

Q302. LESSON 3 HW Solutions

1. WOLVES $\frac{dP}{dt} = k(800 - P)$

$$A] \int \frac{dP}{800-P} = \int k dt$$

$$-\ln|800-P| = kt + C$$

$$\ln|800-P| = -kt + C$$

$$|800-P| = Ce^{-kt}$$

$$800-P = C^*e^{-kt}$$

$$P = 800 - C^*e^{-kt}$$

$$500 = 800 - C^*e^0$$

$$\therefore C^* = 300$$

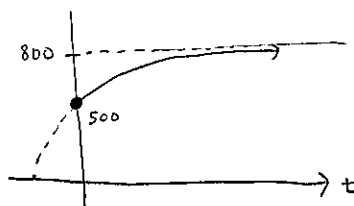
$$P = 800 - 300e^{-kt}$$

$$B] 700 = 800 - 300e^{-2k}$$

$$\frac{1}{3} = e^{-2k}$$

$$k = \frac{\ln(\frac{1}{3})}{-2}$$

$$P = 800 - 300\left(\frac{1}{3}\right)^{\frac{t}{2}}$$



$$C] P(6) = 800 - 300\left(\frac{1}{3}\right)^3$$

$$= 800 - \frac{300}{27} \rightarrow \text{approx } 11$$

$$= 800 - \frac{100}{9}$$

$$\approx 789 \text{ wolves}$$

$$D] \lim_{t \rightarrow \infty} P(t) = \lim_{t \rightarrow \infty} 800 - 300\left(\frac{1}{3}\right)^t$$

$$= 800 \text{ wolves}$$

2. MEL SNAKE

let M = length (inches) of a Mel Snake at time t months

$$\frac{dM}{dt} = k(40 - M)$$

$$t=0 \rightarrow M=2$$

$$t=4 \rightarrow M=21$$

$$A] \int \frac{dM}{40-M} = \int k dt$$

$$-\ln|40-M| = kt + C$$

$$\ln|40-M| = -kt + C$$

$$|40-M| = Ce^{-kt}$$

$$40-M = C^*e^{-kt}$$

$$M = 40 - C^*e^{-kt}$$

$$2 = 40 - C^*e^0 \therefore C^* = 38$$

$$M = 40 - 38e^{-kt}$$

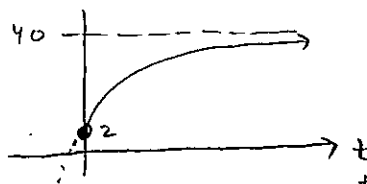
$$B] 21 = 40 - 38e^{-4k}$$

$$\frac{19}{38} = e^{-4k}$$

$$\frac{1}{2} = e^{-4k}$$

$$k = \frac{\ln(\frac{1}{2})}{-4}$$

$$M = 40 - 38\left(\frac{1}{2}\right)^{\frac{t}{4}}$$



$$C] \frac{141}{4} = 40 - 38\left(\frac{1}{2}\right)^{\frac{t}{4}}$$

$$\left(\frac{141}{4} - \frac{160}{4}\right) = -38\left(\frac{1}{2}\right)^{\frac{t}{4}}$$

$$\begin{aligned} -\frac{19}{4} &= -38\left(\frac{1}{2}\right)^{\frac{t}{4}} \\ \frac{19}{4(38)} &= \left(\frac{1}{2}\right)^{\frac{t}{4}} \\ \frac{1}{8} &= \left(\frac{1}{2}\right)^{\frac{t}{4}} \end{aligned}$$

$$\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^{\frac{t}{4}}$$

$$\frac{t}{4} = 3$$

$$t = 12 \text{ months}$$

3. $T = \text{temp. } (^{\circ}\text{C})$ of water at time t minutes

$$\frac{dT}{dt} = h(18 - T) \quad \begin{array}{l} t=0 \rightarrow T=98 \\ t=5 \rightarrow T=38 \end{array}$$

A] $\int \frac{dT}{18-T} = \int h dt$

$$-\ln|18-T| = ht + C$$

$$\ln|18-T| = -ht + C$$

$$|18-T| = Ce^{-ht}$$

$$18-T = C^* e^{-ht}$$

$$T = 18 - C^* e^{-ht}$$

$$98 = 18 - C^* e^0 \quad \therefore C^* = -80$$

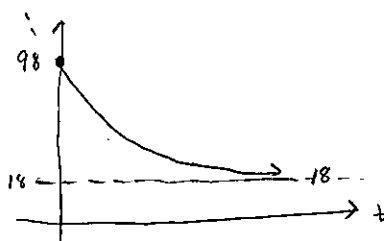
$$T = 18 + 80e^{-ht}$$

B] $38 = 18 + 80e^{-5h}$

$$\frac{20}{80} = e^{-5h}$$

$$h = \frac{\ln(\frac{1}{4})}{-5}$$

$$T = 18 + 80\left(\frac{1}{4}\right)^{t/5}$$



C] $20 = 18 + 80\left(\frac{1}{4}\right)^{t/5}$

$$\left(\frac{2}{80}\right) = \left(\frac{1}{4}\right)^{t/5}$$

$$\frac{1}{40} = \left(\frac{1}{4}\right)^{t/5}$$

$$\ln\left(\frac{1}{40}\right) = \frac{t}{5} \ln\left(\frac{1}{4}\right)$$

$$\frac{t}{5} = \frac{\ln\left(\frac{1}{40}\right)}{\ln\left(\frac{1}{4}\right)}$$

$$t = \frac{5 \ln\left(\frac{1}{40}\right)}{\ln\left(\frac{1}{4}\right)} \text{ min}$$

4. $t=0 \quad V=100$

$$\int \frac{dG}{200-G} = \int -\ln\left(\frac{1}{10}\right) dt$$

$$-\ln|200-G| = -\ln\left(\frac{1}{10}\right)t + C$$

$$\ln|200-G| = \ln\left(\frac{1}{10}\right)t + C$$

$$|200-G| = Ce^{\ln(\frac{1}{10})t}$$

$$200-G = C^*\left(\frac{1}{10}\right)^t$$

$$G = 200 - C^*\left(\frac{1}{10}\right)^t$$

$$100 = 200 - C^* \quad \therefore C^* = 100$$

$$G = 200 - 100\left(\frac{1}{10}\right)^t$$

A] $G(2) = 200 - 100\left(\frac{1}{10}\right)^2 = 199 \text{ cm}^3$

B] $\lim_{t \rightarrow \infty} G(t) = \lim_{t \rightarrow \infty} 200 - 100\left(\frac{1}{10}\right)^t = 200 \text{ cm}^3$