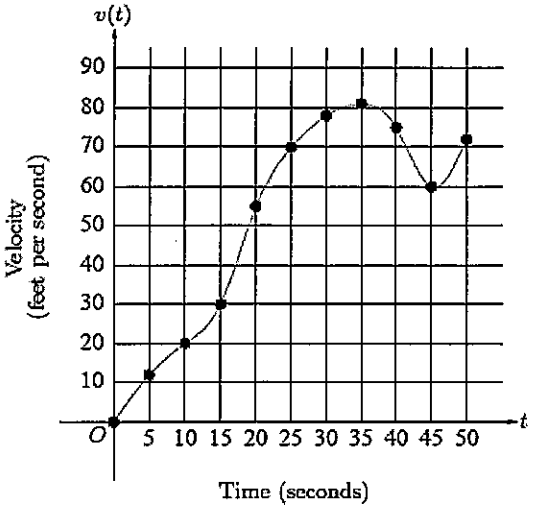
HW1: 1998 #3



t (seconds)	$v(t)$ (feet per second)
0	0
5	12
10	20
15	30
20	55
25	70
30	78
35	81
40	75
45	60
50	72

3. The graph of the velocity $v(t)$, in ft/sec, of a car traveling on a straight road, for $0 \le t \le 50$, is shown above. A table of values for $v(t)$, at 5 second intervals of time t, is shown to the right of the graph.
   (a) During what intervals of time is the acceleration of the car positive? Give a reason for your answer.
   (b) Find the average acceleration of the car, in ft/sec^2, over the interval $0 \le t \le 50$.
   (c) Find one approximation for the acceleration of the car, in ft/sec^2, at $t = 40$. Show the computations you used to arrive at your answer.
   (d) Approximate $\int_0^{50} v(t)\,dt$ with a Riemann sum, using the midpoints of five subintervals of equal length. Using correct units, explain the meaning of this integral.

(a) $a(t) > 0$ on $(0,35) \cup (45,50)$ b/c $v'(t) > 0$ on this interval
(OR) b/c $v(t)$ is increasing on this interval.

(b) ave rate Δ in $v(t) = \dfrac{v(50) - v(0)}{50 - 0} = 1.44 \text{ ft/sec}^2$

(c) $a(40) \approx \dfrac{v(45) - v(35)}{45 - 35} = \dfrac{60 - 81}{10} = -2.1 \text{ ft/sec}^2$ (Average on a small neighborhood method)

(d) $\int_0^{50} v(t)\,dt \approx \text{MRAM} = \Delta t \left[v(5) + v(15) + v(25) + v(35) + v(45) \right]$

$= 10 \left[12 + 30 + 70 + 81 + 60 \right]$

$= 2530 \text{ ft}$

This is the total distance traveled by the car in the first 50 sec. NOTE $v(t) \ge 0$ on $0 \le t \le 50$ (otherwise the integral is displacement)

HW2: 2004FB #3

t (minutes)	0	5	10	15	20	25	30	35	40
$v(t)$ (miles per minute)	7.0	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3

3. A test plane flies in a straight line with positive velocity $v(t)$, in miles per minute at time t minutes, where v is a differentiable function of t. Selected values of $v(t)$ for $0 \le t \le 40$ are shown in the table above.

   (a) Use a midpoint Riemann sum with four subintervals of equal length and values from the table to approximate $\int_0^{40} v(t)\,dt$. Show the computations that lead to your answer. Using correct units, explain the meaning of $\int_0^{40} v(t)\,dt$ in terms of the plane's flight.

   (b) Based on the values in the table, what is the smallest number of instances at which the acceleration of the plane could equal zero on the open interval $0 < t < 40$? Justify your answer.

   (c) The function f, defined by $f(t) = 6 + \cos\left(\frac{t}{10}\right) + 3\sin\left(\frac{7t}{40}\right)$, is used to model the velocity of the plane, in miles per minute, for $0 \le t \le 40$. According to this model, what is the acceleration of the plane at $t = 23$? Indicate units of measure.

   (d) According to the model f, given in part (c), what is the average velocity of the plane, in miles per minute, over the time interval $0 \le t \le 40$?

[SEE ATTACHED SHEET]
(on Back)

2004FB #3

a) $\int_0^{40} v(t)\,dt \approx MRAM = \Delta t\left[9.2 + 7.0 + 2.4 + 4.3\right] = 10\left[9.2 + 7.0 + 2.4 + 4.3\right] = 229$

The integral gives the total distance, in miles, that the plane flew over this 40 min interval $[0, 40]$ b/c $v(t) > 0$.

b)

t	0	5	10	15	20	25	30	35	40
$v(t)$	7	9.2	9.5	7.0	4.5	2.4	2.4	4.3	7.3
Ave $v'(t)$		0.44	0.06	−0.5	−0.5	−0.42	0	0.38	0.6

← ave $a(t)$

$a(t) = v'(t)$

- On the interval $5 < t < 15$ $a(t)$ equal 0.06 and −0.5 at least once by the M.V.T. This also means that $a(t)$ must pass through zero at least one time on this interval (By the Intermediate Value Theorem).
- The average $a(t)$ on the interval $25 < t < 30$ is zero. By the M.V.T, $a(t)$ must equal zero at least one time on this interval.

∴ $a(t) = 0$ at least two times on $0 < t < 40$.

* Essentially we are looking that average $a(t)$ changes sign. *

c) $f'(23) = -0.408$ miles/min^2

There are many ways you can get this answer. Here is what I did

- $y1(x) = 6 + \cos(x/10) + 3\sin(7x/10)$
- Home: $d(y1(x), x) \mid x = 23$

  ↑ type in equals button

  ↑ button below the equals button

This sure beats any copy and paste ☺

d) Av. Velocity Valve $= \frac{1}{40}\int_0^{40} f(t)\,dt = 5.916$ miles per minute.

HW3: 2008 #2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

2. Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \le t \le 9$. Values of $L(t)$ at various times t are shown in the table above.

   (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.

   (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.

   (c) For $0 \le t \le 9$, what is the fewest number of times at which $L'(t)$ must equal 0 ? Give a reason for your answer.

   (d) The rate at which tickets were sold for $0 \le t \le 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ($t = 3$), to the nearest whole number?

a) $L'(5.5) \approx \dfrac{L(7)-L(4)}{7-4} = \dfrac{150-126}{3} = 8$ people per hour

This is "average on a small neighborhood method."

b) $\dfrac{\int_0^4 L(t)\,dt}{4-0} = \dfrac{1}{4}\int_0^4 L(t)\,dt \approx \dfrac{1}{4}\,TRAM = \dfrac{1}{4}\left[\dfrac{120+156}{2}(1)+\dfrac{156+176}{2}(2)+\dfrac{176+126}{2}(1)\right]$

$= 155.25$ people

c)

	0	1	3	4	7	8	9
	120	156	176	126	150	80	0
Ave $L'(t)$		36	10	−50	8	−70	−80

☐ Ave $L'(t) = 36, 10, -50, 8, -70, -80$ at least one time on $0 < t < 9$ by the M.V.T.

☐ The average $L'(t)$ changes sign three times and so $L'(t)$ must equal zero at least three times by the Intermediate Value Theorem

d) $\int_0^3 r(t)\,dt \approx 973$ tickets