AB. Q204. PRACTICE EXAM V. 2.0

A] $f(x) = \ln(2 + \sin x)$ $\pi \le x \le 2\pi$

INTERIOR CRITICAL x-values

$$f'(x) = \frac{1}{2+\sin x} \cdot \cos x = \frac{\cos x}{2 + \sin x} = 0$$

TRUE FOR $\cos x = 0$ $x = \frac{3\pi}{2}$

ENDPOINT CRITICAL x-VALUES

$x = \pi$ $x = 2\pi$

EVALUATION

$$f(\pi) = \ln(2 + \sin \pi) = \ln(2)$$

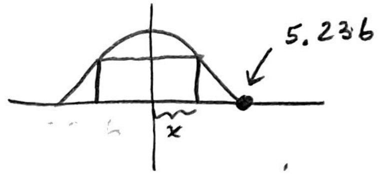
$$f\left(\frac{3\pi}{2}\right) = \ln\left(2 + \sin \frac{3\pi}{2}\right) = \ln(2-1) = \ln(1) = 0$$

$$f(2\pi) = \ln(2 + \sin 2\pi) = \ln(2)$$

Answer ABSOLUTE MAX = $\boxed{\ln 2}$

ABSOLUTE MIN = $\boxed{0}$

OPTIMIZATION



1. $A(x) = \underbrace{(2x)}_{\text{base}}\underbrace{(8\cos(0.3x))}_{\text{height}}$

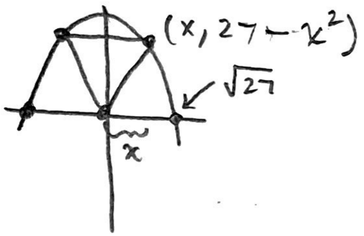
Domain: $0 < x < 5.236$ ← establishes an open interval

$A'(x) = 0$ at $x = 2.868$

$A'(x) > 0$ on $0 < x < 2.868$
$A'(x) < 0$ on $2.868 < x < 5.236$
$\therefore$ $A(x)$ has an absolute max at $x = 2.868$

Answer: $A(2.868) = \boxed{29.925 \text{ UNITS}^2}$

2. $A(x) = \frac{1}{2}$ base height $\therefore$ $A(x) = \frac{1}{2}(2x)(27 - x^2)$



$A(x) = x(27 - x^2) = 27x - x^3$

Domain: $0 < x < \sqrt{27}$

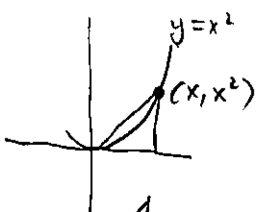
$A'(x) = 27 - 3x^2 = 0$ at $x = \cancel{-3}$ or $\underline{x = 3}$ OUT

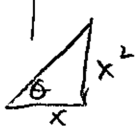
$A'(x) > 0$ on $(0, 3)$
$A'(x) < 0$ on $(3, \sqrt{27})$
$\therefore$ A is max (absolute) at $x = 3$

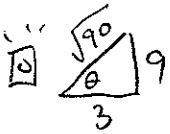
Answer: $A(3) = 27(3) - (3)^3 = \boxed{54 \text{ units}^2}$

3.

3.







$\cos\theta = \frac{3}{\sqrt{90}}$

$\sec\theta = \frac{\sqrt{90}}{3}$

Given | $\frac{dx}{dt} = 10$ m/sec

Find | $\frac{d\theta}{dt}$ when $x=3$

Rel | $\tan\theta = \frac{x^2}{x}$

$\tan\theta = x$

$\sec^2\theta \frac{d\theta}{dt} = \frac{dx}{dt}$

$\left(\frac{90}{9}\right)\frac{d\theta}{dt} = 10$

$\boxed{\frac{d\theta}{dt} = 1 \text{ radian/sec}}$

4.

$\frac{4.5}{6} = \frac{r}{h}$

$4.5h = 6r$

$r = \frac{15}{2}h$

Given | $\frac{dV}{dt} = -50\ m^3/min$

a) Find | $\frac{dh}{dt}$ when $h=5m$

Rel | $V = \frac{1}{3}\pi r^2 h$

UP | $V = \frac{1}{3}\pi\left(\frac{15}{2}h\right)^2 h$

$* \; V = \frac{225}{12}\pi h^3 \; *$

$\frac{dV}{dt} = \frac{225}{4}\pi h^2 \frac{dh}{dt}$

$-50 = \frac{225}{4}\pi(25)\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{-8}{225\pi} \text{ m/min}}$

b) Find $\frac{dr}{dt}$ when $h=5$

R | $r = \frac{15}{2}h$

$\frac{dr}{dt} = \frac{15}{2}\frac{dh}{dt}$

$\frac{dr}{dt} = \frac{15}{2}\cdot\frac{-8}{225\pi} = \boxed{\frac{-4}{15\pi} \text{ m/min}}$

5. $f(x) = xe^{2x} + 2x + 4$

$f'(x) = xe^{2x}\cdot 2 + e^{2x}(1) + 2 = 2xe^{2x} + e^{2x} + 2$

$f(0) = 0(e)^0 + 2(0) + 4 = 4$

$f'(0) = 2(0)e^0 + e^0 + 2 = 3$

$L(x) = 4 + 3(x-0)$

$f(-0.3) \approx L(-0.3) = 4 + 3(-0.3) = \boxed{3.1}$

6. $L(x) = f(1) + f'(1)(x-1)$ $\quad f'(1) = \ln(\cos^2(0)) + e^{\sin(0)}$

$L(x) = 3.156 + (\ 1\ )(x-1\ )$ $\quad = \ln(1) + e^0$

$L(x) = 3.156 + (x-1)$ $\quad = 0 + 1$

$= 1$

$f(1.216) \approx L(1.216) = 3.156 + (0.216) = \boxed{3.372}$

7. $\underset{[-6, \frac{13}{6}]}{\text{Ave rate } \Delta} = \dfrac{f(13/6) - f(-6)}{\frac{13}{6} - (-6)} = \dfrac{14 - 14}{\frac{13}{6} + 6} = 0$

$\text{Inst. rate } \Delta = f'(x) = \begin{cases} 2(x+2) & ; x < 1 \leftarrow [-6, 1) \\ 6 & ; x \geq 1 \leftarrow [1, 13/6] \end{cases}$

$f'(x) = 0$ $\quad 2x + 4 = 0$ $\quad 2x + 4 = 0$

or $\quad \rightarrow$

$\cancel{6 = 0}$ $\quad \boxed{x = -2} \in (-6, 1) \in (-6, 13/6)$

8. $\underset{[-2, 1]}{\text{Ave rate } \Delta} : \dfrac{f(1) - f(-2)}{1 - (-2)} = \dfrac{2.334}{3} = 0.778$

$\text{Inst. rate } \Delta = f'(x) = \cancel{0.785}$ 0.778 ~~at~~ $\underbrace{x = -1.310 \text{ or } x = 0.344}$

which both belong to the interval $(-2, 1)$ ✓

$y1 = \tan^{-1}(e^{x+2}) + \sin(x^2)$

Home: $(y1(1) - y1(-2))/(3)$

Home: $d(y1(x), x) \leftarrow$ Copy $\quad$ * you may notice cosh def... this is a special way to write the function.

$y2 = $ paste

$y3 = 0.785$ $\Big\rangle$ Find intersection [or] $y2 = \text{paste} - 0.778 \Big\rangle$ Find zeros

9. $$\lim_{x\to 0^+} \frac{\ln(x^2+2x)}{\ln x} = \lim_{x\to 0^+} \frac{\frac{1}{x^2+2x}(2x+2)}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{2x+2}{x^2+2x}\cdot\frac{x}{1}$$

"FORMAL"

$\lim_{x\to 0^+} \ln(x^2+2x) \to -\infty$

and

$\lim_{x\to 0^+} \ln(x) \to -\infty$

Indeterminant form $\left\{\frac{\infty}{\infty}\right\}$

Use L'HOPITAL'S Rule

$$= \lim_{x\to 0^+} \frac{2x+2}{x+2}$$

$$= \frac{2}{2}$$

$$= \boxed{1}$$

10. $$\lim_{x\to 0^+} \frac{\tan(x)}{2x} = \lim_{x\to 0^+} \frac{\sec^2 x}{2} = \boxed{\frac{1}{2}}$$

"FORMAL"

$\lim_{x\to 0^+} \tan(x) = 0$

and

$\lim_{x\to 0^+} 2x = 0$

Indeterminant form $\left\{\frac{0}{0}\right\}$

Use L'HOPITAL'S Rule

12. $$\lim_{x\to\infty} \frac{2x^6+8x^4}{4x^6+2} = \frac{2}{4} = \frac{1}{2}$$

"pre calc"

you do not need to do L'HÔPITAL'S Rule.

11. "RAPID FIRE"

$$\lim_{x\to 1} \frac{x^4-x^3-3x^2+5x-2}{x^4-5x^3+9x^2-7x+2} \quad \left\{\frac{0}{0}\right\}$$

$$= \lim_{x\to 1} \frac{4x^3-3x^2-6x+5}{4x^3-15x^2+18x-7} \quad \left\{\frac{0}{0}\right\}$$

$$= \lim_{x\to 1} \frac{12x^2-6x-6}{12x^2-30x+18} = \lim_{x\to 1} \frac{2x^2-x-1}{2x^2-5x+3} \quad \left\{\frac{0}{0}\right\}$$

$$= \lim_{x\to 1} \frac{4x-1}{4x-5} = \frac{3}{-1} = \boxed{-3}$$