

TECHNOLOGY SECTION

1. A function f is defined on $(0.250, 1.250)$ and the derivative of the function is given by

$$f'(x) = x \sin(e^{2x} - x)$$

$$f'(x) = 0 \text{ at } x = 0.669, 0.992, 1.181$$

Round Answer to three decimal places

- A. Find the interval(s) on which f is increasing. Justify your answer.

f is increasing on $(0.250, 0.669] \cup [0.992, 1.181]$ b/c
 $f'(x) > 0$ on $(0.250, 0.669) \cup (0.992, 1.181)$

- B. For what value(s) of x does f have a local minimum? Justify your answer.

f has a local min at $x = 0.992$ b/c $f'(x)$
 goes from negative to positive at $x = 0.992$

- C. On what interval is f concave upward? Justify your answer.

$f''(x) = 0$ at $x = 0.463, x = 0.870, x = 1.099$
 f is concave up on $(0.250, 0.463) \cup (0.870, 1.099)$ b/c $f''(x) > 0$
 on this interval.

- D. How many points of inflection are there on the graph of $f(x)$? Justify your answer.

There are three points of inflection on $(0.250, 1.250)$ b/c
 $f''(x)$ changes sign three times on this interval.

- E. Suppose we extended f to include the endpoints at $a = 0.250$ and $b = 1.250$. Use an endpoint analysis to determine whether each endpoint is a local maximum or local minimum.

There is a local min at $x = 0.250$

There is a local min at $x = 1.250$

$$f(0.522) = 4$$

- F. Write an equation of the line tangent to $f(x)$ at $x = 0.522$.

$$\text{Home: } y = f(0.522) = 4$$

$$y = f(0.522) + f'(0.522)(x - 0.522) \rightarrow y = 4 + 0.383(x - 0.522)$$

- G. Use the tangent line in part (F) to estimate $f(0.3)$

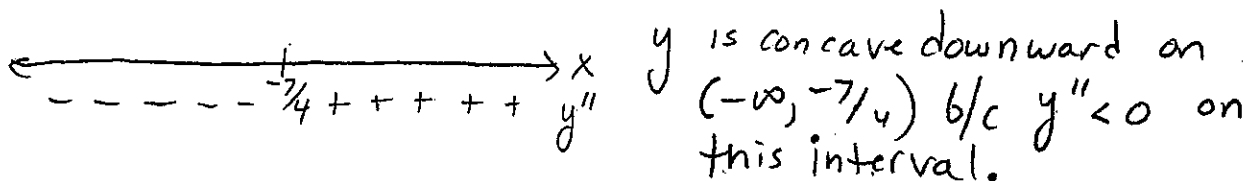
$$f(0.3) \approx L(0.3) = 4 + 0.383(0.3 - 0.522) = \boxed{3.915}$$

NON TECHNOLOGY SECTION

2. Let $y = 4x^3 + 21x^2 + 36x - 20$

A. On what interval is the graph of y concave downward? Justify your answer.

$$y' = 12x^2 + 42x + 36 \quad y'' = 24x + 42 = 0 \quad x = -7/4$$



B. The graph of y has how many points of inflection? Justify your answer.

y has one point of inflection b/c y'' changes sign one time.

3. Let $f(x) = 3x - x^3 + 5$

Use the second derivative test to determine the local extremes of $f(x)$.

$$f'(x) = 3 - 3x^2 = 0 \quad x = \pm 1$$

$$f''(x) = -6x \quad f''(1) = -6 < 0 \quad \therefore f \text{ has a local max at } x = 1$$

$$f''(-1) = 6 > 0 \quad \therefore f \text{ has a local min at } x = -1$$

* so local max is $f(1) = \boxed{7}$

* so local min is $f(-1) = \boxed{3}$

Use the second derivative test to determine the local extremes of $f(x)$.

$$f'(x) = xe^x + e^x \leftarrow \text{Product Rule} \quad xe^x + e^x = 0 \quad e^x(x+1) = 0 \quad e^x \neq 0 \quad \boxed{x = -1}$$

$$f''(x) = xe^x + e^x + e^x \quad f''(-1) = (-1)e^{-1} + e^{-1} + e^{-1} = e^{-1}(2-1) = e^{-1} > 0$$

$\therefore f$ has a local min at $x = -1$ * so $f(-1) = \boxed{-e^{-1}}$ is the local min

5. Let $f'(x) = (x-1)^2(x-2)$ be the derivative of the function of $y = f(x)$.

A. Find the x -value(s) where the function f has a local minimum. Justify your answer.

$$f'(x) = 0 \quad \text{at } x = 1, 2$$

OPTION 1: f has a local min at $x = 2$ b/c $f'(x)$ changes from negative to positive at $x = 2$.

OPTION 2: $f''(1) = 0 \rightarrow$ neither max or min at $x = 1$
 $f''(2) > 0 \rightarrow \therefore f$ has a local min at $x = 2$.

B. Find the x -value(s) where the function f has a point of inflection. Justify your answer.

$$f''(x) = (x-1)^2 + (x-2) \cdot 2(x-1)$$

$$= (x-1)[(x-1) + 2(x-2)]$$

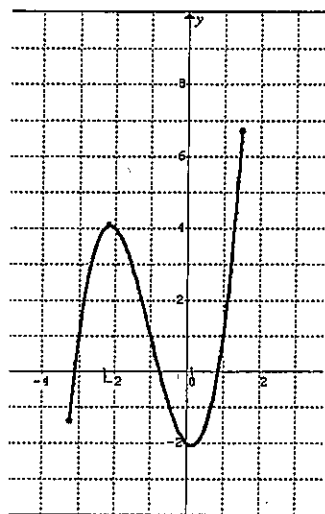
$$= (x-1)[3x-5] = 0$$

$x = 1, 5/3$

A horizontal number line with arrows at both ends. Two tick marks are labeled 1 and $5/3$. Between 1 and $5/3$, there are three plus signs. Between $5/3$ and the next tick mark, there are three minus signs. To the right of the last tick mark, there are three plus signs. Below the line, the label $f''(x)$ is written. To the right of the number line, the text says: "f has a point of inflection at $x = 1, 5/3$ b/c f'' changes sign at these x -values."

GRAPH ANALYSIS SECTION

6. Consider the function $y = f(x)$ shown below.



A. Estimate the x value(s) where $f'(x) = 0$.

$$x = -2.25 \quad x = 0.25$$

B. Estimate the x value(s) where $f'(x) > 0$. [$f(x)$ is increasing]

$$x \in (-3.25, -2.25) \cup (0.25, 1.5)$$

C. Estimate the x value(s) where $f'(x) < 0$. [$f(x)$ is decreasing]

$$x \in (-2.25, 0.25)$$

D. Estimate the x value(s) where $f''(x) = 0$. [f has a point of inflection]

$$x = -1$$

E. Estimate the x value(s) where $f''(x) > 0$. [f is concave up]

$$x \in (-1, 1.5)$$

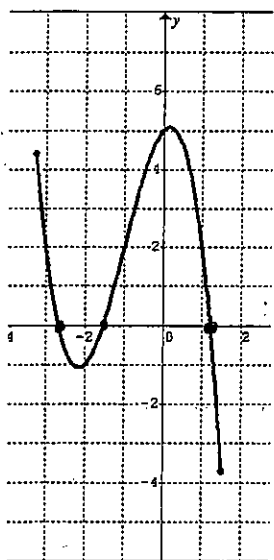
F. Estimate the x value(s) where $f''(x) < 0$. [f is concave down]

$$x \in (-3.25, -1)$$

G. Estimate the x value(s) where $f'(x)$ is decreasing. Same as $f''(x) < 0$
[Concave down]

$$x \in (-3.25, -1)$$

7. Consider the derivative graph function $y = f'(x)$ shown below.



A. Estimate the x -value(s) where f is increasing. [$f'(x) > 0$]

$$x \in [-3.25, -2.75] \cup [-1.5, 1.25]$$

B. Estimate the x -value(s) where f is decreasing. [$f'(x) < 0$]

$$x \in [-2.75, -1.5] \cup [1.25, 1.5]$$

C. Estimate the x -value(s) where f is concave up. ($f'(x)$ increasing)

$$x \in (-2.25, 0.25)$$

D. Estimate the x -value(s) where f is concave down. ($f'(x)$ dec.)

$$x \in (-3.25, -2.25) \cup (0.25, 1.5)$$