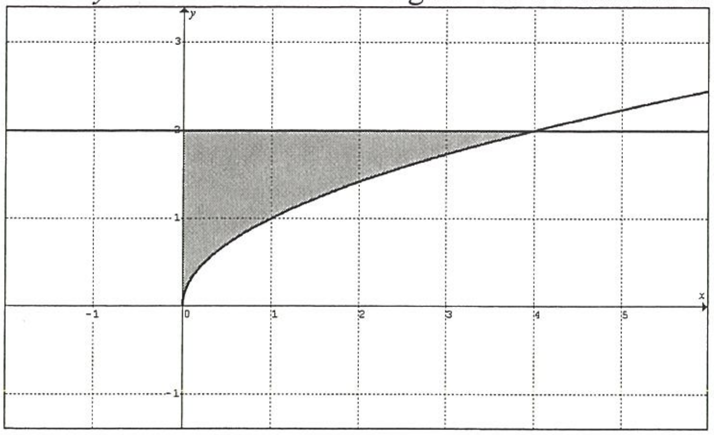
AB: Q402 / BC: Q302

~~BC:Q302~~ **LESSON 1 HOMEWORK**

1. NO CALCULATOR. Let R be the shaded region enclosed by the graphs of $y=\sqrt{x}$, $y=2$, and the y-axis as shown in the figure below. Find the area of region R.



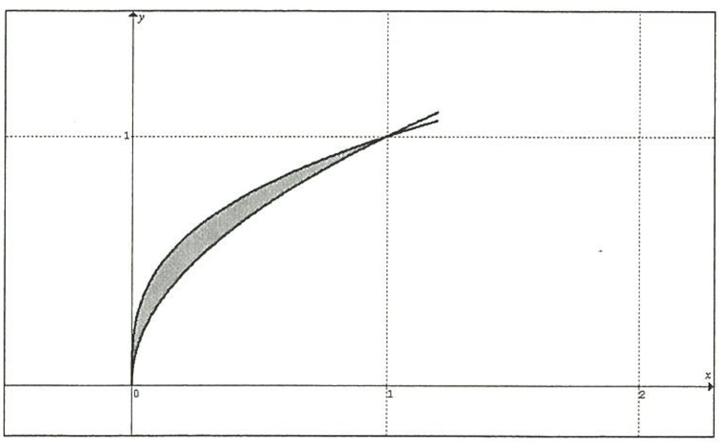
$$A = \int_0^4 \left[2-\sqrt{x}\right] dx$$

$$= \left[2x - \frac{2x^{3/2}}{3}\right]_0^4$$

$$= 8 - \frac{16}{3} = \boxed{\frac{8}{3}}$$

(OR) ALT: $\displaystyle\int_{y=0}^{y=2} \left[y^2 - 0\right] dy = \left[\frac{y^3}{3}\right]_0^2 = \frac{8}{3}$ ✓

2. NO CALCULATOR. Let R be the shaded region enclosed by the graphs of $x=y^3$, $x=y^2$, and the x-axis as shown in the figure below. Find the area of region R.

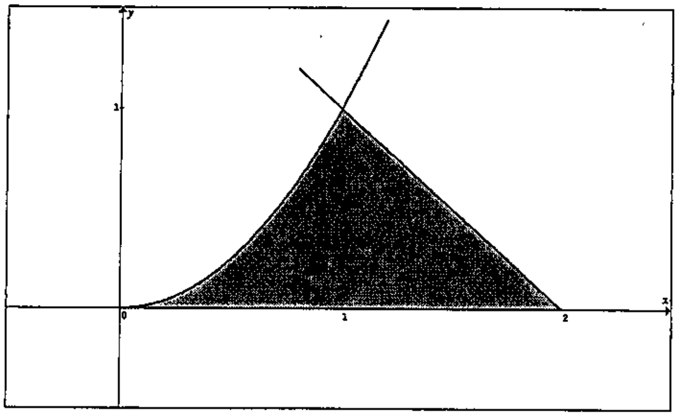


$$A = \int_{y=0}^{y=1} \left[y^2 - y^3\right] dy = \left[\frac{y^3}{3} - \frac{y^4}{4}\right]_0^1$$

$$= \frac{1}{3} - \frac{1}{4} = \boxed{\frac{1}{12}}$$

(OR) ALT: $\displaystyle\int_{x=0}^{x=1} \left[x^{1/3} - x^{1/2}\right] dx = \left[\frac{3x^{4/3}}{4} - \frac{2x^{3/2}}{3}\right]_0^1 = \frac{3}{4} - \frac{2}{3} = \frac{1}{12}$ ✓

3. NO CALCULATOR. Let R be the shaded region enclosed by the graphs of $y = x^2$, $x + y = 2$, and the x-axis as shown in the figure below. Find the area of region R.



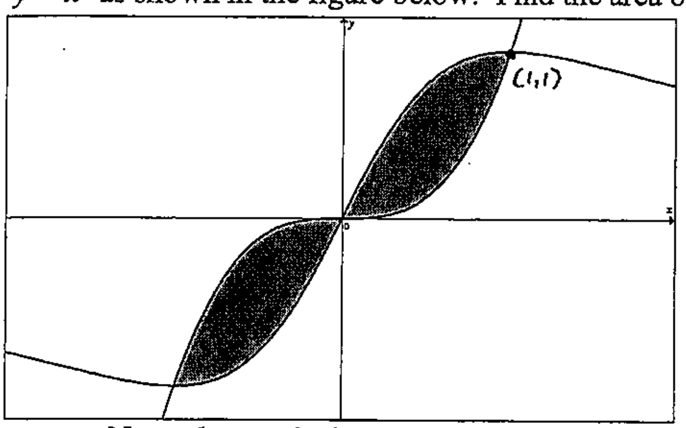
$$A = \int_{y=0}^{y=1} \left[(2-y) - y^{1/2}\right] dy$$

$$= \left[2y - \frac{y^2}{2} - \frac{2y^{3/2}}{3}\right]_0^1$$

$$= 2 - \frac{1}{2} - \frac{2}{3} = \boxed{\frac{5}{6}}$$

(OR) ALT: $\int_{x=0}^{x=1} [x^2]\, dx + \int_1^2 [2-x] dx = \left[\frac{x^3}{3}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2 = \frac{1}{3} + \left[(4-2) - \left(2 - \frac{1}{2}\right)\right] = \frac{5}{6}$

4. NO CALCULATOR. Let R be the shaded region enclosed by the graphs of $y = \frac{2x}{x^2+1}$ and $y = x^3$ as shown in the figure below. Find the area of region R.



Note: the graphs intersect at (1,1)

Symmetry

$$A = 2\int_0^1 \left[\frac{2x}{x^2+1} - x^3\right] dx$$

$$= 2\left[\ln(x^2+1) - \frac{x^4}{4}\right]_0^1$$

$$= 2\left[\left(\ln(2) - \frac{1}{4}\right) - \left(\cancel{\ln(1)}^{0} - 0\right)\right]$$

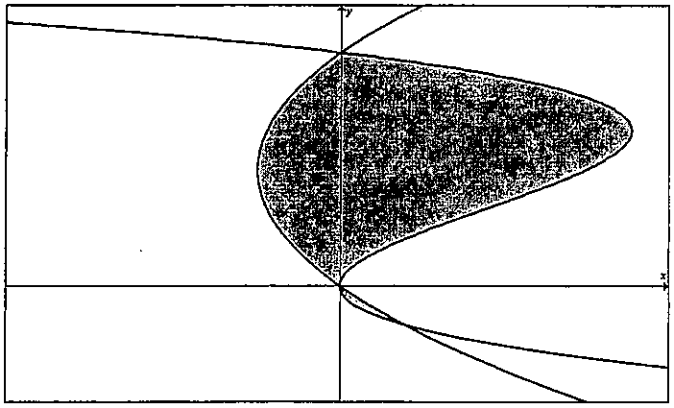
$$= \boxed{2\ln(2) - \frac{1}{2}}$$

$\int \frac{2x}{x^2+1} dx$

$u = x^2 + 1$

$du = 2x\, dx$

5. CALCULATOR PERMITTED. Let R be the shaded region enclosed by the graphs of $x = 2y^2 - 2y$ and $x = 12y^2 - 12y^3$ as shown in the figure below. Find the area of region R.



$f(y) = 2y^2 - 2y \qquad g(y) = 12y^2 - 12y^3$

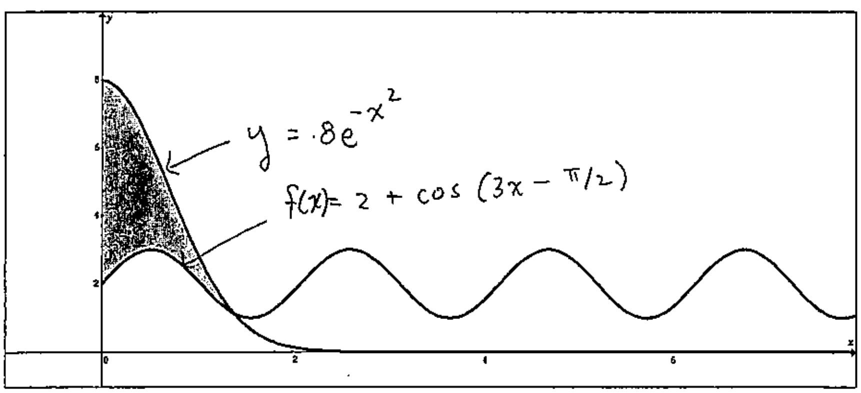
$f(y) = g(y)$ at $y = 1, 0, -\frac{1}{6}$

(If you are up for an extreme arithmetic challenge, try to find the answer to #5 without a calculator)

$$A = \int_{-1/6}^{0} \left[(2y^2 - 2y) - (12y^2 - 12y^3)\right] dy + \int_{0}^{1} \left[(12y^2 - 12y^3) - (2y^2 - 2y)\right] dy = \boxed{\frac{1741}{1296}}$$

$$A = \int_{-1/6}^{0} [f(y) - g(y)]\, dy + \int_{0}^{1} [g(y) - f(y)]\, dy = \frac{1741}{1296}$$

6. CALCULATOR REQUIRED Let R be the shaded region in the first quadrant enclosed by the graphs of $x = \sqrt{-\ln(y/8)}$ and $f(x) = 2 + \cos(3x - \pi/2)$ as shown in the figure below. Find the area of region R. Hint: You must perform preliminary work before using your calculator.



$y = f(x)$ at $x = 1.399$

$X = \sqrt{-\ln(y/8)}$

$x^2 = -\ln(y/8)$

$-x^2 = \ln(y/8)$

$e^{-x^2} = \frac{y}{8}$

$8e^{-x^2} = y$

$$A = \int_{0}^{1.399} \left[\left(8e^{-x^2}\right) - \left(2 + \cos(3x - \pi/2)\right)\right] dx = \boxed{3.455}$$

36
$\times 6$

$180+36 = 216$

$$\int_{-1/6}^{0} \left[(2y^2-2y)-(12y^2-12y^3)\right]dy + \int_{0}^{1} \left[(12y^2-12y^3)-(2y^2-2y)\right]dy$$

3
16
$\times 6$
96

$$\int_{-1/6}^{0} \left[2y^2 - 2y - 12y^2 + 12y^3\right]dy = \int_{-1/6}^{0}\left[-10y^2 - 2y + 12y^3\right]dy = \left(-\frac{10y^3}{3} - y^2 + \frac{12y^4}{4}\right)_{-1/6}^{0}$$

$$= \left(-\frac{10y^3}{3} - y^2 + 3y^4\right)_{-1/6}^{0} = 0 - \left[-\frac{10(-1/6)^3}{3} - (1/6)^2 + 3(1/6)^4\right]$$

$$= -\left[\frac{10}{3}\left(\frac{1}{216}\right) - \frac{1}{36} + 3\left(\frac{1}{1296}\right)\right]$$

$$= -\left[\frac{\left(\frac{10}{3}\right)(6)}{1296} - \frac{6^2}{1296} + \frac{3}{1296}\right]$$

$$= -\left[\frac{+20 - 36 + 3}{1296}\right]$$

$$= \frac{13}{1296}$$

$$\int_{0}^{1} 12y^2 - 12y^3 - 2y^2 + 2y = \int_0^1 10y^2 - 12y^3 + 2y = \left(\frac{10y^3}{3} - \frac{12y^4}{4} + y^2\right)_0^1$$

$$= \left(\frac{10y^3}{3} - 3y^4 + y^2\right)_0^1$$

$$= \frac{10}{3} - 3 + 1$$

$$= \frac{10}{3} - 2$$

$$= \frac{4}{3}$$

$$\frac{(432)(4) + 13}{1296}$$

$$\boxed{\frac{1741}{1296}}$$

$3 \overline{)1296}$ = 432
-12
$0 9$

432
$\times 4$
1728

1728
$+ 13$
1741

Compliments