PRACTICE

AB.Q102.EXAMINATION – FORM A

CH 2 – Limits and Continuity at $x = a$

(100 points)

NO CALCULATORS

[60 minutes]

NAME: Solutions

DATE:

BLOCK:

1[19]. Formally **prove** that $f(x)=\begin{cases}-x^4+3, & x\le 2\\ x^2-17, & x>2\end{cases}$ is or is not continuous at $x=2$.

i) $f(2) = -(2)^4 + 3 = -13$

ii) □ $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} -x^4 + 3 = -13$

□ $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} x^2 - 17 = -13$

$\therefore \lim_{x\to 2} f(x) = -13$

iii) $\lim_{x\to 2} f(x) = f(2)$

$\therefore$ f is continuous at $x=2$

2[19]. Formally **prove** that $f(x)=\begin{cases}3-x, & x\ge -1\\ x^2+1, & x<-1\end{cases}$ is or is not continuous at $x=-1$.

i) $f(-1) = 3-(-1) = 4$

ii) □ $\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} 3-x = 4$

□ $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} x^2+1 = 2$

$\therefore \lim_{x\to -1} f(x)$ (DNE)

iii) $\lim_{x\to -1} f(x) \neq f(-1)$

$\therefore$ f is NOT continuous at $x=-1$

3[19]. Formally **prove** that $f(x)=\begin{cases} x\sin\frac{1}{x^2}; & x\neq 0 \\ 0; & x=0 \end{cases}$ is or is not continuous at $x=0$.

i) $f(0)=0$

ii) $-1 \le \sin\frac{1}{x^2} \le 1$

$-x \le x\sin\frac{1}{x^2} \le x$

$\lim_{x\to 0} x = 0$

$\lim_{x\to 0} -x = 0$

$\therefore \lim_{x\to 0} f(x) = 0$

iii) $\lim_{x\to 0} f(x) = f(0)$

$\therefore$ f is continuous at $x=0$

4[20] Let $f(x)=\dfrac{x-3}{\sqrt{x}-\sqrt{3}}$ and let $g(x)=\begin{cases} f(x); \; x>3 \\ (x-1)\sqrt{\dfrac{x^2}{3}}; \; x\le 3 \end{cases}$.

Formally **prove** that $g(x)$ is or is not continuous at $x=3$.

i) $g(3) = (2)\sqrt{\frac{9}{3}} = 2\sqrt{3}$

ii) □ $\lim_{x\to 3^+} g(x) = \lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \dfrac{x-3}{\sqrt{x}-\sqrt{3}} = \lim_{x\to 3^+} \dfrac{(x-3)(\sqrt{x}+\sqrt{3})}{(\sqrt{x}-\sqrt{3})(\sqrt{x}+\sqrt{3})}$

$= \lim_{x\to 3^+} \dfrac{(x-3)(\sqrt{x}+\sqrt{3})}{x-3}$ Prop limit when you substitute

$= \lim_{x\to 3^+} \sqrt{x}+\sqrt{3} = \sqrt{3}+\sqrt{3} = 2\sqrt{3}$

□ $\lim_{x\to 3^-} g(x) = \lim_{x\to 3^-} (x-1)\sqrt{\dfrac{x^2}{3}} = 2\sqrt{3}$

Prop limit when you substitute

$\therefore \lim_{x\to 3} g(x) = 2\sqrt{3}$

iii) $\lim_{x\to 3} g(x) = g(3)$

$\therefore$ g is Continuous at $x=3$

5[5]. Formally **prove** that $g(x)=\sqrt{x-5}$ is or is not continuous at $x=5$?

Def^n: continuous at left endpoint $x=a$

□ A function f is continuous at left-endpoint $x=a$ iff $\lim_{x\to a^+} f(x)=f(a)$

i) $g(5)=0$

ii) $\lim_{x\to 5^+} g(x)=\lim_{x\to 5^+}\sqrt{x-5}=0$

iii) $\lim_{x\to 5^+} g(x)=g(5)$

$\therefore$ g is continuous at left-endpoint $x=5$

6[9]. **Find** $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for the function $f(x)=\dfrac{2x+5}{|3x-4|}$. **What information** do these limits provide about the graph of f?

$$f(x)=\begin{cases}\dfrac{2x+5}{3x-4}; & x>4/3\\[2mm] \dfrac{2x+5}{-(3x-4)}; & x<4/3\end{cases}$$

$$\lim_{x\to\infty} f(x)=\lim_{x\to\infty}\frac{2x+5}{3x-4}=\lim_{x\to\infty}\frac{2+5/x}{3-4/x}=\frac{2}{3}$$

The graph of $y=f(x)$ approaches the horizontal asymptote $y=2/3$ at the far right end of the x-axis

$$\lim_{x\to-\infty} f(x)=\lim_{x\to-\infty}\frac{2x+5}{-(3x-4)}=\lim_{x\to-\infty}\frac{2+5/x}{-3+4/x}=\frac{-2}{3}$$

The graph of $y=f(x)$ approaches the horizontal asymptote of $y=-2/3$ at the far left end of the x-axis.

7[9]. **Write** a function $f(x)$ (in factored form) which has the following properties:

i. $\lim_{x\to\pm\infty} f(x)=2$

ii. $\lim_{x\to-1} f(x)=\infty$ (must be same direction)

iii. $f(x)$ has a removable (hole) discontinuity at $x=-5$

$$f(x)=\frac{2(x+5)(x+k)^2}{(x+5)(x+1)^2}\quad k\in\mathbb{R}$$

important