L'HÔPITAL'S RULE PRACTICE SET # 1

1) Find $\displaystyle \lim_{x\to 0} \frac{1-\cos x}{x^2} \left\{\tfrac{0}{0}\right\} = \lim_{x\to 0} \frac{\sin x}{2x} \left\{\tfrac{0}{0}\right\} = \lim_{x\to 0} \frac{\cos x}{2} = \boxed{\tfrac{1}{2}}$

2) Find $\displaystyle \lim_{x\to \pi} \frac{\csc x}{1+\cot x} \left\{\tfrac{\infty}{\infty}\right\} = \lim_{x\to \pi} \frac{-\csc x \cot x}{-\csc^2 x} = \lim_{x\to\pi} \cos x = \boxed{-1}$

3) Find $\displaystyle \lim_{x\to 0^+} x \ln x$ $-\{0\cdot\infty\}$ $\displaystyle = \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} \left\{\tfrac{\infty}{\infty}\right\} = \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} -x = \boxed{0}$

4) Find $\displaystyle \lim_{x\to 0} \frac{\sin(x^2)}{x} \left\{\tfrac{0}{0}\right\} = \lim_{x\to 0} \frac{2x\cos(2x)}{1} = \boxed{0}$

5) Find $\displaystyle \lim_{x\to \pi/2} \left(\tfrac{\pi}{2}-x\right)\tan x \;\{0\cdot\infty\} = \lim_{x\to \pi/2} \frac{\frac{\pi}{2}-x}{\cot x} \left\{\tfrac{0}{0}\right\} = \lim_{x\to \pi/2} \frac{-1}{-\csc^2 x} = \boxed{1}$

6) Find $\displaystyle \lim_{x\to 1} \frac{x^3-1}{4x^3-x-3} \left\{\tfrac{0}{0}\right\} = \lim_{x\to 1} \frac{3x^2}{12x^2-1} = \boxed{\tfrac{3}{11}}$

7) Find $\displaystyle \lim_{x\to \infty} \frac{\log_2 x}{\log_2(x+3)} = \lim_{x\to\infty} \frac{\frac{\ln x}{\ln 2}}{\frac{\ln(x+3)}{\ln 2}} = \lim_{x\to\infty} \frac{\ln x}{\ln(x+3)} \left\{\tfrac{\infty}{\infty}\right\} = \lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{x+3}}$

$\displaystyle = \lim_{x\to\infty} \frac{x+3}{x} \left\{\tfrac{0}{0}\right\} = \lim_{x\to\infty} \frac{1}{1} = \lim_{x\to\infty} 1 = \boxed{1}$

* 8) Find $\displaystyle \lim_{x\to \infty} \frac{3x-5}{2x^2-x+2} = \boxed{0}$

* 9) Find $\displaystyle \lim_{x\to \infty} \frac{4x^3-2x^2+5}{3x^3-x} = \boxed{\tfrac{4}{3}}$

* 10) Find $\displaystyle \lim_{x\to \infty} \frac{3x^2+1}{5x} = \boxed{\infty}$

- GREEN → NON CALCULUS SIMPLIFICATION
- BLUE → RESULT FROM AFTER DERIVATIVE

* CAN DO WITHOUT L'HOPITAL'S RULE

LESSON2 HW – LINEARIZATION

1. Let f be a function with $f(0) = 10.2$ and $f'(x) = 2\sin\left(\frac{\pi}{2} - x\right)$.

Use a linearization of f at $x = 0$ and use it to approximate $f(-0.3)$

$L(x) = f(0) + f'(0)(x-0)$

$L(x) = 10.2 + 2(x)$

$f(-0.3) \approx L(-0.3) = 10.2 + 2(-0.3) = \boxed{9.6}$

2. Let $f(x) = \sqrt{x^2 + 9}$. Use a linearization of f at $x = -4$ and use it to approximate $f(-3.9)$.

$f'(x) = \frac{1}{2}(x^2+9)^{-1/2} \cdot 2x = \frac{x}{\sqrt{x^2+9}}$

$f(-4) = \sqrt{16+9} = 5$

$f'(-4) = \frac{-4}{5}$

$L(x) = f(-4) + f'(-4)(x+4)$

$L(x) = 5 - \frac{4}{5}(x+4)$

$f(-3.9) \approx L(-3.9) = 5 - \frac{4}{5}(0.1) = \boxed{4.92}$

3. Let $f(x) = \ln(x+1)$. Use a linearization of f at $x = 0$ and use it to approximate $f(0.2)$.

$f'(x) = \frac{1}{x+1}$

$f(0) = 0$

$f'(0) = 1$

$L(x) = f(0) + f'(0)(x)$

$L(x) = 0 + 1(x)$

$L(x) = x$

$f(0.2) \approx L(0.2) = \boxed{0.2}$

4. Let f be a function with $f(0) = 2$ and $f'(x) = e^x \cos(x)$.

Use a linearization of f at $x = 0$ and use it to approximate $f(0.5)$

$L(x) = f(0) + f'(0)(x-0)$

$L(x) = 2 + 1(x)$

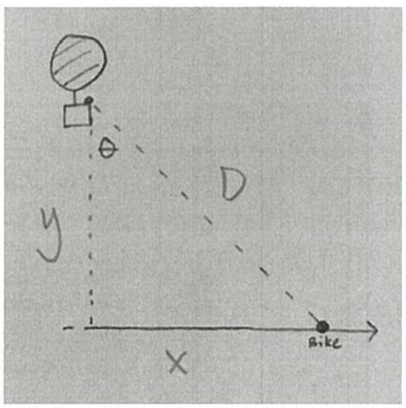
$f(0.5) \approx L(0.5) = 2 + 0.5 = \boxed{2.5}$

LESSON2 HW – RELATED RATES REVIEW

5. A man inside a hot-air balloon basket, directly above a level, straight path, is filming a bicyclist traveling along the path. The balloon is rising vertically at a constant rate of 1 m/sec. The cyclist is moving away from the point directly below the balloon at a constant rate of 5 m/sec. Consider the point in time when the ~~camera~~ balloon is 24 m above the ground and the bicycle is 7 m from the point directly below the balloon?

A. How fast is the distance between the bicycle and the ~~man's camera~~ balloon changing?

B. How fast is the angle of the ~~camera~~ balloon changing?



Given : $\frac{dy}{dt} = 1$ m/s

$\frac{dx}{dt} = 5$ m/s

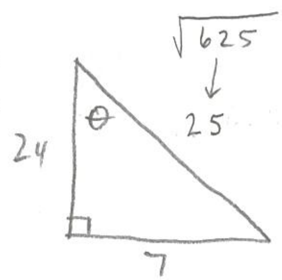
A] Find : $\frac{dD}{dt}$ when $y = 24$, $x = 7$

Rel: $x^2 + y^2 = D^2$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2D\frac{dD}{dt}$$

$$2(7)(5) + 2(24)(1) = 2(25)\frac{dD}{dt}$$

$$\boxed{\frac{dD}{dt} = \frac{59}{25} \text{ m/s}}$$



$\cos\theta = \frac{24}{25}$ $\quad \sec\theta = \frac{25}{24}$

B] Find : $\frac{d\theta}{dt}$ when $y = 24$, $x = 7$

Rel: $\tan\theta = \frac{x}{y}$

$$\sec^2\theta \frac{d\theta}{dt} = \frac{y\left(\frac{dx}{dt}\right) - x\left(\frac{dy}{dt}\right)}{y^2}$$

$$\left(\frac{25}{24}\right)^2 \frac{d\theta}{dt} = \frac{24(5) - (7)(1)}{(24)^2}$$

$$\boxed{\frac{d\theta}{dt} = \frac{113}{625} \text{ radians/sec}}$$