[Solutions]

AB Q101 Additional Practice Problems

Part I.

A. Write the function without the absolute value symbol (put into piecewise format)
B. Graph the function.
C. State the domain and range.

1. $f(x) = 4 - |1 - x|$

$$f(x) = \begin{cases} 4-(1-x); & 1-x \geq 0 \\ 4+(1-x); & 1-x < 0 \end{cases} = \begin{cases} 3+x; & x \leq 1 \\ 5-x; & x > 1 \end{cases}$$

2. $g(x) = \begin{cases} \dfrac{|x+1|}{x+1}; & x \neq -1 \\ 3; & x = 1 \end{cases}$

$$= \begin{cases} \frac{x+1}{x+1}; & x > -1 \\ \frac{-(x+1)}{x+1}; & x < -1 \\ 3; & x = -1 \end{cases} = \begin{cases} 1; & x > -1 \\ -1; & x < -1 \\ 3; & x = -1 \end{cases}$$

3. $h(x) = \begin{cases} 4+|x-2|; & x > 0 \\ 4; & -2 < x \leq 0 \\ x+6; & x < -2 \end{cases}$

$$\begin{cases} 4+x-2; & x \geq 2 \\ 4-(x-2); & 0 < x < 2 \\ 4; & -2 < x \leq 0 \\ x+6; & x < -2 \end{cases} = \begin{cases} 2+x; & x \geq 2 \\ 6-x; & 0 < x < 2 \\ 4; & -2 < x \leq 0 \\ x+6; & x < -2 \end{cases}$$

4. $f(x) = \begin{cases} \ln(x-2); & x > 3 \\ |x-3|; & x < 3 \end{cases}$

$$\begin{cases} \ln(x-2); & x > 3 \\ -(x-3); & x < 3 \end{cases} = \begin{cases} \ln(x-2); & x > 3 \\ 3-x; & x < 3 \end{cases}$$

Part II.

A. Graph the function.
B. Indicate the coordinates of any key points and clearly label any asymptotes.
C. Write the limit statement(s) that expresses any asymptote.

5. $g(x) = \dfrac{10(x+1)^2}{(x-2)^2(x+3)^2}$

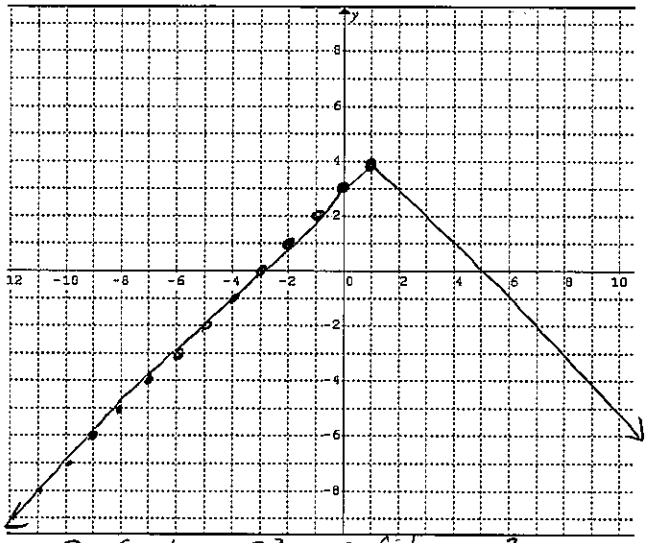
$\frac{5}{16}$ $\quad \frac{5}{24 \cdot 9}$ $\quad \frac{5}{18}$

6. $h(x) = \dfrac{x^2-1}{x^2-9} = \dfrac{(x+1)(x-1)}{(x+3)(x-3)}$

7. $h(x) = \dfrac{x-2}{x(x-1)}$

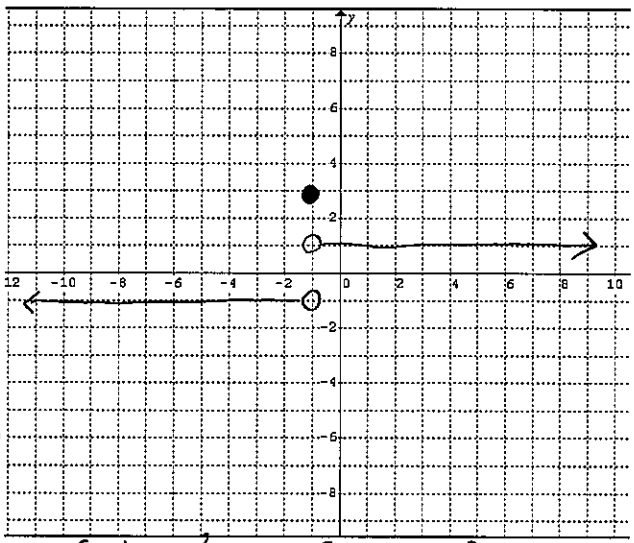
SEE OTHER PAGES FOR GRAPHS

1.



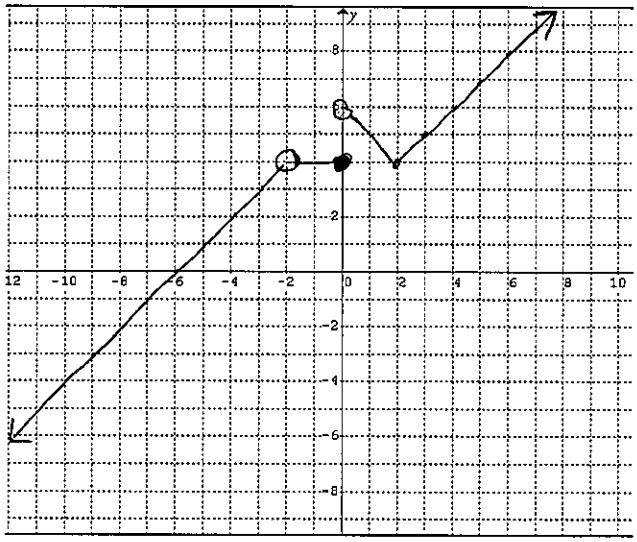
$D: \{x \mid x \in \mathbb{R}\}$ $R: \{y \mid y \leq 4\}$

2.



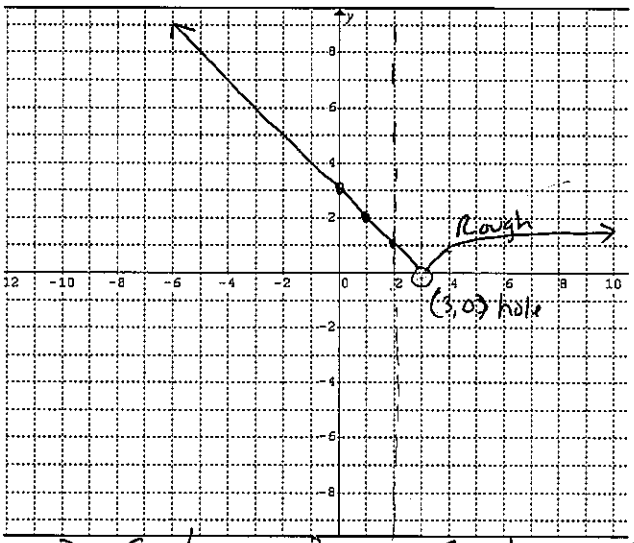
$D: \{x \mid x \in \mathbb{R}\}$ $R: \{-1, 1, 3\}$

3.



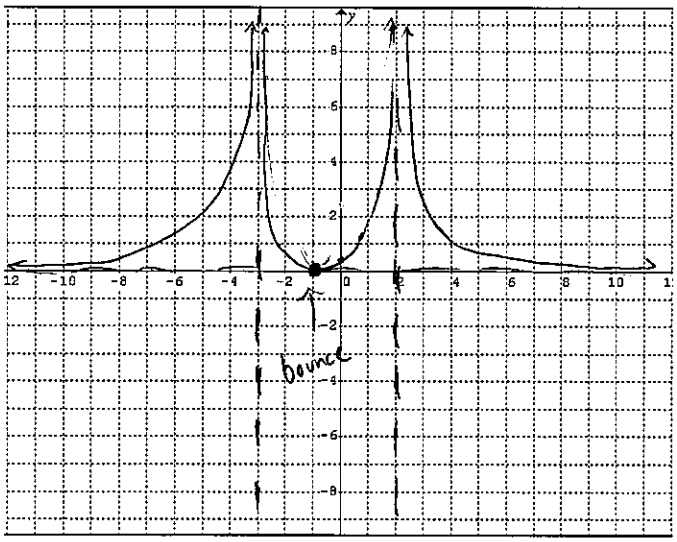
$D: \{x \mid x \neq -2\}$ $R \{y \mid y \in \mathbb{R}\}$

4.



$D: \{x \mid x \neq 3\}$ $R: \{y \mid y > 0\}$

5.



Zero: $(-1, 0)$ Bounce

y-int: $(0, 5/18)$

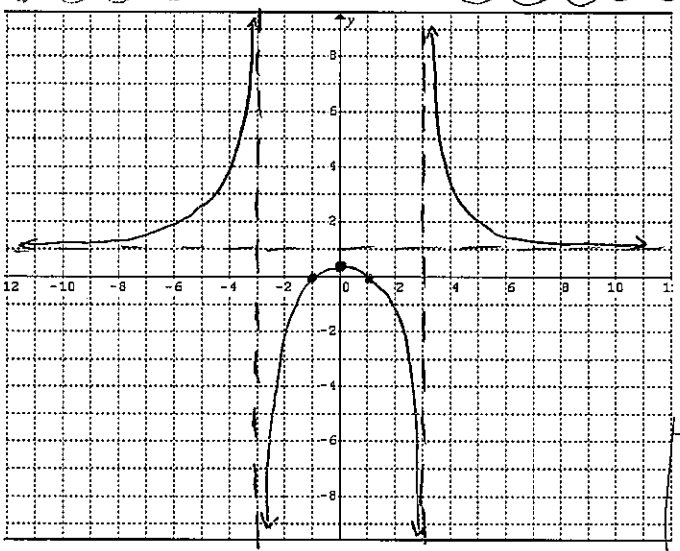
V.A: $x = 2$ even
$x = -3$ even

H.A: $y = 0$ $\lim_{x \to \pm\infty} g(x) = 0$ H.A

H.A pure?
No Because $g(x) = 0$ at $x = -1$

$\lim_{x \to 2} g(x) = \infty$
$\lim_{x \to -3} g(x) = \infty$ V.A

6.



Zero(s): $(1,0)$ $(-1,0)$

y-int: $(0, 1/9)$

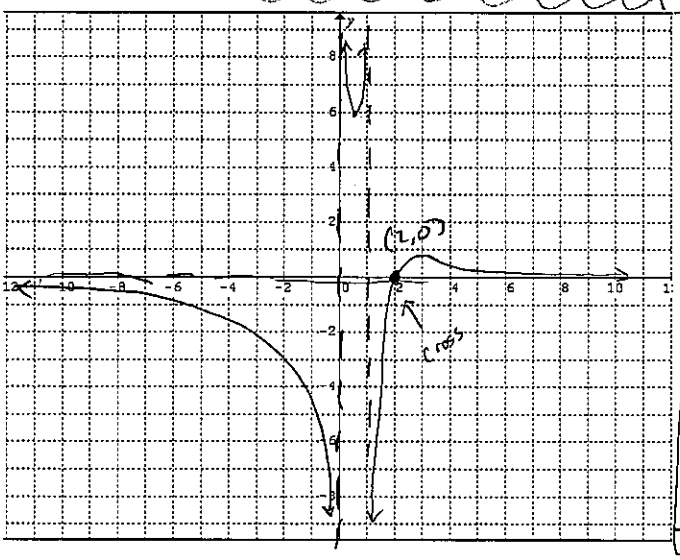
V.A: $x = 3$ odd
$x = -3$ odd

H.A: $y = 1$ $\lim_{x \to \pm\infty} h(x) = 1$ H.A

H.A pure
$x^2 - 9 = x^2 - 1$
$-9 \neq -1$
yes pure $h(x) \neq 1$

$\lim_{x \to 3^-} h(x) = -\infty$
$\lim_{x \to 3^+} h(x) = \infty$
; $\lim_{x \to -3^+} h(x) = -\infty$
$\lim_{x \to -3^-} h(x) = +\infty$ V.A

7.



Zero(s): $(2, 0)$

y-int; none

V.A: $x = 0$ odd
$x = 1$ odd

H.A: $y = 0$ $\lim_{x \to \pm\infty} h(x) = 0$ H.A.

not pure

$h(x) = 0$ @ $x = 2$

$\lim_{x \to 1^+} h(x) = -\infty$
$\lim_{x \to 1^-} h(x) = \infty$
; $\lim_{x \to 0^+} h(x) = \infty$
$\lim_{x \to 0^-} h(x) = -\infty$ V.A.

$h(4) = \frac{2}{4(3)} = \frac{1}{6} > 0$

$h(\frac{1}{2}) = \frac{-3/2}{\frac{1}{2}(-1/2)} = \frac{-3/2}{-1/4} = \frac{3}{2} \cdot \frac{4}{1} = 6$

$(4, 1/6)$
$(\frac{1}{2}, 6)$
} a couple helpful point.

8. $f(x) = \frac{\sqrt{x-2}}{\sqrt{7-x}}$

$x-2 \geq 0$ and $7-x > 0$

$x \geq 2$ and $x < 7$

$\boxed{x \in [2, 7)}$

9. $g(x) = \frac{2x}{x^2-3x-28} = \frac{2x}{(x-7)(x+4)}$

$\boxed{x \neq 7, -4}$ or $\boxed{x \in (-\infty, -4) \cup (-4, 7) \cup (7, \infty)}$

10. $h(x) = \frac{\ln(x+3)}{4-(x+1)^2}$ $x > -3$ and $x \neq 1$

$(x+1)^2 = 4$

$|x+1| = 2$

$x+1 = 2$ or $x+1 = -2$

$x = 1$ or $x = -3$

$\boxed{x \in (-3, 1) \cup (1, \infty)}$

11. $P(x) = \frac{\cot x}{\sqrt{-x}}$

$\sin x = 0$ for $x = \pi k$ k is an integer

$x < 0$

$\boxed{x \in (-\infty, 0) \text{ and } x \neq -\pi k \text{ where } k \text{ is a whole number}}$