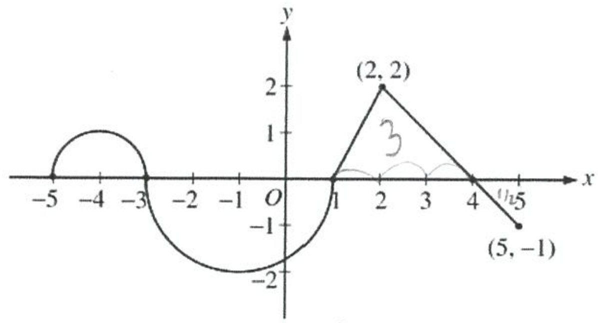
AB Q304 LESSON 1A

GUIDED PRACTICE 1: 2007FB #4



Graph of f'

4. Let f be a function defined on the closed interval $-5 \le x \le 5$ with $f(1) = 3$. The graph of f', the derivative of f, consists of two semicircles and two line segments, as shown above.
   (a) For $-5 < x < 5$, find all values x at which f has a relative maximum. Justify your answer.
   (b) For $-5 < x < 5$, find all values x at which the graph of f has a point of inflection. Justify your answer.
   (c) Find all intervals on which the graph of f is concave up and also has positive slope. Explain your reasoning.
   (d) Find the absolute minimum value of $f(x)$ over the closed interval $-5 \le x \le 5$. Explain your reasoning.

(a) f has a rel. max at $x = -3$ and $x = 4$ b/c $\boxed{+1}$
$f'(x) = 0$ AND $f'(x)$ goes from positive to negative at these x-values. $\boxed{+1}$

(b) f has a point of inflection at $x = -4$, $x = -1$, $x = 2$ $\boxed{+1}$
b/c $f''(x)$ changes sign at these x-values. $\boxed{+1}$
[$f'(x)$ changes its increasing / decreasing behavior]

(c) $(-5,-4) \cup (1,2)$ b/c on this interval both $\boxed{+1}$
and $\begin{cases} f'(x) > 0 & [f' \text{ graph above } x\text{-axis}] \\ f''(x) > 0 & [f'(x) \text{ is increasing}] \end{cases}$ $\boxed{+1}$

(d) Interior critical x-values: $f'(x) = 0$ at $x = -3, 1, 4$
endpoint critical x-values: $x = -5$, $x = 5$

$f(-5) = 3 + 3\pi/2$
$f(-3) \rightarrow$ local max ✗
$\boxed{+1}$ $\boxed{+1}$ $f(1) = 3$ (Given)
$f(4) \rightarrow$ local max ✗
$f(5) = 6 - \frac{1}{2}$

absolute min $= \boxed{3}$ +1

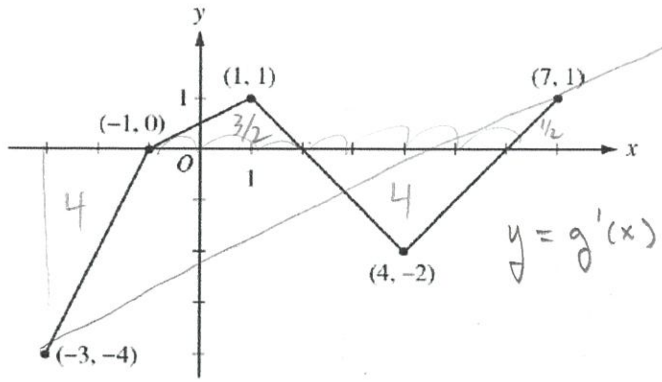
$\rightarrow \int_{-5}^{1} f'(x)\,dx = f(1) - f(-5)$
$\frac{1}{2}\pi - \frac{1}{2}\pi(2)^2 = 3 - f(-5)$
$-\frac{3\pi}{2} = 3 - f(-5) \rightarrow f(-5) = 3 + \frac{3\pi}{2}$

$\rightarrow \int_{1}^{5} f'(x)\,dx = f(5) - f(1)$
$3 - \frac{1}{2} = f(5) - 3 \rightarrow f(5) = 6 - \frac{1}{2}$

D] Ave rate Δ of $g'(x) = \dfrac{g'(7) - g'(-3)}{7-(-3)} = \dfrac{1-(-4)}{10} = \dfrac{5}{10} = \dfrac{1}{2}$ +1

Mean value Thm requires $\begin{cases} g'(x) \text{ to be continuous on } [-3, 7] \checkmark \text{ which it is} \checkmark \\ \text{and } g'(x) \text{ to be diff. on } (-3, 7) \text{ which it fails to be at the corners} \end{cases}$

GUIDED PRACTICE 2: 2008FB#5



Graph of g'

$\rightarrow m = \frac{1}{2}$

M.V.T Cannot guarantee. +1

5. Let g be a continuous function with $g(2) = 5$. The graph of the piecewise-linear function g', the derivative of g, is shown above for $-3 \le x \le 7$.

(a) Find the x-coordinate of all points of inflection of the graph of $y = g(x)$ for $-3 < x < 7$. Justify your answer.

(b) Find the absolute maximum value of g on the interval $-3 \le x \le 7$. Justify your answer.

(c) Find the average rate of change of $g(x)$ on the interval $-3 \le x \le 7$.

(d) Find the average rate of change of $g'(x)$ on the interval $-3 \le x \le 7$. Does the Mean Value Theorem applied on the interval $-3 \le x \le 7$ guarantee a value of c, for $-3 < c < 7$, such that $g''(c)$ is equal to this average rate of change? Why or why not?

+1 +1 (a) $x = 1$, $x = 4$ b/c $g''(x)$ changes sign at these x values

(b) interior critical: $g'(x) = 0$ at $x = -1$, $x = 2$, $x = 6$

endpoint critical: $x = -3$, $x = 7$

$g(-3) = 5 + 4 - 3/2$

$g(-1) \Rightarrow$ local min X

+1 +1 $g(2) = 5$

$g(6) \rightarrow$ local min X

$g(7) = 5 - 4 + 1/2$

$\int_{-3}^{2} g'(x)\,dx = g(2) - g(-3)$

$-4 + \frac{3}{2} = 5 - g(-3)$

$\rightarrow g(-3) = 5 + 4 - \frac{3}{2}$

$\int_{2}^{7} g'(x)\,dx = g(7) - g(2)$

$-4 + \frac{1}{2} = g(7) - 5$

$\rightarrow g(7) = 5 - 4 + \frac{1}{2}$

absolute max $= \boxed{5 + 4 - \frac{3}{2}}$ OR $\frac{15}{2}$ OR 7.5

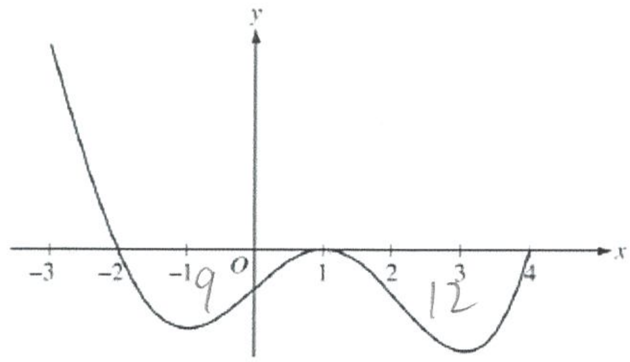
+1

(c) Ave rate Δ of $g(x)$ $\overset{OR}{\equiv}$ Ave value of $g'(x)$

+1 $\dfrac{g(7) - g(-3)}{7-(-3)}$ $\overset{OR}{\equiv}$ $\dfrac{\int_{-3}^{7} g'(x)\,dx}{7-(-3)}$

$\dfrac{5 - 4 + 1/2 - 5 - 4 + 3/2}{10}$ $\overset{OR}{\equiv}$ $\dfrac{-4 + 3/2 - 4 + 1/2}{10} = \dfrac{-8 + 2}{10} = \dfrac{-6}{10} = \boxed{\dfrac{-3}{5}}$ +1

GUIDED PRACTICE 3: 2015 #5



Graph of f'

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval $[-3, 4]$. The graph of f' has horizontal tangents at $x = -1$, $x = 1$, and $x = 3$. The areas of the regions bounded by the x-axis and the graph of f' on the intervals $[-2, 1]$ and $[1, 4]$ are 9 and 12, respectively.

(a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.

(b) On what open intervals contained in $-3 < x < 4$ is the graph of f both concave down and decreasing? Give a reason for your answer.

(c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.

(d) Given that $f(1) = 3$, write an expression for $f(x)$ that involves an integral. Find $f(4)$ and $f(-2)$.

$\boxed{+1}$ $\boxed{+1}$ (a) rel. max at $x = -2$ b/c $f'(x) = 0$ and $f'(x)$ goes from positive to negative at $x = -2$

(b) $\boxed{+1}$ $\boxed{+1}$ f is both concave down and decreasing on $(-2, -1) \cup (1, 3)$ b/c on this interval both

$f''(x) < 0$ "f' graph decreasing"

and

$f'(x) < 0$ "f' graph below x-axis"

(c) $\boxed{+1}$ $\boxed{+1}$ $x = -1$, $x = 1$, $x = 3$ b/c at these x-values $f''(x)$ changes sign. {Verticies of the f' graph}

(d) $\int_1^x f'(t)\,dt = f(x) - f(1) \rightarrow f(x) = 3 + \int_1^x \underbrace{f'(t)\,dt}_{\text{use a dummy variable}}$

$\int_1^4 f'(x)\,dx = f(4) - f(1)$

$-12 = f(4) - 3$

$\therefore f(4) = -9$ $\boxed{+1}$

$\int_{-2}^1 f'(x)\,dx = f(1) - f(-2)$

$-9 = 3 - f(-2)$

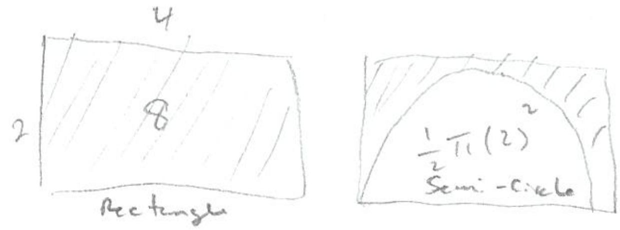
$-12 = -f(-2)$

$f(-2) = 12$ $\boxed{+1}$

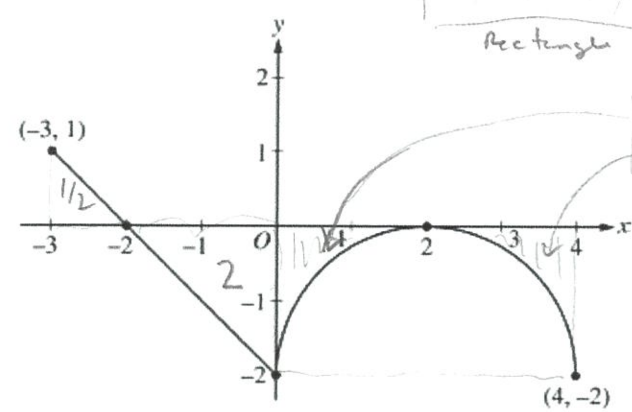
integrand $\boxed{+1}$

$f(x)$ $\boxed{+1}$

GUIDED PRACTICE 4: 2003 #4



$\boxed{8 - 2\pi} = $ Area



Graph of f'

Let f be a function defined on the closed interval $-3 \le x \le 4$ with $f(0) = 3$. The graph of f', the derivative of f, consists of one line segment and a semicircle, as shown above.

(a) On what intervals, if any, is f increasing? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f on the open interval $-3 < x < 4$. Justify your answer.

(c) Find an equation for the line tangent to the graph of f at the point $(0, 3)$.

(d) Find $f(-3)$ and $f(4)$. Show the work that leads to your answers.

+1 +1 (a) $x \in [-3, -2]$ b/c $f'(x) > 0$ on $(-3, -2) \rightarrow$ or on $[-3, -2)$

↑ we technically need brackets but would may get away with ( ).

+1 +1 (b) $x = 0$, $x = 2$ b/c $f''(x)$ changes sign at these x-values

(c) $y - f(a) = f'(a)(x - a) \rightarrow y = f(a) + f'(a)(x - a)$

$f(0) = 3$ $\quad$ $\underline{f'(0) = -2 \text{ (point on graph)}}$

+1 $\boxed{y = 3 - 2(x - 0)}$ or $y = -2x + 3$

(d) $\int_{-3}^{0} f'(x)\,dx = f(0) - f(-3)$ $\qquad$ $\int_{0}^{4} f'(x)\,dx = f(4) - f(0)$

+1 $\frac{1}{2} - 2 = 3 - f(-3)$ $\qquad$ +1 $-(8 - 2\pi) = f(4) - 3$

$\boxed{f(-3) = 3 + 2 - \frac{1}{2}}$ +1 $\qquad$ $\boxed{f(4) = 3 - (8 - 2\pi)}$ +1

OR $\qquad$ OR

$9/2$ $\qquad$ $(2\pi - 5)$

OR

4.5

below x-axis