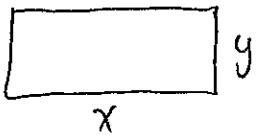
AB. Q2.04 LESSON 1 HW Solutions

1. OBJ: MIN PERIMETER $= 2x + 2y$



$A = x \cdot y$

$16 = x \cdot y$

$y = \frac{16}{x}$

$P(x) = 2x + 2\left(\frac{16}{x}\right)$

$P(x) = 2x + \frac{32}{x}$ D: $0 < x < \infty$

$P'(x) = 2 - \frac{32}{x^2} = 0$

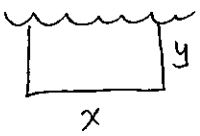
$\frac{32}{x^2} = 2$ $x^2 = 16$ $x = -4$ (at of domain) or $x = 4$

$P''(x) = \frac{64}{x^3}$ $\therefore P$ is min at $x = 4$

$P''(4) = \frac{64}{(4)^3} > 0$

Answer: $P(4) = 2(4) + \frac{32}{4} = 8 + 8 = \boxed{16 \text{ cm}}$

2.



$800 = x + 2y$

$y = \frac{800 - x}{2} = 400 - \frac{x}{2}$

OBJ: MAX AREA = base · height

$A(x) = x \cdot y$

$A(x) = x\left(400 - \frac{x}{2}\right)$

$A(x) = 400x - \frac{x^2}{2}$ D: $0 < x < 800$

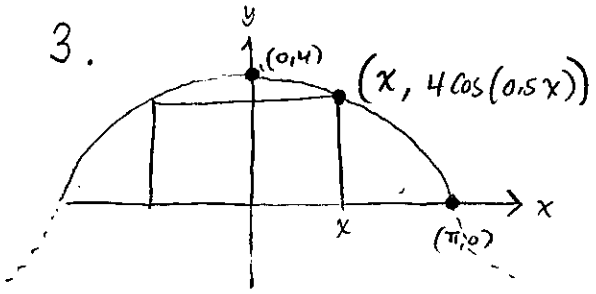
$A'(x) = 400 - x = 0$ $x = 400$

$A''(x) = -1$ $A'(400) < 0$ $\therefore A$ is max at $x = 400$

Answer: $A(400) = 400(400 - 200) = \boxed{80{,}000 \text{ m}^2}$

$x = 400$ m $y = 200$ m

3.



D: $0 < x < \pi$

OBJ: MAX AREA = Base · height

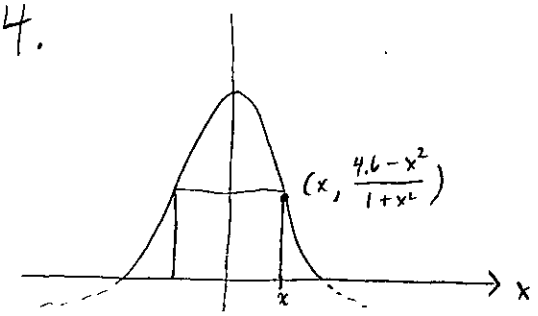
$A(x) = 2x \cdot 4\cos(0.5x)$

$A'(x) = 0$ at $x = 1.721$

A is max at $x = 1.721$ b/c
A' goes from positive to negative at $x = 1.721$

Answer: $A(1.721) = \boxed{8.978 \text{ units}^2}$

4.



OBJ: MAX AREA = Base · height

$A(x) = 2x\left(\frac{4.6-x^2}{1+x^2}\right)$

D: $0 < x < 2.145$

$A'(x) = 0$ at $x = 0.751$

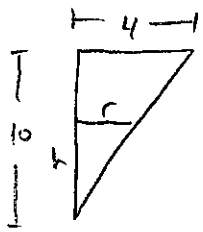
A is max at $x = 0.751$ b/c
A' goes from positive to negative at $x = 0.751$

Answer: $A(0.751) = \boxed{3.876 \text{ units}^2}$

#5. Given $\frac{dV}{dt} = -5 \text{ ft}^3/\text{min}$

Find $\frac{dh}{dt}$ when $h = 6$

Rel $V = \frac{1}{3}\pi r^2 h$



$\frac{10}{4} = \frac{h}{r}$

$10r = 4h$

$r = \frac{2}{5}h$

$* \quad V = \frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 h$

$* \quad V = \frac{4\pi}{75}h^3 \quad *$

$\frac{dV}{dt} = \frac{4\pi}{25}h^2\frac{dh}{dt}$

$-5 = \frac{4\pi}{25}(6)^2\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{-125}{36(4)\pi}$

$= \boxed{\frac{-125}{144\pi} \text{ ft/min}}$

#6 A: $y - f(a) = f'(a)(x-a)$

$y = f(a) + f'(a)(x-a)$

$\boxed{L(x) = 12 - 4(x+1)}$

B: $f(-1.02) \approx L(-1.02) = 12 - 4(-0.02) = 12.08$

C: Ave rate of change on $[1,3]$ $= \frac{f(3)-f(1)}{3-1} = \frac{12-0}{2} = \boxed{6}$

D: Inst. rate of change $= f'(x) = 3x^2 - 7 = 6$

$3x^2 = 13$

$x^2 = \frac{13}{3}$ $\quad x = -\sqrt{\frac{13}{3}}$ (out of domain) or $\boxed{x = \sqrt{\frac{13}{3}}}$

#7 $L(x) = f(0) + f'(0)(x-0)$

$f'(0) = 2\sin\left(\frac{\pi}{2} - 0\right) = 2(1) = 2$

$\boxed{L(x) = 10.2 + (2)(x)}$

$f(-0.3) \approx L(-0.3) = 10.2 + 2(-0.3) = \boxed{9.6}$