AB Q204

CH8 Lesson 3 L'HOPITAL'S RULE (8.2)

HW Solutions

#5 $\lim_{x\to 0} \frac{1-\cos x}{x^2} \left\{\frac{0}{0}\right\} = \lim_{x\to 0} \frac{\sin x}{2x} \left\{\frac{0}{0}\right\} = \lim_{x\to 0} \frac{\cos x}{2} = \boxed{\frac{1}{2}}$

#13 $\lim_{x\to\pi} \frac{\csc x}{1+\cot x} \left\{\frac{\infty}{\infty}\right\} = \lim_{x\to\pi} \frac{-\csc x \cot x}{-\csc^2 x} = \lim_{x\to\pi} \frac{\cot x}{\csc x}$

$= \lim_{x\to\pi} \cos x = \boxed{-1}$

#17 $\lim_{x\to 0^+} (x \cdot \ln x) \{-0 \cdot \infty\} = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} \left\{\frac{\infty}{\infty}\right\}$

$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} -x = \boxed{0}$

#21 $\lim_{x\to 0} (e^x + x)^{1/x} \{1^\infty\}$ let $y = (e^x + x)^{1/x}$

$\ln y = \frac{1}{x}\ln(e^x + x)$

$\lim_{x\to 0} \ln y = \lim_{x\to 0} \frac{1}{x} \ln(e^x + x) \left\{\frac{0}{0}\right\} = \lim_{x\to 0} \frac{\frac{1}{e^x+x}(e^x+1)}{1}$

$= \frac{(e^0+1)}{(e^0+0)} = \frac{2}{1} = 2$

Now! $\lim_{x\to 0} (e^x + x)^{1/x} = \lim_{x\to 0} y = \lim_{x\to 0} e^{\ln y} = e^{\lim_{x\to 0} \ln y} = \boxed{e^2}$

#25 $\lim_{x\to 0^+} \left(1+\frac{1}{x}\right)^x \{1^0\}$ let $y = \left(1+\frac{1}{x}\right)^x$

$\ln y = x \ln\left(1+\frac{1}{x}\right)$

$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} x \ln\left(1+\frac{1}{x}\right) \{0 \cdot \infty\} = \lim_{x\to 0^+} \frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}} \left\{\frac{\infty}{\infty}\right\}$

$= \lim_{x\to 0^+} \frac{\frac{1}{\left(1+\frac{1}{x}\right)} \cdot \left(-\frac{1}{x^2}\right)}{\frac{-1}{x^2}} = \lim_{x\to 0^+} \frac{1}{1+\frac{1}{x}} = \lim_{x\to 0^+} \frac{1}{\frac{x+1}{x}}$

simplify

$= \lim_{x\to 0^+} \frac{x}{x+1} = 0$

Now! $\lim_{x\to 0^+} \left(1+\frac{1}{x}\right)^x = \lim_{x\to 0^+} y = \lim_{x\to 0^+} e^{\ln y} = e^{\lim_{x\to 0^+} \ln y} = e^0 = \boxed{1}$

#31 $\lim\limits_{x\to\infty} (1+x)^{1/x}$ $\{\infty^0\}$ let $y = (1+x)^{1/x}$

$\ln y = \frac{1}{x}\ln(1+x)$

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \tfrac{1}{x}\ln(1+x) \ \left\{\tfrac{\infty}{\infty}\right\} = \lim_{x\to\infty} \frac{\frac{1}{1+x}\cdot(1)}{1} = 0$$

$$\lim_{x\to\infty}(1+x)^{1/x} = \lim_{x\to\infty} y = \lim_{x\to\infty} e^{\ln y} = e^{\lim\limits_{x\to\infty}\ln y} = e^0 = \boxed{1}$$

#33 $$\lim_{\theta\to 0}\frac{\sin\theta^2}{\theta}\left\{\tfrac{0}{0}\right\} = \lim_{\theta\to 0}\frac{\cos(\theta^2)\cdot 2\theta}{1} = 0$$

#35 $$\lim_{x\to\infty}\frac{\log_2 x}{\log_3(x+3)} = \lim_{x\to\infty}\left(\frac{\ln 3}{\ln 2}\right)\frac{\ln x}{\ln(x+3)}\left\{\tfrac{\infty}{\infty}\right\} = \frac{\ln 3}{\ln 2}\lim_{x\to\infty}\frac{\frac{1}{x}}{\frac{1}{x+3}}$$

$$= \frac{\ln 3}{\ln 2}\lim_{x\to\infty}\frac{x+3}{x}\left\{\tfrac{\infty}{\infty}\right\} = \frac{\ln 3}{\ln 2}\lim_{x\to\infty}\frac{1}{1} = \frac{\ln 3}{\ln 2}$$

#37 $$\lim_{y\to\pi/2}\left(\tfrac{\pi}{2}-y\right)\tan y\ \{0\cdot\infty\} = \lim_{y\to\pi/2}\frac{\pi/2-y}{\cot y}\left\{\tfrac{0}{0}\right\}$$

$$= \lim_{y\to\pi/2}\frac{-1}{-\csc^2 y} = \lim_{y\to\pi/2}\sin^2 y = 1$$

~~#39~~ $$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\sqrt{x}}\right) = \lim_{x\to 0^+}\left(\frac{1}{x}-\frac{\sqrt{x}}{x}\right) = \lim_{x\to 0^+}\left(\frac{1-\sqrt{x}}{x}\right)\to\infty$$

#41 Review of Precalc $\boxed{0}$

#43 $\lim\limits_{x\to\infty}(1+2x)^{1/2\ln x}$ $\{\infty^0\}$ let $y = (1+2x)^{1/2\ln x}$

$\ln y = \frac{1}{2\ln x}\ln(1+2x)$

$$\lim_{x\to\infty}\ln y = \lim_{x\to\infty}\frac{\ln(1+2x)}{2\ln(x)}\left\{\tfrac{\infty}{\infty}\right\} = \lim_{x\to\infty}\frac{\frac{1}{1+2x}\cdot\cancel{2}}{\frac{\cancel{2}}{x}} = \lim_{x\to\infty}\frac{x}{1+2x}$$

$$= \lim_{x\to\infty}\frac{1}{2} = \frac{1}{2}$$

$$\lim_{x\to a}(1+2x)^{1/2\ln x} = \cdots = \boxed{e^{1/2}}$$

#45 $\boxed{e}$ SEE BELOW

#47 $\lim_{x\to 1^+} x^{1/(1-x)}$ $\{1^\infty\}$ let $y = x^{1/(1-x)}$

$\ln y = \frac{\ln x}{1-x}$

$\lim_{x\to 1^+} \ln y = \lim_{x\to 1^+} \frac{\ln x}{1-x} \left\{\frac{0}{0}\right\} = \lim_{x\to 1^+} \frac{\frac{1}{x}}{-1} = \lim_{x\to 1^+} -\frac{1}{x} = -1$

$\lim_{x\to 1^+} x^{1/(1-x)} = \dots = \boxed{e^{-1}}$

#49 $\lim_{x\to 1} \frac{x^3-1}{4x^3-x-3} \left\{\frac{0}{0}\right\} = \lim_{x\to 1} \frac{3x^2}{12x^2-1} = \boxed{\frac{3}{11}}$

45 $\lim_{x\to 0^+} (1+x)$ let $y = (1+x)^{1/x}$ $\ln y = \frac{1}{x}\ln(1+x)$

$\lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \frac{\ln(1+x)}{x} \left\{\frac{0}{0}\right\} = \lim_{x\to 0^+} \frac{\frac{1}{1+x}}{1} = 1$

NOW: $\lim_{x\to 0^+} (1+x)^{1/x} = \lim_{x\to 0^+} y = \lim_{x\to 0^+} e^{\ln y} = e^{\lim_{x\to 0^+} \ln y} = e^1 = \boxed{e}$