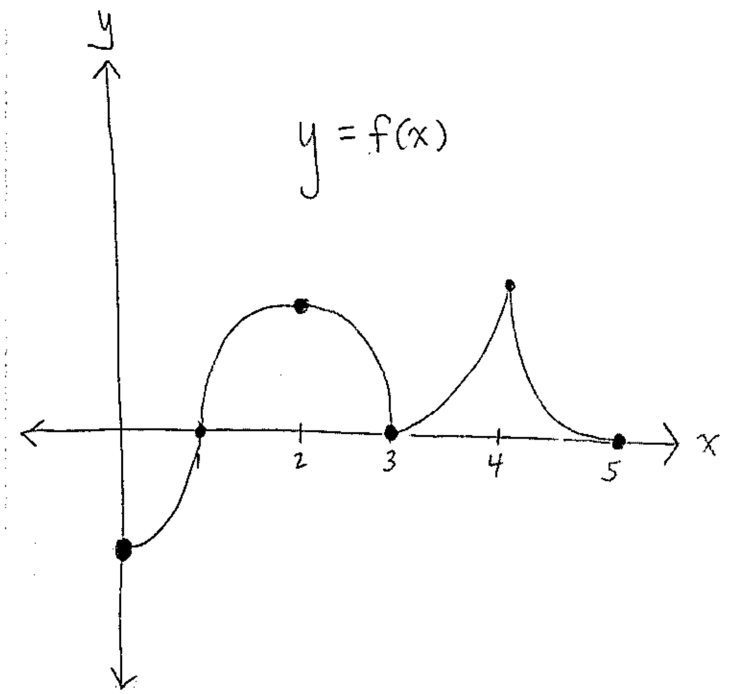
D. How many points of inflection are on f?

f has <u>5</u> points of inflection b/c $f''(x)$ changes sign 5 times.

(f' changes its increasing/decreasing behavior 5 times.)

GRAPH THEORY

5 ~~14~~. Below is Steven's graph of $y = f(x)$.



THE CHART REPRESENTS STEVEN'S GRAPH

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3	$3<x<4$	4	$4<x<5$	5
$f(x)$	−	−	0	+	+	+	0	+	+	+	0
$f'(x)$	0	+	DNE	+	0	−	DNE	+	DNE	−	0
$f''(x)$		+	DNE	−	−	−	DNE	+	DNE	+	

FILL IN EACH BLANK IN THE CHART ABOVE WITH ONE OF THE FOLLOWING:

+ for positive

− for negative

0 for zero

DNE for Does not Exist