PRACTICE EXAM

AB.Q103.EXAMINATION – FORM A

Ch 2.4, 3.1, 3.2: Derivative Foundation

NO CALCULATORS

[60 minutes]

NAME: Solutions

DATE:

BLOCK:

1[10]. Consider the function $k(x)=\begin{cases}2x+4; x\le 1\\ x^2-4x+9; x>1\end{cases}$.

Formally prove that k is or is not **continuous at $x = 1$.**

i) $k(1) = 2(1)+4 = 6$

ii) □ $\lim_{x\to 1^+} h(x) = \lim_{x\to 1^+} x^2-4x+9 = 6$

□ $\lim_{x\to 1^-} h(x) = \lim_{x\to 1^-} 2x+4 = 6$

$\therefore \lim_{x\to 1} h(x) = 6$

iii) $\lim_{x\to 1} h(x) = k(1)$

$\therefore$ h is continuous at $x = 1$

2[20]. Suppose $f(x)=\begin{cases}2x-3; x\ge 1\\ x^2-2; x<1\end{cases}$.

Formally prove that $f(x)$ is or is not **differentiable at** $x = 1$.

$$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = \ ?$$

□ $f'_+(1) = \lim_{h\to 0^+} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^+} \frac{2(1+h)-3-[-1]}{h}$

$= \lim_{h\to 0^+} \frac{2h}{h} = \lim_{h\to 0^+} 2 = 2$

□ $f'_-(1) = \lim_{h\to 0^-} \frac{f(1+h)-f(1)}{h} = \lim_{h\to 0^-} \frac{(1+h)^2-2-[-1]}{h}$

$= \lim_{h\to 0^-} \frac{1+2h+h^2-2+1}{h} = \lim_{h\to 0^-} \frac{h(2+h)}{h} = \lim_{h\to 0^-} 2+h = 2$

$\therefore f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h} = 2$

i.e f is differentiable at $x = 1$

3[5]. Consider the <u>continuous and differentiable</u> function $f(x) = \begin{cases} 2x+4; x \geq 1 \\ x^2+5; x<1 \end{cases}$. 3

Find the **average rate of change** of f on [-2,3]. Show work.

$$\underset{[-2,3]}{\text{ave rate } \Delta} = \frac{f(3)-f(-2)}{3-(-2)} = \frac{10-9}{5} = \frac{1}{5}$$

4[20]. Let $g(x)$ be a smooth and continuous function that is not explicitly defined, but whose select values are shown in the table below. The domain for $g(x)$ is $[-4,6]$.

x	-4	-3	-2	0	3	4	5	6
$g(x)$	2	5	0	-2	4	6	-12	-15
$g'(x)$	?	?	?	?	1.8	?	?	?

A. **Estimate** $g'(-3)$, $g'(4.5)$. Show work.

$$g'(-3) \approx \frac{g(-2)-g(-4)}{-2-(-4)} = \frac{0-2}{2} = -1$$

$$g'(4.5) \approx \frac{g(5)-g(4)}{5-4} = \frac{-12-6}{1} = -18$$

average on a small neighborhood method

B. Write an equation of the line tangent to $g(x)$ at $x=3$.

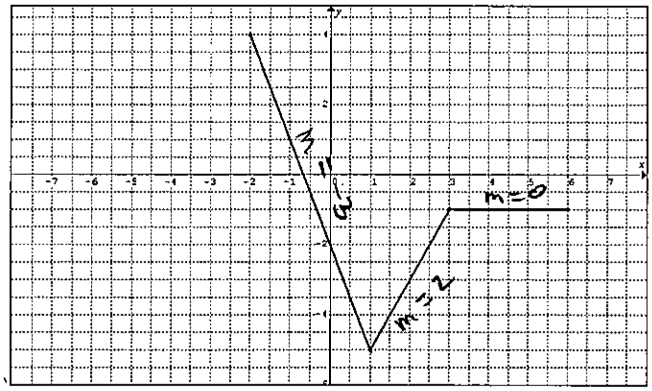
$g'(3) = 1.8 \leftarrow$ given $\quad g(3) = 4 \leftarrow$ given

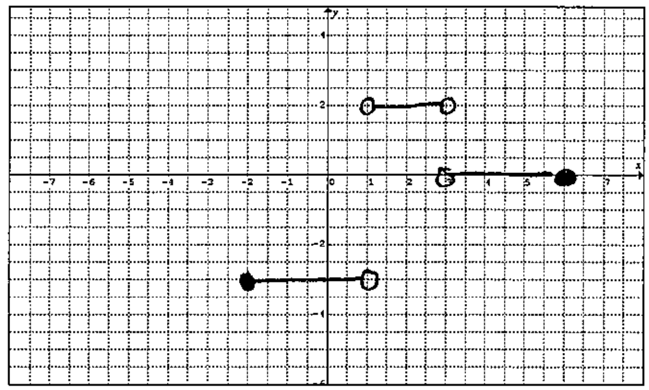
$$\boxed{y-4 = 1.8(x-3)}$$

C. Find the average rate of change in g on $[-4,6]$. Show work.

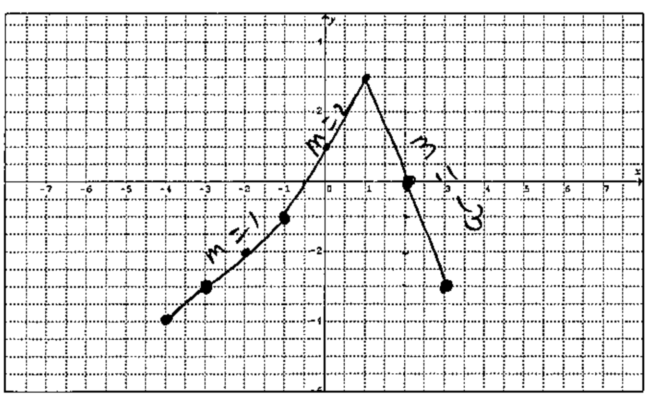
$$\underset{[-4,6]}{\text{av rat } \Delta} = \frac{f(6)-f(-4)}{6-(-4)} = \frac{-15-2}{10} = \frac{-17}{10}$$

5[10]. The graph of $f(x)$ is given below on the left. **Draw** the function $f'(x)$.

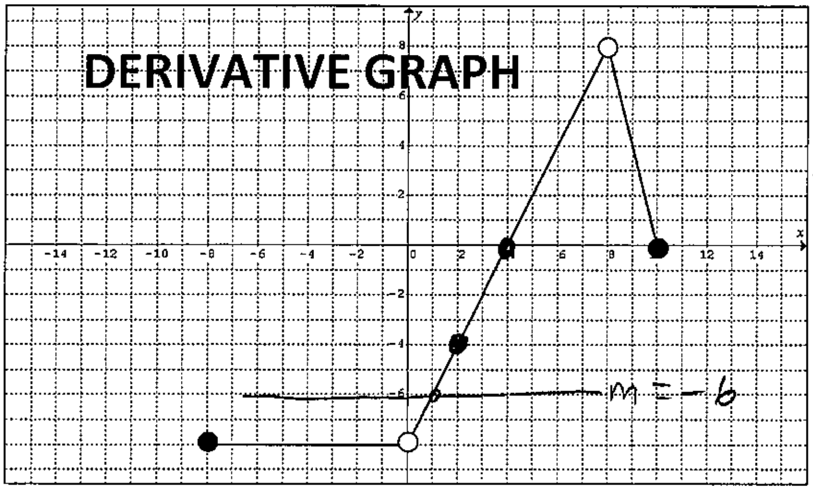




6[10]. **Draw** the function $g(x)$ which is continuous for all points on its domain. The domain of $g(x)$ is [-4, 3], $g(2) = 0$ and $g'(x) = \begin{cases} 1; x < -1 \\ 2; -1 < x < 1. \\ -3; x > 1 \end{cases}$



7[10]. The graph of the ***derivative*** of $f(x)$ is given below.



This is the graph of $f'(x)$

A. If $f(2) = 3$, write an equation of the tangent to the f at $x = 2$

$f'(2) = -4 \leftarrow$ Value on graph of $f'(x)$

$$\boxed{y - 3 = -4(x - 2)}$$

B. For what value(s) of x will f have a horizontal tangent?

$\underline{f'(x) = 0}$ at $x = 4$ and at $x = 10$

see where graph touches x axis

C. For what value(s) of x will f have a tangent line parallel to $y = -6x - 15$

$m = -6$ when $x = 1$

8[15]. Let $f(x) = \dfrac{1}{x+1}$.

A. Use the **definition for the derivative at** $x = a$ to find $f'(2)$.

$$f'(2) = \lim_{h\to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h\to 0} \frac{\frac{1}{3+h} - \frac{1}{3}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{3-(3+h)}{3(3+h)}}{h} = \lim_{h\to 0} \frac{\frac{-h}{3(3+h)}}{h}$$

$$= \lim_{h\to 0} \frac{-1}{3(3+h)} = -\frac{1}{9}$$

B. Write an **equation** for the line **tangent** to $f(x)$ at $x = 2$.

$$f(2) = \frac{1}{3}$$

$$y - \frac{1}{3} = -\frac{1}{9}(x - 2)$$