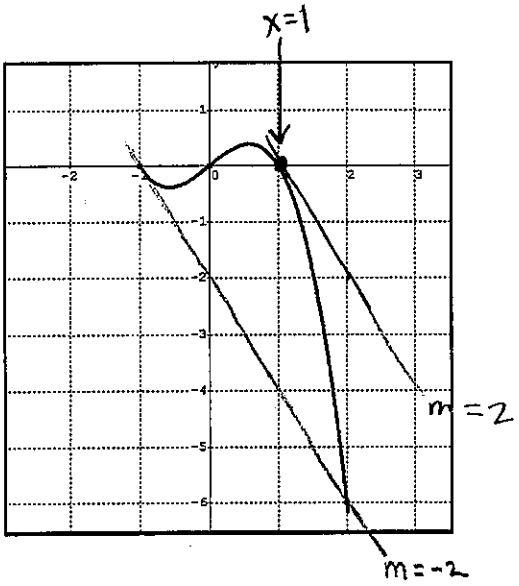
M.V.T Solutions

3. Consider the function $y = x - x^3$ on the interval [-1, 2]. ***Calculators Permitted***

A. Find the average rate of change of y on [-1, 2] $y = f(x)$

$$\text{Ave rate } \Delta = \frac{f(2)-f(-1)}{2-(-1)} = \frac{(2-8)-(-1+1)}{3} = \frac{-6}{3} = \boxed{-2}$$

B. Find the value of x that satisfies the conclusion to the mean value theorem for derivatives.
Make a sketch supporting your findings.



$$f'(x) = 1 - 3x^2 = \text{ave rate } \Delta$$

$$1 - 3x^2 = -2$$

$$-3x^2 = -3$$

$$x^2 = 1$$

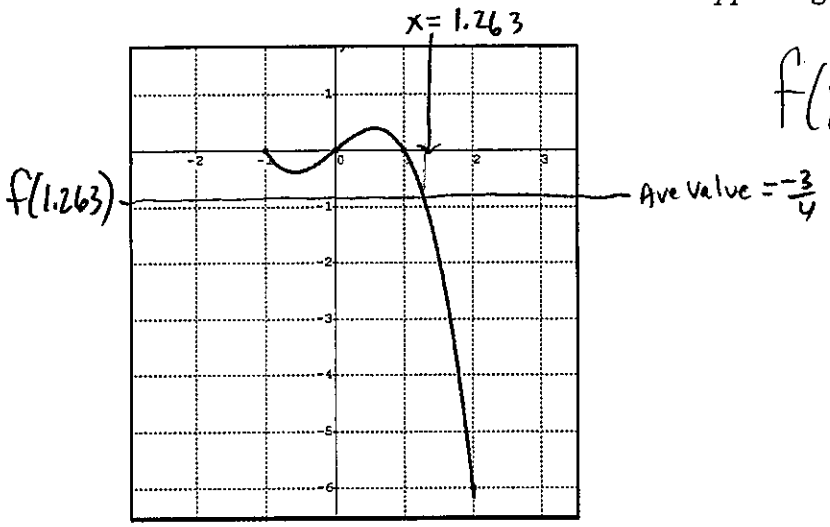
$$x = 1 \in (-1, 2)$$

NOTE: $x=-1$ works BUT $-1 \notin (-1, 2)$

C. Find the average value of y on [-1, 2].

$$\text{Ave Value} = \frac{\int_{-1}^{2} f(x)\,dx}{2-(-1)} = \frac{1}{3}\left[\frac{x^2}{2} - \frac{x^4}{4}\right]_{-1}^{2} = \frac{1}{3}\left[(2-4)-\left(\frac{1}{2}-\frac{1}{4}\right)\right] = \frac{1}{3}\cdot\frac{-9}{4} = \boxed{\frac{-3}{4}}$$

D. Find the value of x that satisfies the conclusion to the mean value theorem for integrals.
Make a sketch supporting your findings.



$$f(x) = x - x^3 = \text{ave value}$$

$$x - x^3 = -\frac{3}{4}$$

$$x - x^3 + \frac{3}{4} = 0$$

$$\boxed{x = 1.263}$$

M.V.T Solutions

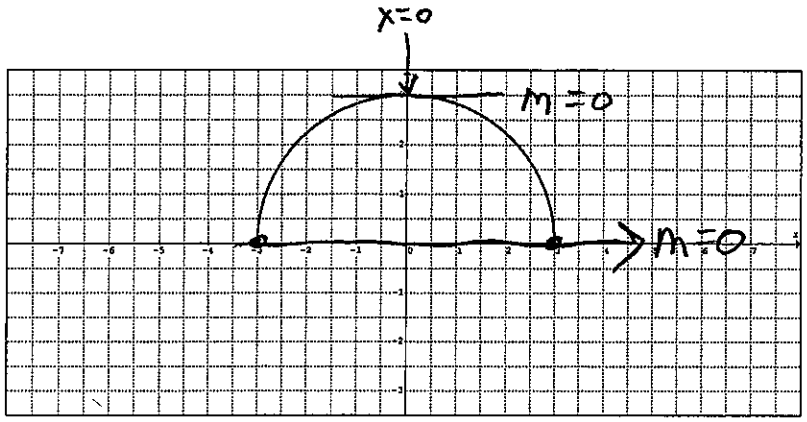
4. Consider the function $y = \sqrt{9 - x^2}$ on the interval [-3, 3]. ***Calculators Permitted***

A. Find the average rate of change of y on [-3, 3]

$$\text{Ave rate } \Delta = \frac{y(3) - y(-3)}{3 - (-3)} = \frac{0}{9} = 0$$

B. Find the value of x that satisfies the conclusion to the mean value theorem for derivatives.

Make a sketch supporting your findings.



$$y'(x) = \frac{1}{2}(9 - x^2)^{-1/2}(-2x)$$

$$y'(x) = \frac{-x}{\sqrt{9 - x^2}} = 0$$

$$x = 0$$

or by observation of semi-circle.

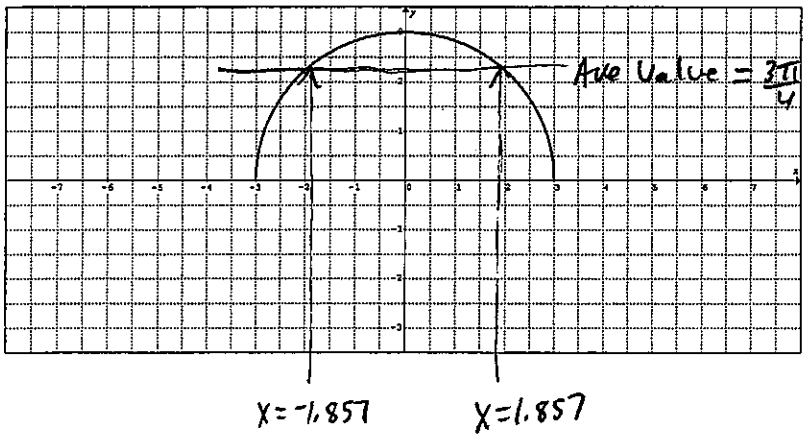
C. Find the average value of y on [-3, 3].

$$\text{Ave Value} = \frac{\int_{-3}^{3} \sqrt{9 - x^2}}{3 - (-3)} = \frac{\frac{\pi(3)^2}{2}}{6} = \frac{3\pi}{4} \approx 2.356$$

← appeal to geometry (area of semi circle)

D. Find the value of x that satisfies the conclusion to the mean value theorem for integrals.

Make a sketch supporting your findings.



$$\sqrt{9 - x^2} = \frac{3\pi}{4}$$

$$\sqrt{9 - x^2} - \frac{3\pi}{4} = 0$$

$$x = -1.857 \text{ or } 1.857$$