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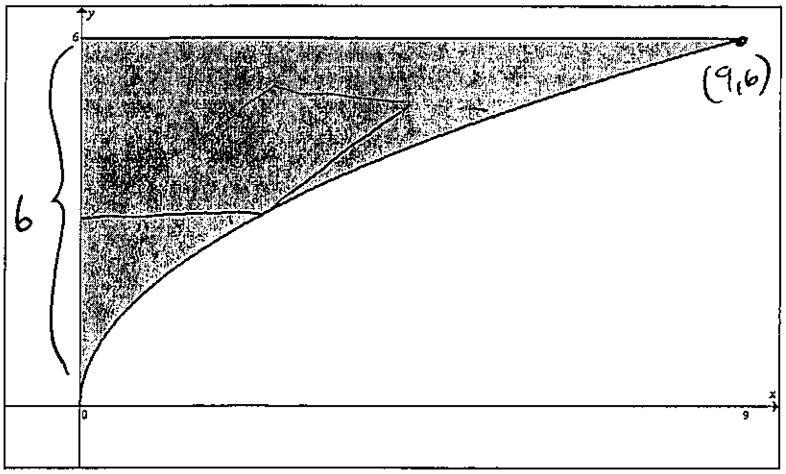

$x^{1/2}$ $\frac{4}{3}x^{3/2}$

1. Let R be the region in the first quadrant bounded by the graph of $y = 2\sqrt{x}$, the horizontal $y = 6$, and the $y =$ axis, as shown in the figure above.

$x = \frac{y^2}{4}$

(a) Find the area of R.
(b) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when R is rotated about the horizontal line $y = 7$.
(c) Region R is the base of a solid. For each y, where $0 \le y \le 6$, the cross section of the solid taken perpendicular to the y-axis is a rectangle whose ~~base~~ height is 3 times the length of its base in region R. Write, but do not evaluate, an integral expression that gives the volume of the solid.
(d) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of region R.

a) $\int_0^9 (6 - 2\sqrt{x})\,dx = \left[6x - \frac{4}{3}x^{3/2}\right]_0^9 = 54 - \frac{4}{3}(27) = 54 - 36 = \boxed{18}$

b) $V = \pi\int_0^9 \left[(7-2\sqrt{x})^2 - (7-6)^2\right] dx$

c) $A_{\square} = (\text{base})(3\,\text{base}) = 3(\text{base})^2 = 3\left(\frac{y^2}{4}\right)^2$

$V = 3\int_{y=0}^{y=6} \left(\frac{y^2}{4}\right)^2 dy$

$\frac{dy}{dx} = \frac{1}{\sqrt{x}}$

d) $P = 6 + 9 + \int_0^9 \sqrt{1 + \left(\frac{1}{\sqrt{x}}\right)^2}\, dx$

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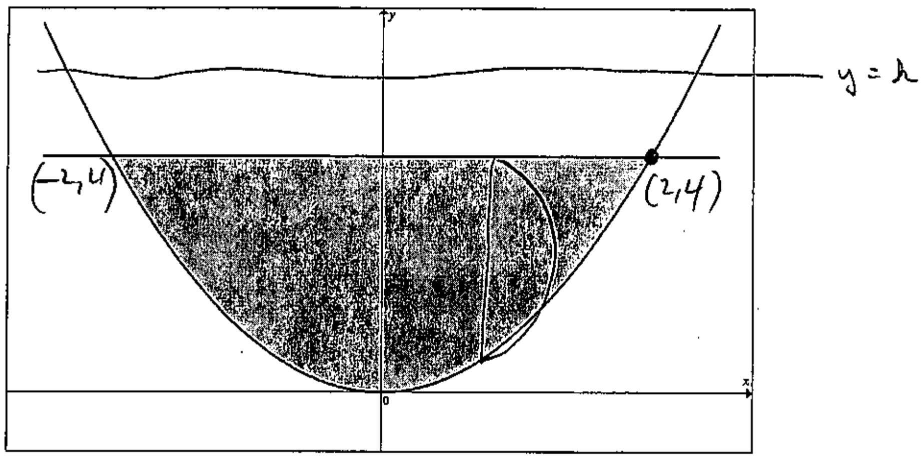

2. The shaded region, R, is bounded by the graphs of $y = x^2$ and the line $y = 4$, as shown in the figure above.

(a) Find the area of R.
(b) Find the volume of solid generated by revolving R about the x-axis.
(c) There exists a number k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (b). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.
(d) The region R is the base a solid. For this solid, every cross section perpendicular to the x-axis is a semi-circle. Write, but do not evaluate, an integral expression used to find the volume of the solid.
(e) Write, but do not evaluate, an expression involving one or more integrals that gives the perimeter of region R.

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a) $A = \int_{-2}^{2}(4-x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \left(8-\frac{8}{3}\right) - \left(-8+\frac{8}{3}\right) = 16 - \frac{16}{3} = \boxed{\frac{32}{3}}$

b) $V = \pi\int_{-2}^{2}\left[(4)^2 - (x^2)^2\right]dx$

$= \pi\left[16x - \frac{x^5}{5}\right]_{-2}^{2}$

$= \pi\left[\left(32 - \frac{32}{5}\right) - \left(-32 + \frac{32}{5}\right)\right]$

$= \pi\left[64 - \frac{64}{5}\right]$

$= \boxed{\frac{256}{5}\pi}$

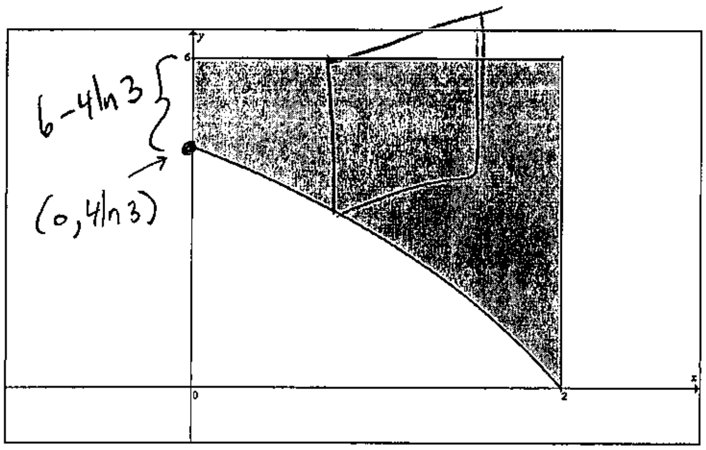
c) $\pi\int_{-2}^{2}\left[(k-x^2)^2 - (k-4)^2\right]dx = \frac{256}{5}\pi$

D) $A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi\left(\frac{4-x^2}{2}\right)^2 = \frac{\pi}{8}(4-x^2)^2$

$V = \frac{\pi}{8}\int_{-2}^{2}(4-x^2)^2\,dx$

E) $P = 4 + \int_{-2}^{2}\sqrt{1+(2x)^2}\,dx$

A graphing **calculator is required** for some parts of the problem



3. In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3-x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

$f(x) = 4\ln(3-x)$ $\qquad f'(x) = \frac{-4}{3-x}$

(a) Find the area of R.
(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.
(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the *x*-axis is a square. Find the volume.
(d) Find the perimeter of the region R.

(a) $A = \int_0^2 [6 - f(x)]\,dx = \boxed{6.817}$

(b) $V = \pi\int_0^2 [(8-f(x))^2 - (8-6)^2]\,dx = \boxed{168.180}$ or 53.533π

(c) $A = (\text{base})^2 = (6 - f(x))^2$

$V = \int_0^2 (6 - f(x))^2\,dx = \boxed{26.267}$

(d) $P = (6 - 4\ln 3) + 2 + 6 + \int_0^2 \sqrt{1 + \left(\frac{-4}{3-x}\right)^2}\,dx$

$\approx 14 - 4\ln(3) + 4.861 \approx \boxed{14.467}$

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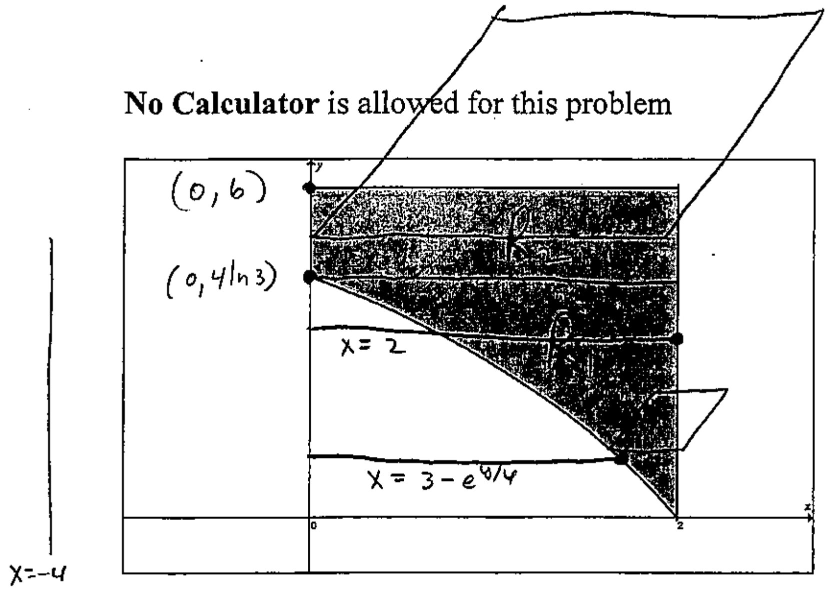

$$f(x) = 4\ln(3-x)$$

$$f'(x) = \frac{-4}{3-x}$$

$$y = 4\ln(3-x)$$

$$\frac{y}{4} = \ln(3-x)$$

$$e^{y/4} = 3 - x \quad \rightarrow \quad x = 3 - e^{y/4}$$

4. (3 continued...) In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3-x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

(e) The region R is the base of a solid. For this solid, each cross section perpendicular to the y-axis is a square. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of this solid.

(f) Suppose a horizontal line cuts through the shaded region R at the point where $y = 4\ln(3-x)$ intersects the y-axis. This horizontal line creates two regions. Suppose the bottom region is revolved about the line $x = -4$. Set up, but do not evaluate, an expression involving one or more integrals used to find the volume of the solid.

(e) $A = (\text{base})^2 \quad R_1: \; A = \left(2 - (3 - e^{y/4})\right)^2$

$\qquad\qquad\qquad\quad R_2: \; A = (2)^2$

$$\boxed{V = \int_{y=0}^{y=4\ln 3} \left[2 - (3 - e^{y/4})\right]^2 dy + \int_{y=4\ln 3}^{6} [2]^2 \, dy}$$

(f) Bottom Region R_1

$$\boxed{V = \pi \int_{y=0}^{y=4\ln 3} \Big[(\underbrace{2+4}_{\text{out radius}})^2 - (\underbrace{3 - e^{y/4} + 4}_{\text{in radius}})^2 \Big] dy}$$

[OR] (BC-SHELLS):
$$V = \int_{x=0}^{x=2} 2\pi \underbrace{(x+4)}_{\text{radius}} \underbrace{\left(4\ln 3 - 4\ln(3-x)\right)}_{\text{height}} dx$$

5. Find the length of $y = x^2$ on $-1 \le x \le 2$.

$$L_{-1}^{2} = \int_{-1}^{2} \sqrt{1 + (2x)^2}\, dx = 6.126$$

6. Find the length of $y^2 + 2y = 2x + 1$ from the point (-1, -1) to the point (7, 3).

$$L_{-1}^{3} = \int_{-1}^{3} \sqrt{1 + (y+1)^2}\, dy = 9.294$$

7. Find the length of $y = \int_0^x \tan t \, dt$ on $0 \le x \le \pi/6$.

$$\frac{dy}{dx} = \tan x \qquad L_{0}^{\pi/6} = \int_{0}^{\pi/6} \sqrt{1 + (\tan x)^2}\, dx = 0.5493$$

8. Find the length of $x = \int_0^y \sqrt{\sec^4 t - 1}\, dt$ on $-\pi/4 \le y \le \pi/4$.

$$\frac{dx}{dy} = \sqrt{\sec^4 y - 1} \qquad L_{-\pi/4}^{\pi/4} = \int_{-\pi/4}^{\pi/4} \sqrt{1 + (\sec^4 y - 1)}\, dy = \int_{-\pi/4}^{\pi/4} \sec^2 y \, dy = \Big[\tan y\Big]_{-\pi/4}^{\pi/4}$$

$$= \tan\left(\frac{\pi}{4}\right) - \tan\left(-\frac{\pi}{4}\right) = 1 - (-1) = \boxed{2}$$

9. Find the length of $y = \int_0^x \sqrt{\cos 2t}\, dt$ on $0 \le x \le \pi/4$.

$$\frac{dy}{dx} = \sqrt{\cos 2x} \qquad L_{0}^{\pi/4} = \int_{0}^{\pi/4} \sqrt{1 + \cos 2x}\, dx = \int_{0}^{\pi/4} \underbrace{\sqrt{2\cos^2 x}}_{\text{Identity}}\, dx = \sqrt{2}\int_{0}^{\pi/4} \cos x \, dx$$

$$= \sqrt{2}\Big[\sin x\Big]_{0}^{\pi/4} = \boxed{1}$$