Q201 EXAM REVIEW SOLUTIONS

3.7 #1 $\quad y^2 = x^2 - x - 8$

A] $2y\frac{dy}{dx} = 2x - 1 \qquad \frac{dy}{dx} = \frac{2x-1}{2y}$

B] $y = 2 \qquad 4 = x^2 - x - 8 \longrightarrow x^2 - x - 12 = 0 \rightarrow (x-4)(x+3) = 0$

$(4,2) \quad (-3,2) \qquad\qquad x = 4, -3$

C] $\frac{dy}{dx}\Big|_{(4,2)} = \frac{8-1}{4} = \frac{7}{4} \longrightarrow y - 2 = 7/4\,(x-4)$

$\frac{dy}{dx}\Big|_{(-3,2)} = \frac{-6-1}{4} = \frac{-7}{4} \longrightarrow y - 2 = -7/4\,(x+3)$

D] HORIZONTAL TANGENT $\Rightarrow \frac{dy}{dx} = 0 \qquad \frac{2x-1}{2y} = 0$ when $2x - 1 = 0 \rightarrow x = 1/2$

<BUT> $y^2 = (\frac{1}{2})^2 - (\frac{1}{2}) - 8 \rightarrow y^2 = \frac{-33}{4}$ <BUT> $y^2 \neq \frac{-33}{4}$

E] $\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d}{dx}\left[\frac{2x-1}{2y}\right] = \frac{2y(2) - (2x-1)2\left[\frac{dy}{dx}\right]}{(2y)^2} = \frac{4y - (2x-1)2\left[\frac{2x-1}{2y}\right]}{(2y)^2}$ (SUB)

F] $\frac{d^2y}{dx^2}\Big|_{(-3,-2)} = \frac{-8 - (-6-1)(2)\left[\frac{-6-1}{-4}\right]}{(-4)^2} = \frac{-8 - (-7)(2)(\frac{-7}{-4})}{16} = \frac{-8 + \frac{49}{2}}{16}$

$= \frac{-16+49}{32} = \boxed{\frac{33}{32}}$

#2 $\quad x^3 - xy^3 = 18xy$

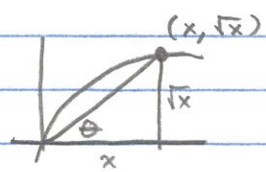
$3x^2 - x(3y^2)\frac{dy}{dx} + y^3(-1) = 18x\frac{dy}{dx} + y(18)$

$(-3xy^2 - 18x)\frac{dy}{dx} = 18y + y^3 - 3x^2 \longrightarrow \boxed{\frac{dy}{dx} = \frac{18y + y^3 - 3x^2}{-3xy^2 - 18x}}$

or

$\frac{3x^2 - 18y - y^3}{18x + 3xy^2}$

4.6 #3



Given| $\frac{dx}{dt} = 8$ m/s

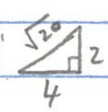
Find| $\frac{d\theta}{dt}$ when $x = 4$

Relationship| $\tan\theta = \frac{\sqrt{x}}{x}$ — simplifies $\rightarrow \tan\theta = x^{-1/2}$

$\sec^2\theta\,\frac{d\theta}{dt} = -\frac{1}{2}x^{-3/2}\,\frac{dx}{dt}$

••• HARDER WAY •••

$\sec^2\theta\,\frac{d\theta}{dt} = \frac{x\cdot\frac{1}{2}x^{-1/2}\frac{dx}{dt} - \sqrt{x}\,\frac{dx}{dt}}{x^2}$



$\left(\frac{\sqrt{20}}{4}\right)^2\frac{d\theta}{dt} = -\frac{1}{2}(4)^{-3/2}(8)$

$\cos\theta = \frac{4}{\sqrt{20}} \qquad \frac{20}{16}\frac{d\theta}{dt} = -\frac{1}{2}\left(\frac{1}{8}\right)(8)$

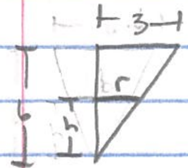
$\sec\theta = \frac{\sqrt{20}}{4} \qquad d\theta/dt = -\frac{1}{2}\cdot\frac{16}{20} = \frac{-8}{20} = \frac{-2}{5}$ radians/sec

4. A] CONE: Given| $\frac{dV}{dt} = -10 \text{ in}^3/\text{min}$

Find| dh/dt when $h = 5$

Rel| $V = \frac{1}{3}\pi r^2 h$

UPDATE| $V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 \cdot h$



$* \; V = \frac{\pi}{12} h^3$

$\frac{3}{6} = \frac{r}{h}$

$6r = 3h$

$r = \frac{h}{2}$

$\frac{dV}{dt} = \frac{\pi}{4} h^2 \frac{dh}{dt}$

$-10 = \frac{\pi}{4}(5)^2 \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-40}{25\pi} = \frac{-8}{5\pi}$ in/min ← h is decreasing

B] CAN: Given| $\frac{dV}{dt} = +10 \text{ in}^3/\text{min}$

Find| $\frac{dh}{dt}$ when h = DOES NOT MATTER

Rel| $V = \pi r^2 h$ BUT "r" always $= 3$

$* \; V = 9\pi h$

$\frac{dV}{dt} = 9\pi \frac{dh}{dt}$ ... see does not depend on h

$10 = 9\pi \frac{dh}{dt}$

$\frac{dh}{dt} = \frac{10}{9\pi}$ in/min ← h is increasing

5. A.P QUESTION

a) $xy^2 - x^3y = 6$

$$x\left(2y\frac{dy}{dx}\right) + y^2(1) - x^3\left(\frac{dy}{dx}\right) + y\left(-3x^2\right) = 0$$

$$\frac{dy}{dx}\left(2xy - x^3\right) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3} \qquad 2xy - x^3 \neq 0$$

b) $(1)y^2 - (1)^3y = 6 \qquad y^2 - y = 6 \qquad y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$y = 3,\ y = -2$

POINT	$\frac{dy}{dx}$	
$(1,3)$:	$\frac{3(3)-9}{6-1} = 0$	$\boxed{y = 3}$
$(1,-2)$:	$\frac{3(-2)-4}{-4-1} = \frac{-10}{-5} = 2$	$\boxed{y+2 = 2(x-1)}$

c) vertical when $\frac{dy}{dx}$ DNE when $2xy - x^3 = 0$

$$2xy = x^3$$
$$2y = x^2$$
$$y = x^2/2$$

$$x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6$$

$$\frac{x^5}{4} - \frac{x^5}{2} = 6$$

$$\frac{x^5}{4} - \frac{2x^5}{4} = 6$$

$$-\frac{x^5}{4} = 6$$

$$-x^5 = 24$$

$$x = \sqrt[5]{-24}$$

6] $2\cos(xy^2) + y = x^2 y$

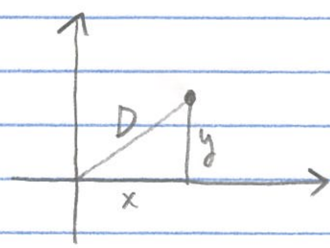
$$-2\sin(xy^2)\left[x \cdot 2y\frac{dy}{dx} + y^2(1)\right] + \frac{dy}{dx} = x^2\frac{dy}{dx} + y(2x)$$

$$-4xy\sin(xy^2)\frac{dy}{dx} - 2y^2\sin(xy^2) + \frac{dy}{dx} = x^2\frac{dy}{dx} + 2xy$$

$$\frac{dy}{dx}\left(-4xy\sin(xy^2) + 1 - x^2\right) = 2xy + 2y^2\sin(xy^2)$$

$$\frac{dy}{dx} = \frac{2xy + 2y^2\sin(xy^2)}{-4xy\sin(xy^2) + 1 - x^2}$$

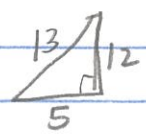
7]



Given] $\frac{dx}{dt} = -1$ m/sec $\quad \frac{dy}{dt} = 5$ m/sec

Find] $\frac{dD}{dt}$ when $x = 5$, $y = 12$

Rel] $x^2 + y^2 = D^2$



$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2D\frac{dD}{dt}$$

$$\cancel{2}(5)(-1) + \cancel{2}(12)(5) = \cancel{2}(13)\frac{dD}{dt}$$

$$\frac{dD}{dt} = \frac{-5 + 60}{13} = \frac{55}{13} \text{ m/s}$$