Solutions

AB: Q203

BC: Q201 – EXAMINATION REVIEW (Lessons 1 – 3)

TECHNOLOGY SECTION: *Round answers to three decimal places.*

1. The velocity of a particle moving along a horizontal is given as $v(t) = 8\cos(t) + \ln(\sin(t) + t^2)$ on ~~$0 < t \le 8$~~. $0.1 < t \le 8$

A. On what time interval is the particle moving to the right? Justify.

$v(t) = 0$ at $t = 1.746$ and $t = 4.343$

It is ok if you used a parenthesis

moving right on $(\cancel{0}\,0.1, 1.746) \cup (4.343, 8]$ b/c $v(t) > 0$ on this interval

B. What are the velocity and acceleration at time $t = 5$? Round answers to three decimal places.

$v(5) = 5.449$

$a(5) = 8.099$

C. Is the particle speeding up or slowing down at $t = 3.5$? Justify.

$v(3.5) = -5.015$

$a(3.5) = 3.316$

The particle is slowing down at $t = 3.5$ b/c the velocity and acceleration have different signs at $t = 3.5$

2. The derivative of f is given by $f'(x) = e^{x^2} - 5x^3 + x$ on $0 \le t < 3$

A. On what interval is f decreasing? Justify. $f'(x) = 0$ at $x = 0.824$ and $x = 1.836$

f is decreasing on $[0.824, 1.836]$ b/c $f'(x) < 0$ on $(0.824, 1.836)$

B. At what x-value(s) does f have a relative maximum? Justify. $f'(x) = 0$ at $x = 0.824$

f has a relative max at $x = 0.824$ b/c $f'(x)$ goes from positive to negative at $x = 0.824$

C. On what interval is f concave upward? Justify.

f is concave upward on $(0, 0.344) \cup (1.559, 3)$ b/c $f''(x) > 0$ on this interval.

NO TECHNOLOGY SECTION

1. Let f be defined by $f(x) = \ln(2 + \sin x)$ for $\pi \le x \le 2\pi$.
Find the absolute maximum value and the absolute minimum value of f using the closed interval test.

out of domain

$f'(x) = \frac{1}{2+\sin x} \cdot \cos x = 0 \qquad \cos x = 0 \qquad x = \cancel{\frac{\pi}{2}}, \frac{3\pi}{2}$

endpoints at $x = \pi, 2\pi$

$f(\frac{3\pi}{2}) = \ln(1) = 0$

$f(\pi) = \ln(2)$

$f(2\pi) = \ln(2)$

The abs. max is $\ln 2$

The abs min is 0

SKIP THIS PROBLEM

2. A. When is the graph of $f(x)$ concave upward if $f''(x) = (x-1)(x+2)^2 e^{x^2}$. Justify.
B. How many points of inflection are on f? Justify.

$f''(x) = 0 \qquad x = 1, -2$

x: -2, 1

$f''(x)$: $-$, $-$, $+$

A. f is concave up on $(1, \infty)$ b/c $f''(x) > 0$ on this interval

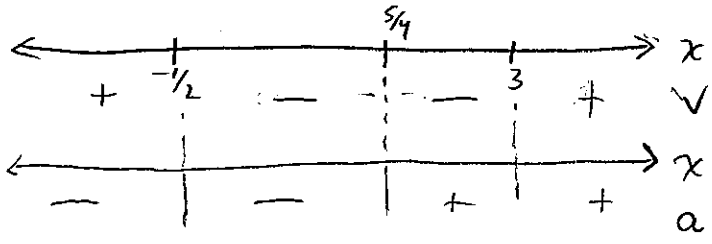
B. There is one point of inflection at $x = 1$ b/c $f''(x)$ changes sign at this x value

3. A particle moves along a horizontal line. It's position at time t is given as
$s(t)=\frac{2}{3}t^3-\frac{5}{2}t^2-3t$.

On what time interval is the particle slowing down? Justify.

$v(t) = 2t^2-5t-3 = 0 \qquad (2t+1)(t-3) = 0 \qquad t=-\frac{1}{2}\ , t=3$

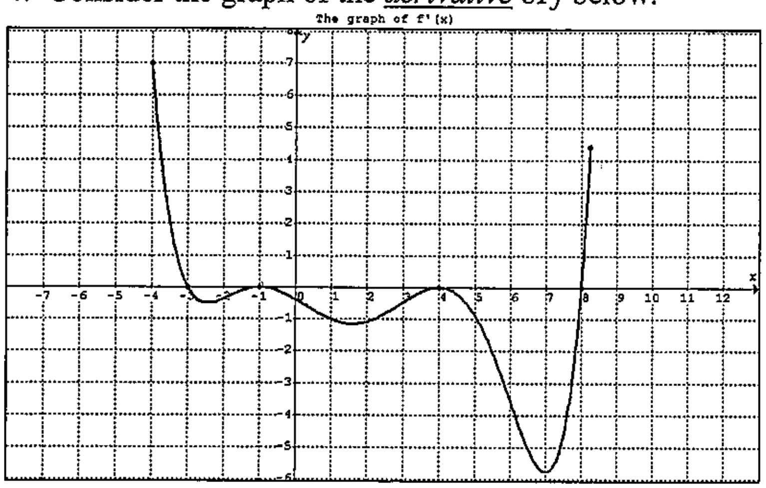
$a(t) = 4t-5=0 \qquad t=\frac{5}{4}$



The particle is slowing down on $(-\infty,-\frac{1}{2}) \cup (\frac{5}{4},3)$ b/c $v(t)$ and $a(t)$ have opposite signs on this interval

- on $(-\infty,-\frac{1}{2})$ $v(t)>0$ and $a(t)<0$
- on $(\frac{5}{4},3)$ $v(t)<0$ and $a(t)>0$

4. Consider the graph of the *derivative* of *f* below.



A. For what *x* – values does *f* have a local minimum? Justify.

f has a local min at $x=8$ b/c $f'(x)$ goes from negative to positive at $x=8$. $f'(x)=0$ at $x=8$. Also f has a min at $x=-4$ b/c f is increasing away from the left endpoint

B. On what interval is *f* increasing? Justify.

f is increasing on $[-4,-3] \cup [8, 8.25]$ (ish) b/c $f'(x)>0$ on $(-4,-3)\cup(8,8.25)$

C. On what interval is *f* concave upward? Justify.

f is concave up on $(-2.5,-1)\cup(1.5,4)\cup(7,8.25)$ b/c $f'(x)$ is increasing on this interval.

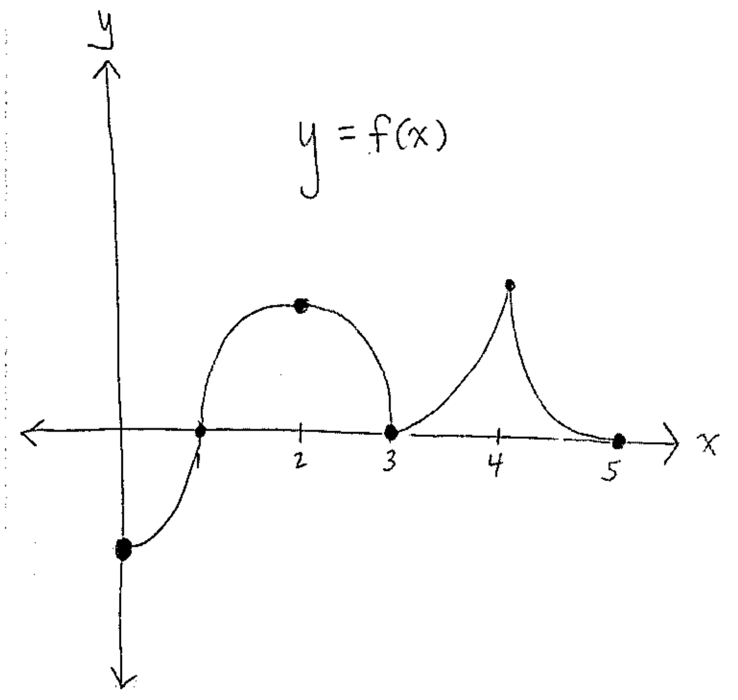
D. How many points of inflection are on *f*?

f has 5 points of inflection b/c $f''(x)$ changes sign 5 times.

(f' changes its increasing/decreasing behavior 5 times.)

GRAPH THEORY

5 ~~4~~. Below is Steven's graph of $y = f(x)$.



THE CHART REPRESENTS STEVEN'S GRAPH

x	0	$0<x<1$	1	$1<x<2$	2	$2<x<3$	3	$3<x<4$	4	$4<x<5$	5
$f(x)$	−	−	0	+	+	+	0	+	+	+	0
$f'(x)$	0	+	DNE	+	0	−	DNE	+	DNE	−	0
$f''(x)$		+	DNE	−	−	−	DNE	+	DNE	+	

FILL IN EACH BLANK IN THE CHART ABOVE WITH ONE OF THE FOLLOWING:

+ for positive

− for negative

0 for zero

DNE for Does not Exist