Q302. CH6 Lesson 2 HW Solutions

#1 $\frac{dy}{dx} = e^{y}(3x^2 - 6x) \qquad f(1) = 0$

(a) $L(x) = f(1) + f'(1,0)(x-1)$

$L(x) = 0 + [e^{0}(3-6)](x-1)$

$L(x) = -3(x-1)$

$f(1.2) \approx L(1.2) = -3(1.2-1) = -3(0.2) = -0.6$

(b) $\frac{dy}{e^{y}} = (3x^2 - 6x)dx$

$\int e^{-y}dy = \int (3x^2 - 6x)dx$

$-e^{-y} = x^3 - 3x^2 + c$ (OR) $-\frac{1}{e^{y}} = x^3 - 3x^2 + c$

$e^{-y} = -x^3 + 3x^2 + c$

$-y = \ln(-x^3 + 3x^2 + c)$

$y = -\ln(-x^3 + 3x^2 + c)$

$0 = -\ln(-1 + 3 + c)$

$-1 + 3 + c = 1$

$c = -1$

$\therefore \boxed{y = -\ln(-x^3 + 3x^2 - 1)}$

(OR):

$e^{y} = \frac{-1}{x^3 - 3x^2 + c}$

$y = \ln\left(\frac{-1}{x^3 - 3x^2 + c}\right)$

$y = \ln\left(\frac{1}{-x^3 + 3x^2 + c}\right)$

$c = -1$

$y = \ln\left(\frac{1}{-x^3 + 3x^2 - 1}\right)$

$y = \ln(1) - \ln(-x^3 + 3x^2 - 1)$

$\boxed{y = -\ln(-x^3 + 3x^2 - 1)}$

B] $f''(x,y) = \frac{d}{dx}[e^{y}(3x^2 - 6x)] = \frac{d}{dx}[3x^2e^{y} - 6xe^{y}]$

$\frac{d^2y}{dx^2} = 3x^2e^{y}\frac{dy}{dx} + 6xe^{y} - 6xe^{y}\frac{dy}{dx} - 6e^{y}$

$\left.\frac{d^2y}{dx^2}\right|_{x=1,\, y=0,\, \frac{dy}{dx}=-3} = 3(1)(1)(-3) + 6(1)(1) - 6(1)(1)(-3) - 6(1)$

$= \boxed{9} > 0$

$\therefore$ The approx in part (b) is an under-estimation

#2 $\frac{dy}{dx} = x(2-y)$

(a) $\int \frac{dy}{2-y} = \int x\,dx \rightarrow -\ln|2-y| = \frac{x^2}{2} + c \rightarrow \ln|2-y| = \frac{-x^2}{2} + c$

$|2-y| = Ce^{-x^2/2} \rightarrow 2-y = c^* e^{-x^2/2} \rightarrow y = 2 - c^* e^{-x^2/2}$

$5 = 2 - C^* \cancel{e^0}^{\,1} \quad \therefore C^* = -3$

$\boxed{y = 2 + 3e^{-x^2/2}}$

(b) $\lim_{x\to\infty} 2 + 3e^{-x^2/2} = \lim_{x\to\infty} 2 + \cancelto{0}{\frac{3}{e^{x^2/2}}} = 2$

(c) $y(0.5) = 2.5$

$y'(0.5, 2.5) = 0.5(2-2.5) = 0.5(-0.5) = -0.25$

$L(x) = 2.5 - 0.25(x - 0.5)$

$y(1.5) \approx L(1.5) = 2.5 - 0.25(1.5 - 0.5) = 2.25$

distribute makes easier

(d) $\frac{d}{dx}\left(x(2-y)\right) = \frac{d}{dx}\left(2x - xy\right)$

$= 2 - \left(x\frac{dy}{dx} + y\right)\Big|_{x=0.5\ \ y=2.5\ \ \frac{dy}{dx}=-0.25}$

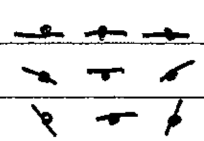
$= 2 - \left(\frac{1}{2}\left(-\frac{1}{4}\right) + \frac{5}{2}\right)$

$= 2 + \frac{1}{8} - \frac{5}{2} = \frac{16}{8} + \frac{1}{8} - \frac{20}{8} = -\frac{3}{8} < 0$

$\therefore$ y is concave down at (0.5, 2.5)

$\therefore$ The approximation found in part (c) is an <u>over</u> estimation of y at $x = 1.5$

(e)



$f'(1,0) = 2 \quad f'(0,y) = 0$

$f'(1,2) = 1$

#3 A. $\frac{dy}{dx} = \frac{x}{y^2}$

$\int y^2 dy = \int x\, dx$

$\frac{y^3}{3} = \frac{x^2}{2} + c$

$y^3 = \frac{3}{2}x^2 + c$

$\boxed{y = \sqrt[3]{\frac{3}{2}x^2 + c}}$

B. $\frac{dy}{dx} = \frac{xy^2}{1+x^2}$

$\frac{dy}{y^2} = \frac{x}{1+x^2}$ ∘∘ $u = 1+x^2$, $du = 2x\,dx$, $dx = \frac{du}{2x}$

$-y^{-1} = \frac{1}{2}\ln|1+x^2| + c$

$\boxed{y = \frac{-2}{\ln|1+x^2| + c}}$

$2550 \times 2 = 5100$

#4 $\frac{dy}{dx} = 100y - 5y^2 \qquad y(0) = 3$

$\left.\frac{dy}{dx}\right|_{x=0,\,y=3} = 100(3) - 5(3)^2 = 300 - 45 = \boxed{255}$

$\frac{d}{dx}(100y - 5y^2) = \left.100\frac{dy}{dx} - 10y\frac{dy}{dx}\right|_{x=0,\,y=3,\,\frac{dy}{dx}=255}$

$= 100(255) - 10(3)(255)$

$= \boxed{17{,}850}$

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$\frac{dy}{y^2} = \frac{x}{1+x^2}$ ... $u = 1+x^2$, $du = 2x\,dx$, $dx = \frac{du}{2x}$

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$\begin{array}{r} 2550 \\ \times\ 2 \\ \hline 5100 \end{array}$

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