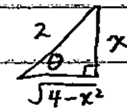
Lesson 3 PRACTICE Solutions

#42:

$$\int \frac{8\,dx}{x^2\sqrt{4-x^2}} = \int \frac{8(2\cos\theta)\,d\theta}{4\sin^2\theta\sqrt{4-4\sin^2\theta}} = 2\int \frac{\cancel{\cos\theta}\,d\theta}{\sin^2\theta\,\cancel{\cos\theta}}$$

let $x = 2\sin\theta$ $\quad dx = 2\cos\theta\,d\theta$



$$= 2\int \csc^2\theta\,d\theta = -2\cot\theta + C$$

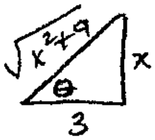
$$= -2\,\frac{\sqrt{4-x^2}}{x} + C$$

Supp #35

$$\int \frac{dx}{\sqrt{9+x^2}} = \int \frac{3\sec^2\theta\,d\theta}{\sqrt{9+9\tan^2\theta}} = \int \frac{3\sec^2\theta\,d\theta}{3\sqrt{1+\tan^2\theta}} = \int \sec\theta\,d\theta$$

Let $x = 3\tan\theta$

$dx = 3\sec^2\theta\,d\theta$



$$= \ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \ln\left|\frac{\sqrt{x^2+9}}{3} + \frac{x}{3}\right| + C$$

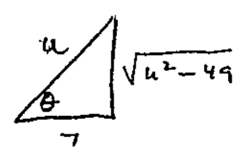
$$= \ln\left|\sqrt{x^2+9} + x\right| + C$$

Supp #37

$$\int \frac{dx}{\sqrt{4x^2-49}} = \int \frac{dx}{\sqrt{(2x)^2-7^2}} = \frac{1}{2}\int \frac{du}{\sqrt{u^2-7^2}} = \frac{7}{2}\int \frac{\sec\theta\tan\theta\,d\theta}{\sqrt{49\sec^2\theta-49}}$$

Let $u = 7\sec\theta$

$du = 7\sec\theta\tan\theta\,d\theta$



$$= \frac{1}{2}\int \frac{\sec\theta\,\cancel{\tan\theta}\,d\theta}{\cancel{\tan\theta}} = \frac{1}{2}\int \sec\theta\,d\theta = \frac{1}{2}\ln\left|\sec\theta + \tan\theta\right| + C$$

$$= \frac{1}{2}\ln\left|\frac{u}{7} + \frac{\sqrt{u^2-49}}{7}\right| + C = \frac{1}{2}\ln\left|\frac{2x}{7} + \frac{\sqrt{4x^2-49}}{7}\right| + C$$

$$= \frac{1}{2}\ln\left|2x + \sqrt{4x^2-49}\right| + C$$

MISC: $\int \sin^3 x\,dx = \int \sin^2 x \cdot \sin x\,dx = \int (1-\cos^2 x)\sin x\,dx$

$u = \cos x \quad du = -\sin x\,dx$

$= -\int (1-u^2)\,du = -\left[u - \frac{u^3}{3} + c\right] = -u + \frac{u^3}{3} + c$

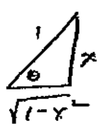
$= -\cos x + \frac{\cos^3 x}{3} + c$

Supp # 39

$\int \frac{x^3}{\sqrt{1-x^2}}\,dx = \int \frac{\sin^3\theta \cos\theta\,d\theta}{\sqrt{1-\sin^2\theta}} = \int \sin^3\theta\,d\theta$

Let $x = \sin\theta$

$dx = \cos\theta\,d\theta$



⊛ $\int \sin^3\theta\,d\theta = \int (\sin^2\theta)\cdot\sin\theta\,d\theta = \int (1-\cos^2\theta)\cdot\sin\theta\,d\theta$

$u = \cos\theta \quad du = -\sin\theta\,d\theta \quad d\theta = du/{-\sin\theta}$

$= -\int (1-u^2)\,du = -u + \frac{u^3}{3} + c = -\cos\theta + \frac{\cos^3\theta}{3} + c$

$= \boxed{-\sqrt{1-x^2} + \frac{(1-x^2)^{3/2}}{3} + c}$ ← my answer

$= \sqrt{1-x^2}\left(-1 + \frac{1}{3}\sqrt{1-x^2}^{\,2}\right) + c = \sqrt{1-x^2}\left[-1 + \frac{1}{3}(1-x^2)\right] + c$

$= \sqrt{1-x^2}\left[-\frac{2}{3} - \frac{x^2}{3}\right] = \sqrt{1-x^2}\left[\frac{-2-x^2}{3}\right] = \boxed{\frac{-(x^2+2)\sqrt{1-x^2}}{3} + c}$

CALCULATOR'S ANSWER