AB. Q204. LESSON 3 HW SOLUTIONS

① $f(x) = e^{\sin x + 3} - \sqrt{x}$ on $[4, 9]$

f is continuous on $[4,9]$ f is differentiable on $(4,9)$

AVE RATE Δ $[4,9] = \dfrac{f(9) - f(4)}{9 - 4} = 3.981$ * MAKE SURE ◇ ENTER FOR DECIMAL

$f'(x) = 3.981$ at $x = 5.233$, $x = 7.777$ } on $(4, 9)$

② $f(x) = \cos(\sin(x^2))$ $[0, 1.2]$

f is cont. on $[0,1.2]$ f is diff. on $(0, 1.2)$

AVE RATE Δ $[0,1.2] = \dfrac{f(1.2) - f(0)}{1.2 - 0} = -0.377$

$f'(x) = -0.377$ at $x = 0.594$, $x = 1.173$ } on $(0, 1.2)$

③ $g(x) = x^3 + 4x$ on $[-3, 6]$

g is cont. on $[-3,6]$ g is diff. on $(-3,6)$

AVE RATE Δ $[-3,6] = \dfrac{g(6) - g(-3)}{6 - (-3)} = \dfrac{(216 + 24) - (-27 - 12)}{9}$

$= \dfrac{240 + 39}{9} = \dfrac{279}{9} = 31$

$g'(x) = 3x^2 + 4 = 31$

$x^2 = 9$ $x = -3$ or $\boxed{x = 3}$ on $(-3, 6)$

$x = -3$ not on $(-3,6)$

3
36
× 6
216

31
9 | 279

④ $g(x) = x^3 - 2x^2 + x + 3 \quad [-1, 1]$

g is cont. on $[-1,1]$ g is diff on $(-1,1)$

AVE RATE Δ $[-1,1]$ $= \dfrac{g(1)-g(-1)}{1-(-1)} = \dfrac{(1-2+1+3)-(-1-2-1+3)}{2}$ (3, −1)

$= \dfrac{4}{2} = 2$

$g'(x) = 3x^2 - 4x + 1 = 2$

$3x^2 - 4x - 1 = 0$

NOT FACTORABLE

$x = \dfrac{4 \pm \sqrt{16 - 4(3)(-1)}}{2(3)} = \dfrac{4 \pm \sqrt{16+12}}{6}$

$= \dfrac{4 \pm \sqrt{28}}{6} = \dfrac{4 \pm 2\sqrt{7}}{6} = \dfrac{2 \pm \sqrt{7}}{3}$

$\boxed{x = \dfrac{2-\sqrt{7}}{3}}$ on $(-1,1)$

$x = \dfrac{2+\sqrt{7}}{3}$ too big not on $(-1,1)$

... $\sqrt{7} > 2 \rightarrow 2+2 > 4 \rightarrow \dfrac{4}{3} > 1$

$2 < \sqrt{7} < 3 \rightarrow 3 < -\sqrt{7} < 2$

$\dfrac{-1}{3} < \dfrac{2-\sqrt{7}}{3} < 0$

⑤ $f(x) = \frac{1}{(x-1)^2}$ on $[0,2]$

f is <u>not</u> continuous on $[0,2]$

Namely f is not continuous at $x = 1$

⑥ $f(x) = x^{2/3}$ $[-8, 8]$

f is cont. on $[-8,8]$

$f'(x) = \frac{2}{3} x^{-1/3} = \frac{2}{3x^{1/3}}$

f is <u>NOT</u> differentiable on $(-8, 8)$

Namely f is not differentiable at $x = 0$

⑦ f is cont and f is diff

AVE RATE Δ $[10,12]$ $= \frac{-20-(-6)}{12-10} = \frac{-14}{2} = -7$

$f'(x) = -7$ on $(10,12)$ M.V.T

AVE RATE Δ $[12,15]$ $= \frac{-1-(-20)}{15-12} = \frac{19}{3}$

$f'(x) = \frac{19}{3}$ on $(12,15)$ M.V.T

$f'(x)$ goes from -7 to $19/3$ so $f'(x) = 0$ at least one time on $[10,15]$ I.V.T

(8.) $f(x) = x^3 - 7x + 6$

A] $y - f(-1) = f'(-1)(x+1)$

$f(-1) = -1 + 7 + 6 = 12$

$f'(x) = 3x^2 - 7$

$f'(-1) = 3 - 7 = -4$

$\boxed{y - 12 = -4(x+1)}$

or $L(x) = 12 - 4(x+1)$

B] $f(-1.02) \approx L(-1.02) = 12 - 4(1.02+1)$

$= 12 - 4(-0.02) = \boxed{12.08}$

C] AVE RATE Δ $[1,3] = \dfrac{f(3) - f(1)}{3-1} = \dfrac{(27 - 21 + 6) - (1 - 7 + 6)}{2} = \boxed{6}$

(12, 0)

D] $f'(x) = 3x^2 - 7 = 6 \qquad x^2 = \frac{13}{3}$

$\boxed{x = \sqrt{\frac{13}{3}}}$ or $x = -\sqrt{\frac{13}{3}}$ OUT

(9.) $f'(x) = 21$ on $(-4, -3)$ M.V.T

$f'(x) = -31$ on $(-3, -2)$ M.V.T

$f'(x) = 0$ on $[-4, -2]$ I.V.T

$f'(x) = -1$ on $(0, 1)$ M.V.T

$f'(x) = 21$ on $(1, 2)$ M.V.T

$f'(x) = 0$ on $[0, 2]$ I.V.T

$f'(x) = 0$ at least <u>two</u> times on $[-4, 2]$

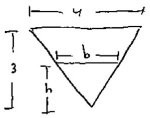
10.

Given: $\frac{dV}{dt} = \frac{5}{2}$ ft^3/min

Find: $\frac{dh}{dt}$ when $h = 2$

Rel: $V = \frac{1}{2} b \cdot h \cdot 15$

$V = \frac{15}{2} b \cdot h$



$\frac{4}{3} = \frac{b}{h}$

$4h = 3b$

$b = \frac{4}{3}h$

UPDATE: $V = \frac{15}{2} \cdot \left(\frac{4}{3}h\right) \cdot h$

$* \; V = 10h^2 \; *$

$\frac{dV}{dt} = 20h \frac{dh}{dt}$

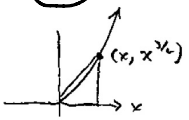
$\frac{5}{2} = 20(2)\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} = \frac{1}{16} \text{ ft/min}}$

11.

Given: $\frac{dD}{dt} = 11$ m/sec

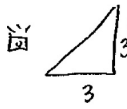
Find: $\frac{dx}{dt}$ when $x = 3$



Rel: $x^2 + (x^{3/2})^2 = D^2$

$x^2 + x^3 = D^2$

$2x\frac{dx}{dt} + 3x^2\frac{dx}{dt} = 2D\frac{dD}{dt}$



$2(3)\frac{dx}{dt} + 3(3)^2\frac{dx}{dt} = 2(6)(11)$

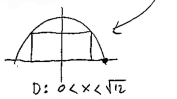
$\sqrt{3^2 + 3^3}$

$\sqrt{9 + 27} = 6$

$\frac{dx}{dt}(6 + 27) = 132$

$\boxed{\frac{dx}{dt} = \frac{132}{33} = 4 \text{ units/sec}}$

12.



- MAX AREA = $A(x) = 2x(12 - x^2) = 24x - 2x^3$

$A'(x) = 24 - 6x^2 = 0 \quad x^2 = 4 \quad x = 2$ or $x = -2$ not in Domain

$A''(x) = -12x$

$A''(2) = -24 < 0 \quad \therefore A$ is max at $x = 2$

Answer: $A(2) = 4(12 - 4) = \boxed{32}$ units^2

Base = 4 Height = 8

$\therefore A$ is local max at $x = 2$

$\therefore A$ is abs. max at $x = 2$