AB. Q204. LESSON 1 – Homework solutions (Version 2.0)

A. $f(x) = 2x^3 - 3x^2 - 12x + 1$ on $-2 \le x \le 3$

Interior critical x values

$$f'(x) = 6x^2 - 6x - 12 = 0$$

$$6(x^2 - x - 2) = 0$$

$$6(x-2)(x+1) = 0$$

$$x = 2, \ x = -1$$

ENDPOINT CRITICAL X VALUES

$$x = -2, \ x = 3$$

Evaluation

$$f(-2) = 2(-3)^2 - 3(-2)^2 - 12(-2) + 1 = -3$$

$$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1 = 8$$

$$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1 = -19$$

$$f(3) = 2(3)^3 - 3(3)^2 - 12(3) + 1 = -8$$

ANSWER:

ABSOLUTE MAX $= \boxed{8}$

ABSOLUTE MIN $= \boxed{-19}$

B. $f(x) = 2 + \ln(\sin x + 2)$ on $1 < x < 2$

$$f'(x) = \frac{1}{\sin x + 2} \cdot \cos x = \frac{\cos x}{\sin x + 2} = 0 \quad \text{at} \quad x = \frac{\pi}{2}$$

(only one critical point)

NOTE: $\sin x + 2 > 0$

$f'(x) > 0$ for $1 < x < \frac{\pi}{2}$
$f'(x) < 0$ for $\frac{\pi}{2} < x < 2$

$\therefore$ f has an absolute max (or simply a max) at $x = \frac{\pi}{2}$ Answer: $f(\frac{\pi}{2}) = \boxed{2 + \ln(3)}$

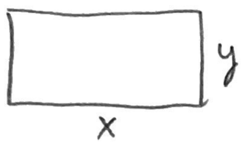
C. $f(x) = x^2 \sin(x + 2.1)$ on $4 < x < 7$

$f'(x) = 0$ at $x = 6.072$ (only one critical point)

$f'(x) > 0$ on $4 < x < 6.072$
$f'(x) < 0$ on $6.072 < x < 7$

$\therefore$ f has an absolute max at $x = 6.072$

1. OBJ: MIN PERIMETER $= 2x + 2y$



$A = xy$

$16 = xy$

$y = \frac{16}{x}$

$P = 2x + 2y$

$P(x) = 2x + \frac{32}{x}$ Domain: $0 < x < \infty$

$P'(x) = 2 - \frac{32}{x^2} = 0$

$x^2 = 16$ $x = \cancel{-4}$ (out of domain) or $x = 4$ (only one critical point)

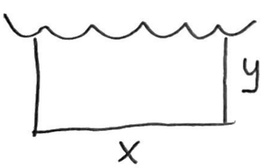
$P'(x) < 0$ on $0 < x < 4$

$P'(x) > 0$ on $4 < x < \infty$

$\therefore$ P has an absolute min at $x = 4$

Answer: $P(4) = 2(4) + \frac{32}{4} = \boxed{16 \text{ cm}}$

2. OBJ: MAX AREA = base · height



$800 = x + 2y$

$y = \frac{800 - x}{2}$

$y = 400 - \frac{x}{2}$

$A = x \cdot y$

$A(x) = x\left(400 - \frac{x}{2}\right)$

$A(x) = 400x - \frac{x^2}{2}$ Domain: $0 < x < 800$

$A'(x) = 400 - x = 0$ at $x = 400$

$A'(x) > 0$ on $0 < x < 400$

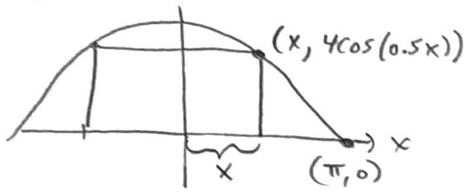
$A'(x) < 0$ on $400 < x < 800$

$\therefore$ A has absolute max at $x = 400$

ANSWER: $A(400) = 400(400 - 200) = \boxed{80{,}000 \text{ m}^2}$

$x = 400$ m $y = 200$ m

3.



Domain : $0 < x < \pi$

OBJ: MAX AREA = Base · Height

$$A(x) = (2x) \cdot (4\cos(0.5x))$$

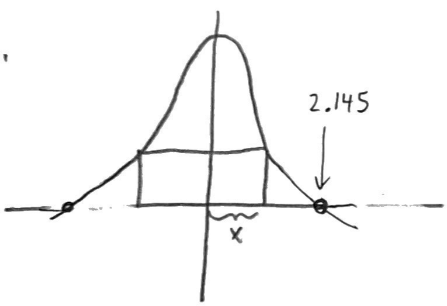
$$A'(x) = 0 \quad \text{at} \quad x = 1.721$$

$A'(x) > 0$ on $0 < x < 1.721$
$A'(x) < 0$ on $1.721 < x < \pi$

∴ A has absolute max at $x = 1.721$

Answer: $A(1.721) = \boxed{8.978 \text{ units}^2}$

4.



Domain : $0 < x < 2.145$

OBJ: MAX AREA = B · H

$$A(x) = (2x) \cdot \left(\frac{4.6 - x^2}{1 + x^2}\right)$$

$$A'(x) = 0 \quad \text{at} \quad x = 0.751$$

$A'(x) > 0$ on $0 < x < 0.751$
$A'(x) < 0$ on $0.751 < x < 2.145$

∴ A has absolute max at $x = 0.751$

Answer: $A(0.751) = \boxed{3.876 \text{ units}^2}$

5

$$V = \frac{1}{3}\pi r^2 h$$

$$* \quad V = \frac{1}{3}\pi\left(\frac{2}{5}h\right)^2 h$$

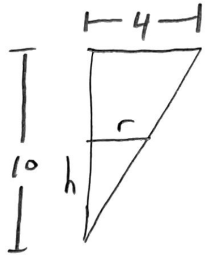
$$* \quad V = \frac{4\pi}{75}h^3$$

$$\frac{dV}{dt} = \frac{4\pi}{25}h^2\frac{dh}{dt}$$

$$-5 = \frac{4\pi}{25}(6)^2\frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-125}{36(4)\pi}$$

$$\boxed{\frac{dh}{dt} = \frac{-125}{144\pi}\ \text{ft/min}}$$



$$\frac{10}{4} = \frac{h}{r}$$

$$10r = 4h$$

$$r = \frac{2}{5}h$$