AB.Q204. DERIVATIVE APPLICATION L2 PRACTICE/REVIEW SOLUTIONS

1. A

f is cont on $[0,1]$ ✓
f is diff on $(0,1)$ ✓
M.V.T APPLIES

$$\text{Ave rate}\Delta_{[0,1]} = \frac{f(1)-f(0)}{1-0} = \frac{2-(-1)}{1} = 3$$

$$\text{ins rate}\Delta = f'(x) = 2x+2$$

$$2x+2 = 3$$

$$\boxed{x = \tfrac{1}{2}} \in (0,1) ✓$$

B

f is cont on $[2,4]$ ✓
f is diff on $(2,4)$ ✓
M.V.T APPLIES

$$\text{A.rate}\Delta_{[2,4]} = \frac{f(4)-f(2)}{4-2} = \frac{\ln(3)-\cancel{\ln(1)}^{0}}{2} = \frac{\ln 3}{2}$$

$$\text{I. rate }\Delta = f'(x) = \frac{1}{x-1}$$

$$\frac{1}{x-1} = \frac{\ln(3)}{2} \rightarrow x-1 = \frac{2}{\ln 3}$$

$$\boxed{x = \frac{2}{\ln 3}+1} \in (2,4) ✓$$

C.

$$\text{Ave rate }\Delta_{[1.5,\,3]} = \frac{f(3)-f(1.5)}{3-1.5} = -1.194$$

$$\text{Ins rate }\Delta = f'(x)$$

$$f'(x) = -1.194 \quad \text{at} \begin{cases} x = 1.764 \in (1.5,3) \\ x = 2.540 \in (1.5,3) \end{cases}$$

* NOTES *

OPTION 1
y2 = [paste f'(x)]
y3 = −1.194
Find intersections

OPTION 2
y2 = [paste f'(x)] + 1.194
Find zeros

2.

$$L(x) = f(-4) + f'(-4)(x+4)$$

$$f(-4) = \sqrt{25} = 5$$

$$f'(x) = \tfrac{1}{2}(x^2+9)^{-1/2}\cdot 2x = \frac{x}{\sqrt{x^2+9}}$$

$$f'(-4) = \frac{-4}{5}$$

$$\boxed{L(x) = 5 - \tfrac{4}{5}(x+4)}$$

$$f(-3.9) \approx L(-3.9) = 5 - \tfrac{4}{5}(0.1) = 5 - 0.08 = \boxed{4.92}$$

3.

$$L(x) = f(0) + f'(0)(x-0)$$

$$f(0) = \ln(1) = 0$$

$$f'(x) = \frac{1}{x+1}$$

$$f'(0) = 1$$

$$\boxed{L(x) = x}$$

$$f(0.2) \approx L(0.2) = \boxed{0.2}$$

4.

$$L(x) = f(0) + f'(0)(x-0)$$

$$f(0) = 2$$

$f'(x) = e^x\cos x \leftarrow$ given

$$f'(0) = e^0\cdot\cos(0) = 1$$

$$\boxed{L(x) = 2 + x}$$

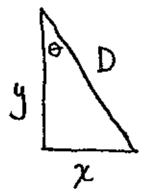
$$f(0.5) \approx L(0.5) = 2 + 0.5 = \boxed{2.5}$$

35
24
59

24
5
126
-7
113

1
66
2
132

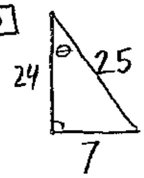
5.



Given $\frac{dy}{dt} = 1 \ m/s \quad \frac{dx}{dt} = 5 \ m/s$

A] Find $\frac{dD}{dt}$ when $y=24, x=7$

Rel $x^2+y^2=D^2$



$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2D\frac{dD}{dt}$$

$$\cancel{2}(7)(5)+\cancel{2}(24)(1) = \cancel{2}(25)\frac{dD}{dt}$$

$$\boxed{\frac{dD}{dt} = \frac{59}{25} \ m/s}$$

$\cos\theta = \frac{24}{25}$

$\sec\theta = \frac{25}{24}$

B] Find $\frac{d\theta}{dt}$ when $y=24, \ x=7$

Rel $\tan\theta = \frac{x}{y}$

$$\sec^2\theta\frac{d\theta}{dt} = \frac{y\left(\frac{dx}{dt}\right)-x\left(\frac{dy}{dt}\right)}{y^2}$$

$$\left(\frac{25}{24}\right)^2\frac{d\theta}{dt} = \frac{24(5)-7(1)}{(24)^2}$$

$$\frac{d\theta}{dt} = \frac{113}{\cancel{(24)^2}}\cdot\frac{\cancel{(24)^2}}{(25)^2}$$

$$\boxed{\frac{d\theta}{dt} = \frac{113}{625} \ rad/sec}$$

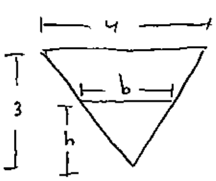
6.

Given $\frac{dV}{dt} = \frac{5}{2} \ ft^3/min$

Find $\frac{dh}{dt}$ when $h=2$

Rel $V = \frac{1}{2}b \cdot h \cdot 15$

$V = \frac{15}{2}b \cdot h$



UPDATE $V = \frac{15}{2}\cdot\left(\frac{4}{3}h\right)\cdot h$

$* \ V = 10h^2 \ *$

$\frac{4}{3} = \frac{b}{h}$

$4h = 3b$

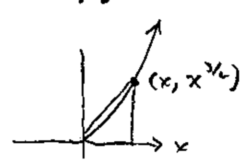
$b = \frac{4}{3}h$

$$\frac{dV}{dt} = 20h\frac{dh}{dt}$$

$$\frac{5}{2} = 20(2)\frac{dh}{dt}$$

$$\boxed{\frac{dh}{dt} = \frac{1}{16} \ ft/min}$$

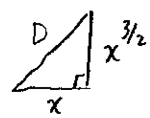
7.



Given $\frac{dD}{dt} = 11$ w/sec

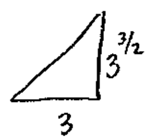
Find $\frac{dx}{dt}$ when $x=3$

Rel $x^2+(x^{3/2})^2 = D^2$



$$x^2+x^3=D^2$$

$$2x\frac{dx}{dt} + 3x^2\frac{dx}{dt} = 2D\frac{dD}{dt}$$



$$2(3)\frac{dx}{dt} + 3(3)^2\frac{dx}{dt} = 2(6)(11)$$

$\sqrt{3^2+3^3}$

$\sqrt{9+27} = 6$

$$\frac{dx}{dt}(6+27) = 132$$

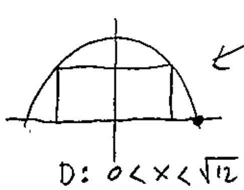
$$\boxed{\frac{dx}{dt} = \frac{132}{33} = 4 \text{ units/sec}}$$

8. MAX AREA $= A(x) = 2x(12-x^2) = 24x-2x^3$

$A'(x) = 24-6x^2 = 0 \quad x^2=4 \quad \underline{x=2}$ or $\cancel{x=-2}$ out of Domain

$A''(x) = -12x \qquad \therefore A$ is max at $x=2$

$A''(2) = -24 < 0$



ANSWER: $A(2) = 4(12-4) = \boxed{32}$ units^2

Base = 4 Height = 8