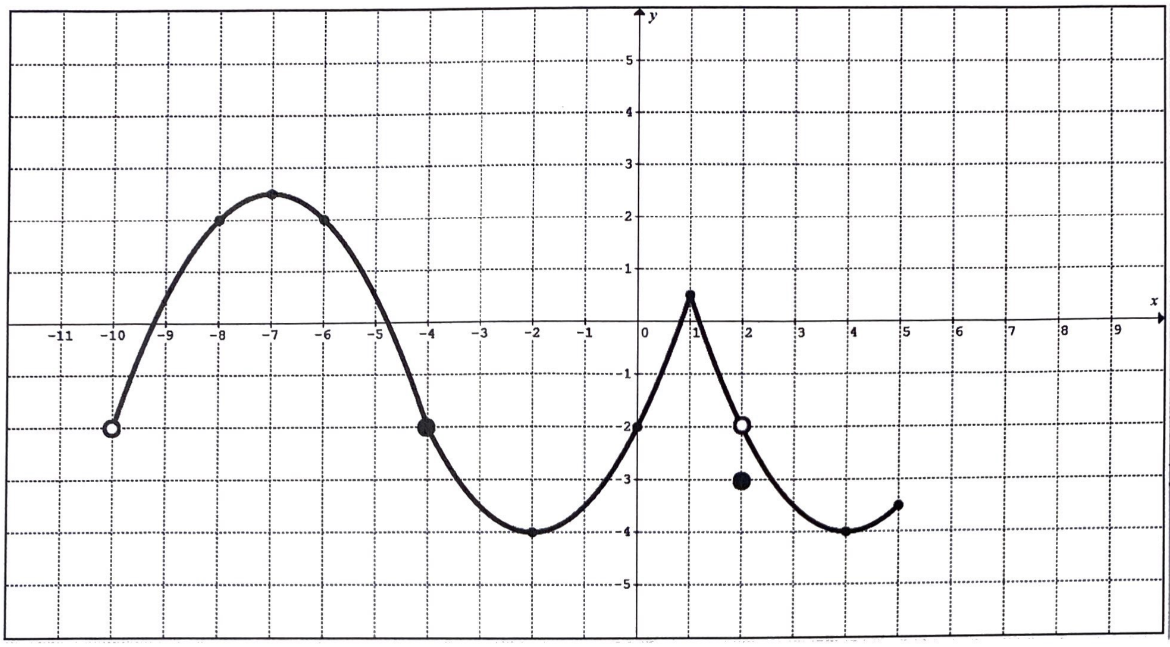
Solutions

AB.Q100.PRACTICE.EXAM



QUESTION #1: Above is the graph of the function $y = f(x)$.

☐ Vertices, integer coordinates, and the endpoint (5, -3.5) have been highlighted.

☐ $f'(-4) = -2$

A. Write the appropriate justification for x-values on $-10 < x < 5$ where the function f is not continuous.

f is not continuous at $x = 2$

i) $f(2) = -3$

ii) $\lim_{x \to 2} f(x) = -2$

iii) $\lim_{x \to 2} f(x) = f(2)$

B. Find the values of x for which the function f is increasing. Justify with slope.

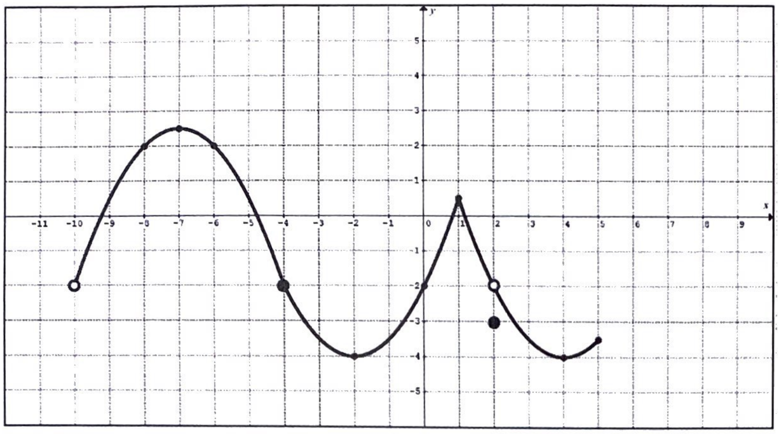
f is increasing on the intervals $(-10, -7]$, $[-2, 1]$, $[4, 5]$ b/c the slope of f is positive on the intervals $(-10, -7)$, $(-2, 1)$, $(4, 5)$

C. Find the values of x on $-10 < x < 5$ for which the function f has a relative maximum. Justify with slope.

f has a relative max at $x = -7$, $x = 1$. At these x-values f is continuous and the slope of f goes from positive to negative.

D. Find or estimate the values of x for which the function f is concave downward. Justify with slope.

f is concave down on the interval $(-10, -4)$ because the slope of f is decreasing on this interval



QUESTION #1 (CONTINUED): Above is the graph of the function $y = f(x)$.

☐ Vertices, integer coordinates, and the endpoint (5, -3.5) have been highlighted.

☐ $f'(-4) = -2$

E. Find the average slope over $-8 \le x \le 0$

$$\frac{f(0)-f(-8)}{0-(-8)} = \frac{-2-2}{0+8} = \frac{-4}{8} = -\frac{1}{2}$$

F1. Find or estimate $f'(-5)$

$$f'(-5) \approx \frac{f(-4)-f(-6)}{-4-(-6)} = \frac{-2-2}{-4+6} = \frac{-4}{2} = -2$$

F2. Find or estimate $\frac{dy}{dx}$ at $x = 4.5$

$$f'(4.5) \approx \frac{f(5)-f(4)}{5-4} = \frac{-3.5-(-4)}{5-4} = 0.5$$

* Given

F3. Find or estimate $f'(1)$

$f'(1)$ DNE "corner" $\frac{\Delta y}{\Delta x}$ Does not Converge as $x \to 1$

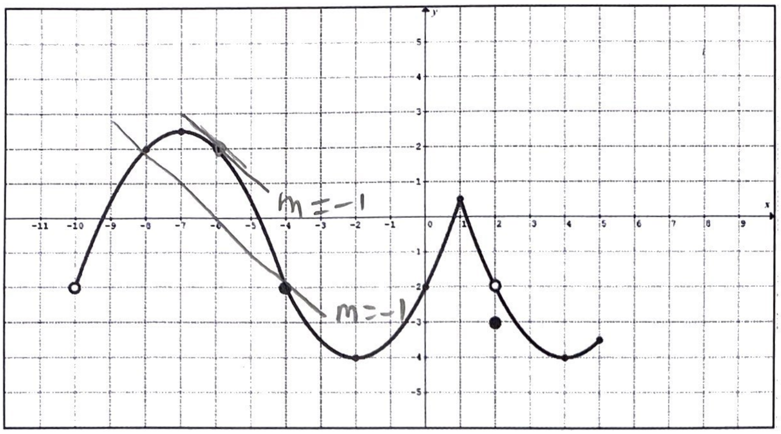
F4. Find or estimate $f'(2)$

$f'(2)$ DNE "f is not continuous at $x=2$" $\frac{\Delta y}{\Delta x}$ Does not converge as $x \to 2$

G. Find or estimate the x-values where f has a horizontal tangent

$f'(x) = 0$ at $x=-7$, $x=-2$, $x=4$

* NOT AT X=1



QUESTION #1 (CONTINUED): Above is the graph of the function $y = f(x)$.

☐ Vertices, integer coordinates, and the endpoint (5, -3.5) have been highlighted.

☐ $f'(-4) = -2$

H. Explain why the Mean Value Theorem applies on [-8, -4] and estimate or find the x-value(s) where the instantaneous slope is equal to the average slope on [-8, -4]. Show this graphically.

f is continuous on $[-8,-4]$ ✓
f has a slope on $(-8,-4)$ ✓

Ave slope on $[-8,-4] = \dfrac{f(-4)-f(-8)}{-4-(-8)} = \dfrac{-2-2}{-4+8} = -1$

$f'(-6) = -1$ Answer: $\boxed{x=-6}$

I. Write a point-slope equation of the line tangent to f at $x = -4$.

pt $(-4,-2)$ $\quad f(-4) = -2$
Slope: -2 (Given) $\quad f'(-4) = -2$

$\boxed{y+2=-2(x+4}$

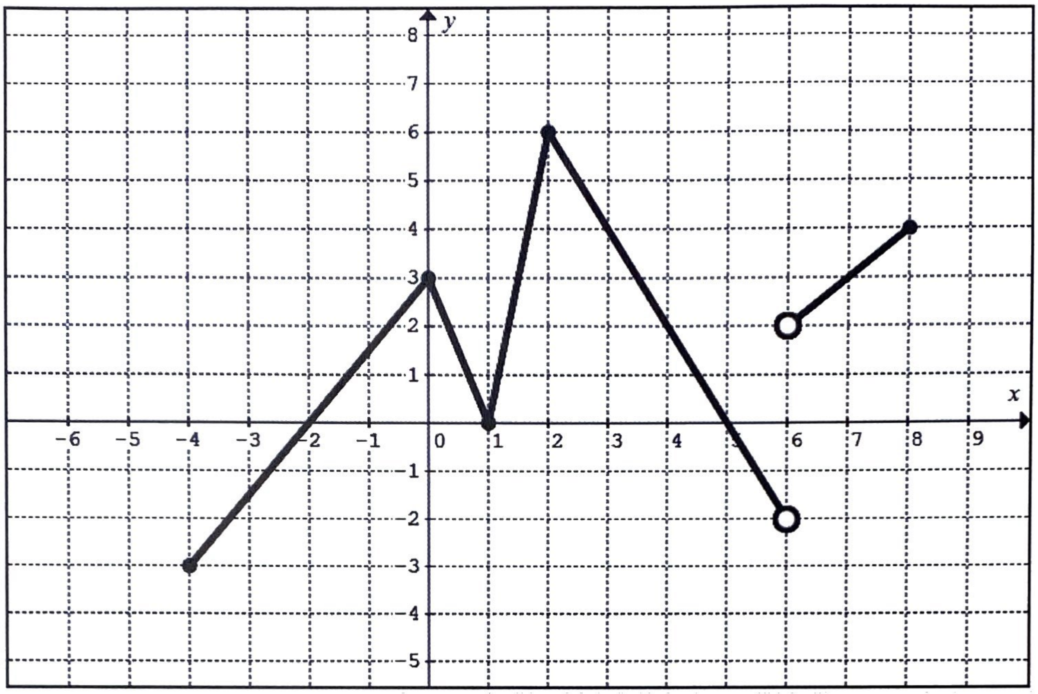
J. True or False: $f'_-(1) > f'_+(1)$ EXPLAIN $\boxed{\text{TRUE}}$

$f'_-(1)$ "slope as we approach $x=1$ from the left" is positive
$f'_+(1)$ "slope as we approach $x=1$ from the right" is negative

K. True or False: The graph of f has a smooth slope transition at the point $(-4,-2)$ EXPLAIN $\boxed{\text{TRUE}}$

We are told that $f'(-4)=-2$ which means we have a defined value for the slope at $x=-4$
*
So f has a smooth slope transition at $x=-4$

* $\frac{\Delta y}{\Delta x}$ does converge as $x \to -4$



QUESTION #2: Above is the graph of g' (the **derivative** of the graph of g).

This graph is made up of line segments.

It is know that the function g is continuous on $-4 \le x \le 8$ with $g(4) = 20$. ↙ given

A. For what values of x on $-4 < x < 8$ does the function g have a relative maximum? Justify with slope. g has a relative max at x=5 b/c at this x-value g is continuous and the slope of g changes from positive to negative.

B. For what values of x on $-4 < x < 8$ does the function g have a relative minimum?
(Does not ask for justification)

$\boxed{x = -2 \text{ and also } x = 6}$

at these x values g is continuous and g'(x) goes from negative to positive

C. For what values of x on $-4 < x < 8$ ~~does~~ is function g concave downward? Justify with slope.
g is Concave down on the intervals (0,1), (2,6) b/c the slope of g is decreasing on these intervals.

D. True or False: $g(3) < g(5)$: EXPLAIN $\boxed{\text{TRUE}}$
Since g is increasing from x=3 to x=5 (g' is positive) it concludes that g(3) must be smaller than g(5)

E. Write an equation in point-slope form of a line tangent to $y = g(x)$ at $x = 4$.

$\boxed{y - 20 = 2(x - 4)}$

↑ Given ↑ point on g' graph

QUESTION #3:

Draw a quick sketch of a function $y = q(x)$ that possesses the following properties:

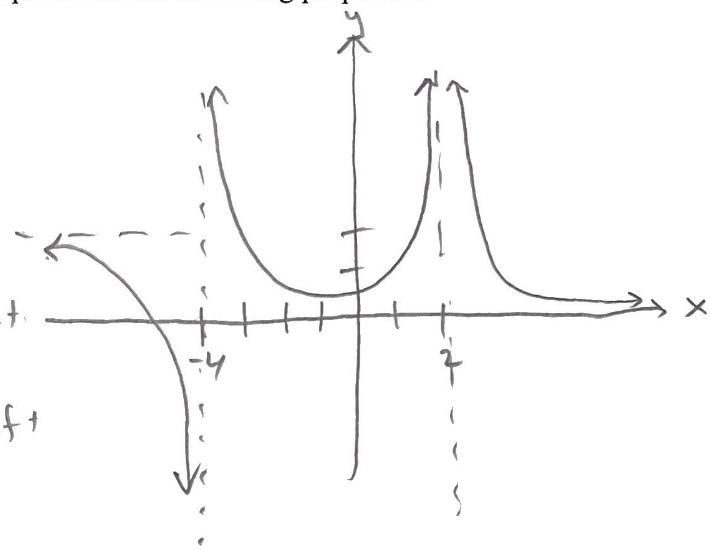
$\lim_{x\to\infty} q(x) = 0$ horizontal asy right end behavior

$\lim_{x\to-\infty} q(x) = 2$ hor. asy left end behavior

$\lim_{x\to 2} q(x) = \infty$ vertical asymptote

$\lim_{x\to -4^+} q(x) = \infty$ vert. asy inc. w/o bound from right

$\lim_{x\to -4^-} q(x) = -\infty$ vert asy dec w/o bound from left



QUESTION #4:

Draw a quick sketch of the functions $y = \sin x$ and $y = e^x$.

SEE HW SOLUTIONS

QUESTION #5:

A. Evaluate $\cos\left(\frac{2\pi}{3}\right)$ $= \boxed{-\frac{1}{2}}$

B. Evaluate $\csc\left(\frac{5\pi}{3}\right)$ $= \boxed{\frac{-2}{\sqrt{3}}}$ or $\frac{-2\sqrt{3}}{3}$

C. Solve the equation: $\cos x - 2\sin x \cos x = 0$ on the domain $D:\{x \mid 0 \le x \le 2\pi\}$

$\cos x(1 - 2\sin x) = 0$ $\quad \cos x = 0$ or $1 - 2\sin x = 0$

$x = \pi/2, 3\pi/2$ $\quad \sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$$\boxed{x = \pi/2, \frac{3\pi}{2}, \pi/6, \frac{5\pi}{6}}$$