$\frac{49}{14} - \frac{2}{14}$

PRACTICE

1: Let $y = f(x)$ be a function with $f(1) = 4$ such that for all points (x, y) on the graph of $y = f(x)$ the slope is given by $\dfrac{3x^2+1}{2y}$.

A. Find the slope of the graph of $y = f(x)$ at the point where $x = 1$.
B. Write an equation for the line tangent to the graph of $y = f(x)$ at $x = 1$ and use it to approximate $f(1.2)$
C. Starting at the point (1, 4), use Euler's Method with step size $\Delta x = -1.0$ to approximate $f(-1)$

A] $f'(1,4) = \dfrac{3(1)^2+1}{2(4)} = \boxed{\dfrac{1}{2}}$

B] $L(x) = f(1) + f'(1,4)(x-1)$

$L(x) = 4 + \frac{1}{2}(x-1)$

$f(1.2) \approx L(1.2) = 4 + \frac{1}{2}(0.2) = \boxed{4.1}$

C] $\begin{pmatrix} x_0 & y_0 \\ 1, & 4 \end{pmatrix}$ $y_1 = 4 + \left[\dfrac{3(1)^2+1}{2(4)}\right](-1) = 4 - \frac{1}{2} = \frac{7}{2} \rightarrow (0, 7/2)$

$(0, 7/2)$ $y_2 = \frac{7}{2} + \left[\dfrac{3(0)^2+1}{2(7/2)}\right](-1) = \frac{7}{2} - \frac{1}{7} = \frac{47}{14} \rightarrow (-1, \frac{47}{14})$

$f(-1) \approx E(-1) = \boxed{\dfrac{47}{14}}$

$\frac{61}{30}$ $\quad 1830 \quad 61 \quad 1891$ $\quad \frac{0.03}{2}$ $\quad \frac{61}{3} \cdot \frac{1}{100} \cdot \frac{1}{2}$

0.015

2: Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

A. Let $y = f(x)$ be a particular solution to the given differential equation with initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$, with step size 0.1, to approximate $f(0.2)$. Show the work that leads to your answer.

B. Starting at the point (0, 3), use Euler's Method with step size $\Delta x = 1/3$ to approximate $f(1)$.

C. Find the particular solution $y = f(x)$ to the given differential equation with initial condition $f(0) = 3$.

A] $\underset{x_0\ y_0}{(0,3)}$ $\quad y_1 = 3 + \left(\frac{(0)(3)}{2}\right)(0.1) = 3 \rightarrow (0.1, 3)$

$(0.1, 3)$ $\quad \hat{y}_2 = 3 + \left(\frac{(0.1)(3)}{2}\right)(0.1) = 3.015 \rightarrow (0.2, 3.015)$

$f(0.2) \approx E(0.2) = \boxed{3.015}$

B] $(0,3)$ $\quad y_1 = 3 + \left(\frac{(0)(3)}{2}\right)\left(\frac{1}{3}\right) = 3 \rightarrow (1/3, 3)$

$(1/3, 3)$ $\quad \hat{y}_2 = 3 + \left(\frac{\frac{1}{3}\cdot 3}{2}\right)\left(\frac{1}{3}\right) = 3 + \frac{1}{6} = \frac{19}{6} \rightarrow (2/3, 19/6)$

$(2/3, 19/6)$ $\quad \hat{y}_3 = \frac{19}{6} + \left(\frac{\frac{2}{3}\left(\frac{19}{6}\right)}{2}\right)\left(\frac{1}{3}\right) = \frac{19}{6} + \frac{19}{54} = \frac{171}{54} + \frac{19}{54} = \frac{190}{54}$

$f(1) \approx E(1) = \boxed{\frac{95}{27}}$

C] $\int \frac{dy}{y} = \int \frac{x}{2}\,dx$ $\qquad y = c^* e^{x^2/4}$

$3 = c^* e^0 \therefore c^* = 3$

$\ln|y| = \frac{x^2}{4} + C$

$|y| = Ce^{x^2/4}$

$\boxed{y = 3e^{x^2/4}}$

3. Consider the differential equation $\frac{dy}{dx} = y + x$ with $y(0) = 1$.

A. Use a linearization centered at $x = 0$ to approximate $y(1.2)$.

B. Use Euler's method starting at $x = 0$ with step size $\Delta x = 0.4$ to approximate $y(1.2)$.

$$\begin{array}{r} 1.8 \\ 0.4 \\ \hline .72 \end{array} \qquad \begin{array}{r} 1.4 \\ .72 \\ \hline 2.12 \end{array}$$

$$\begin{array}{r} 2.12 \\ .80 \\ \hline 2.92 \\ 0.4 \\ \hline 1.168 \\ 2.12 \\ \hline 3.288 \end{array}$$

A] $L(x) = y(0) + y'(0,1)(x-0)$

$L(x) = 1 + [1+0]\,x$

$L(x) = 1 + x$

$y(1.2) \approx L(1.2) = 1 + 1.2 = \boxed{2.2}$

B] $(0,1)$ $\quad \hat{y}_1 = 1 + [1+0](0.4) = 1.4$

$(0.4, 1.4)$ $\quad \hat{y}_2 = 1.4 + [1.4+0.4](0.4)$

$= 1.4 + [1.8](0.4) = 2.12$

$(0.8, 2.12)$ $\quad \hat{y}_3 = 2.12 + [2.12+0.8](0.4)$

$= 2.12 + 1.168 = 3.288$

$y(1.2) \approx E(1.2) = \boxed{3.288}$