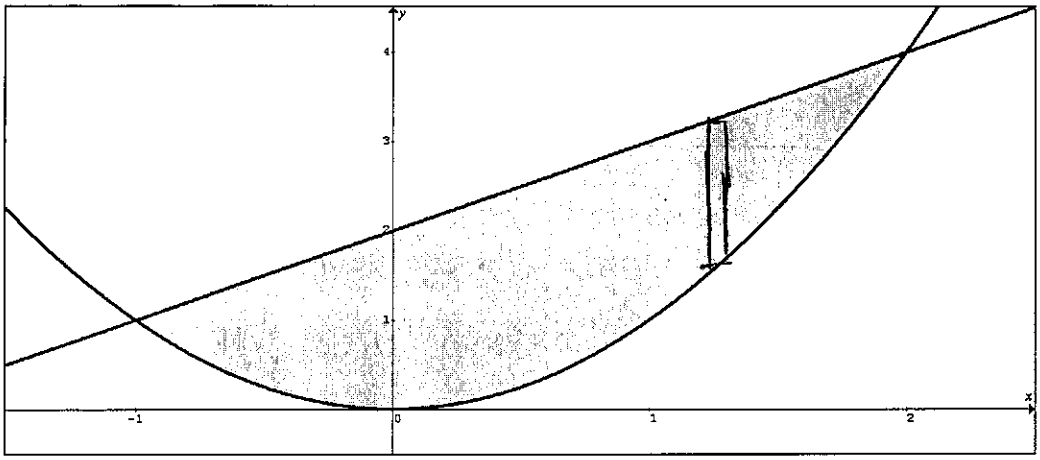
Lesson 1 Solutions



PRACTICE 1: Consider the region R bounded by the graphs of $y = x^2$ and $y = x + 2$ as shown by the shaded region above.

A. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $x = 20$.

$$V = \int_{-1}^{2} 2\pi (20 - x)(x + 2 - x^2)\,dx$$

B. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the x axis.

$$V = \pi \int_{-1}^{2} \left[(x+2)^2 - (x^2)^2\right] dx$$

C. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $y = 20$.

$$V = \pi \int_{-1}^{2} \left[(20 - x^2) - (20 - (x+2))^2\right] dx$$

D. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $y = -1$.

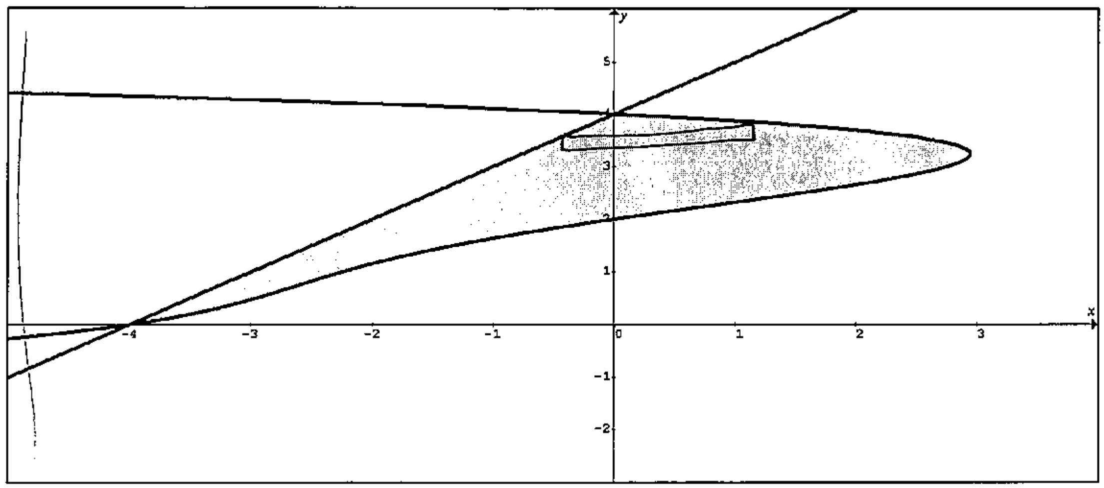
$$V = \pi \int_{-1}^{2} \left[(x + 2 + 1)^2 - (x^2 + 1)^2\right] dx$$

E. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $x = -1$.

$$V = \int_{-1}^{2} 2\pi (x + 1)(x + 2 - x^2)\,dx$$

F. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $x = -40$.

$$V = \int_{-1}^{2} 2\pi (x + 40)(x + 2 - x^2)\,dx$$



PRACTICE 2: Consider the region R bounded by the graphs of $x = \frac{-y^4}{4} + \frac{3y^3}{2} - \frac{5y^2}{2} + 3y - 4$ and $y = x + 4$ as shown by the shaded region above. $\longrightarrow x = y - 4$

Both graphs pass through the points $(-4, 0)$ and $(0, 4)$

A. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $x = 20$.

washer: $$V = \pi \int_0^4 \left[\left(20 - (y-4)\right)^2 - \left(20 - \left(-\frac{y^4}{4} + \frac{3y^2}{2} - \frac{5y^2}{2} + 3y - 4\right)\right)^2 \right] dy$$

B. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the ~~y-axis.~~ x-axis

shell: $$V = \int_0^4 2\pi (y)\left(-\frac{y^4}{4} + \frac{3y^3}{2} - \frac{5y^2}{2} + 3y - 4 - (y-4)\right) dy$$

C. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $y = 20$.

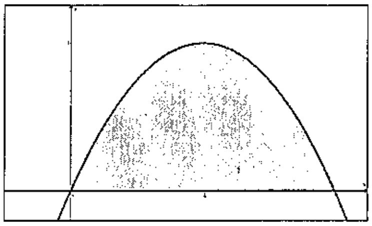
Shell: $$V = \int_0^4 2\pi (20 - y)\left(-\frac{y^4}{4} + \frac{3y^3}{2} - \frac{5y^2}{2} + 3y - 4 - (y-4)\right) dy$$

D. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $y = -1$.

Shell: $$V = \int_0^4 2\pi (y+1)\left(-\frac{y^4}{4} + \frac{3y^3}{2} - \frac{5y^2}{2} + 3y - 4 - (y-4)\right) dy$$

Shell: E. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid generated when the region R is revolved about the line $x = -5$.

wash: $$V = \pi \int_0^4 \left[\left(-\frac{y^4}{4} + \frac{3y^3}{2} - \frac{5y^2}{2} + 3y - 4 + 5\right)^2 - (y - 4 + 5)^2 \right] dy$$



NO CALCULATOR

$$\begin{array}{r} 32 \\ \times\ 5 \\ \hline 160 \end{array} \qquad \begin{array}{r} 16 \\ 15 \\ \hline 80 \\ 16 \\ \hline 240 \end{array} \qquad \begin{array}{r} 32 \\ 3 \\ \hline 96 \end{array} \qquad \begin{array}{r} 160 \\ +\ 96 \\ \hline 256 \end{array}$$

PRACTICE 3: Consider the region R bounded by the graph of $y = 2x - x^2$ and the x-axis as shown by the shaded region above.

A. **Find** the volume of the solid generated when the region R is revolved about the y axis.
B. **Find** the volume of the solid generated when the region R is revolved about the x axis.

B] $$V = \pi \int_0^2 (2x - x^2)^2 \, dx = \pi \int_0^2 (4x^2 - 4x^3 + x^4) \, dx$$

$$= \pi \left[\frac{4x^3}{3} - x^4 + \frac{x^5}{5} \right]_0^2 = \pi \left[\frac{32}{3} - 16 + \frac{32}{5} \right]$$

$$= \pi \left[\frac{32(5)}{15} - \frac{16(15)}{15} + \frac{32(3)}{15} \right] = \pi \left[\frac{16}{15} \right] = \boxed{\frac{16\pi}{15}}$$

A] $$V = 2\pi \int_0^2 (x)(2x - x^2) \, dx = 2\pi \int_0^2 (2x^2 - x^3) \, dx$$

$$= 2\pi \left[\frac{2x^3}{3} - \frac{x^4}{4} \right]_0^2 = 2\pi \left[\frac{16}{3} - \frac{\cancel{16}^{\,4}}{\cancel{4}} \right]$$

$$= 2\pi \left[\frac{16}{3} - \frac{12}{3} \right] = 2\pi \left[\frac{4}{3} \right] = \boxed{\frac{8\pi}{3}}$$