

3.8 3.9 Review Solutions

$$\textcircled{1} \quad \frac{dy}{dx} = e^{\cos^{-1}(\tan(2x))} \cdot \frac{-1}{\sqrt{1 - (\tan(2x))^2}} \cdot \sec^2(2x) \cdot 2 + 1$$

$$\textcircled{2} \quad \frac{dy}{dx} = 7^{\cos^{-1}(\frac{1}{x})} \cdot \ln(7) \cdot \frac{-1}{\sqrt{1 - (\frac{1}{x})^2}} \cdot \frac{-1}{x^2}$$

← This is $(\frac{1}{x})'$

$$= \ln(7) \cdot 7^{\cos^{-1}(\frac{1}{x})} \cdot \frac{-1}{\sqrt{\frac{x^2-1}{x^2}}} \cdot \frac{-1}{x^2}$$

$$= \ln(7) \cdot 7^{\cos^{-1}(\frac{1}{x})} \cdot \frac{1}{\frac{\sqrt{x^2-1}}{x}} \cdot \frac{1}{x^2}$$

$$= \boxed{\frac{\ln(7) \cdot 7^{\cos^{-1}(\frac{1}{x})}}{x\sqrt{x^2-1}}}$$

$$\textcircled{3} \quad \frac{dy}{dx} = \frac{1}{2x^4+7x} \cdot (8x^3+7)$$

$$\textcircled{4} \quad \text{Product Rule: } \frac{dy}{dx} = e^{3x} \cdot \sec^2(\sqrt{x}) \cdot \frac{1}{2} x^{-1/2} + \tan \sqrt{x} \cdot e^{3x} \cdot 3$$

$$\textcircled{5} \quad \frac{dy}{dx} = \frac{-1}{1+(\sqrt{x})^2} \cdot \frac{1}{2} x^{-1/2} = \frac{-1}{1+x} \cdot \frac{1}{2\sqrt{x}}$$

$$= \boxed{\frac{-1}{2\sqrt{x}(x+1)}}$$

$$\textcircled{6} \quad \frac{dy}{dx} = \frac{1}{|\tan(2x+1)|\sqrt{\tan^2(2x+1)-1}} \cdot \sec^2(2x+1) \cdot 2$$

$$\textcircled{7} \quad \frac{dy}{dx} = e^{\tan^{-1}(\sin^{-1}x)} \cdot \frac{1}{1+(\sin^{-1}x)^2} \cdot \frac{1}{\sqrt{1-x^2}}$$

⑧ Product Rule :

$$\frac{dy}{dx} = e^{\tan^{-1}(5x)} \cdot \frac{1}{\csc^{-1}(2x)} \cdot \frac{-1}{|2x|\sqrt{(2x)^2-1}} \cdot 2 + \ln(\csc^{-1}(2x)) \cdot e^{\tan^{-1}(5x)} \cdot \frac{1}{1+(5x)^2} \cdot 5$$

⑨ Quotient Rule :

$$\frac{dy}{dx} = \frac{\sin^{-1}\left(\frac{1}{x^2}\right) \cdot (-\sin(e^x)) \cdot e^x - \cos(e^x) \frac{1}{\sqrt{1-\left(\frac{1}{x^2}\right)^2}} \cdot \frac{-2}{x^3}}{\left(\sin^{-1}\left(\frac{1}{x^2}\right)\right)^2}$$

$$\textcircled{10} \quad \frac{dy}{dx} = \frac{1}{\ln(5x)+x} \left[\frac{1}{5x} \cdot 5 + 1 \right]$$

$$\textcircled{11} \quad f(x) = \frac{1}{5} \ln \left(\frac{(x-3)^4 (x^2+1)}{(2x+5)^3} \right)$$

$$= \frac{1}{5} \left[\ln(x-3)^4 + \ln(x^2+1) - \ln(2x+5)^3 \right]$$

$$= \frac{1}{5} \left[4 \ln(x-3) + \ln(x^2+1) - 3 \ln(2x+5) \right]$$

$$f'(x) = \frac{4}{5} \cdot \frac{1}{x-3} + \frac{1}{5} \cdot \frac{1}{x^2+1} \cdot 2x - \frac{3}{5} \cdot \frac{1}{2x+5} \cdot 2$$

$$\textcircled{12} \quad f(x) = \frac{\ln(\sin x + 5)}{\ln(7)} \quad f'(x) = \frac{1}{\ln(7)} \cdot \frac{1}{\sin x + 5} \cdot \cos x$$

$$\textcircled{13} \quad f(x) = \frac{\ln(\sec^2 x)}{\ln(10)} = \frac{2 \ln(\sec x)}{\ln(10)}$$

$$f'(x) = \frac{2}{\ln(10)} \cdot \frac{1}{\sec x} \cdot \sec x \tan x = \boxed{\frac{2 \tan x}{\ln(10)}}$$

$$(14) \quad \ln y = \cos^{-1} x \cdot \ln(x)$$

$$\frac{1}{y} \frac{dy}{dx} = \cos^{-1} x \cdot \frac{1}{x} + \ln x \left(\frac{-1}{\sqrt{1-x^2}} \right)$$

$$\frac{dy}{dx} = y \left(\frac{\cos^{-1} x}{x} - \frac{\ln x}{\sqrt{1-x^2}} \right)$$

$$\frac{dy}{dx} = x^{\cos^{-1} x} \left(\frac{\cos^{-1} x}{x} - \frac{\ln x}{\sqrt{1-x^2}} \right)$$

$$(15) \quad \ln y = \sin^{-1} x \cdot \ln(\sin x)$$

$$\frac{1}{y} \frac{dy}{dx} = \sin^{-1} x \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x) \cdot \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = (\sin x)^{(\sin^{-1} x)} \left[\frac{\sin^{-1} x \cos x}{\sin x} + \frac{\ln(\sin x)}{\sqrt{1-x^2}} \right]$$

$$(16) \quad f'(x) = 21x^6 + 2$$

$$f'(x) > 0 \quad \forall x$$

$\therefore f$ is always increasing

$\therefore f$ is one to one

$\therefore$ The inverse of f is also a function

$$g'(10) = \frac{1}{f'(1)} = \frac{1}{21(1)^6 + 2} = \boxed{\frac{1}{23}}$$

$$3a^7 + 2a + 5 = 10$$

$$3a^7 + 2a - 5 = 0$$

$$\therefore a = 1$$

$$(17) \quad f'(x) = 2 + \cos x$$

$$\frac{d}{dx} [f^{-1}(\pi+1)] = \frac{1}{f'(\frac{\pi}{2})} = \frac{1}{2 + \cos(\frac{\pi}{2})} = \boxed{\frac{1}{2}}$$

$$2a + 1 \sin(a) = \pi + 1$$

$$a = \frac{\pi}{2}$$

$$(18) \quad x \frac{dy}{dx} + y + \frac{1}{\sqrt{1-y^2}} \frac{dy}{dx} = 2y \frac{dy}{dx}$$

$$\frac{dy}{dx} \left(x + \frac{1}{\sqrt{1-y^2}} - 2y \right) = -y$$

$$\frac{dy}{dx} = \frac{-y}{x + \frac{1}{\sqrt{1-y^2}} - 2y} = \frac{-y}{\frac{x\sqrt{1-y^2}}{\sqrt{1-y^2}} + \frac{1}{\sqrt{1-y^2}} - \frac{2y\sqrt{1-y^2}}{\sqrt{1-y^2}}}$$

$$= \frac{-y}{\frac{x\sqrt{1-y^2} + 1 - 2y\sqrt{1-y^2}}{\sqrt{1-y^2}}} = \frac{-y\sqrt{1-y^2}}{x\sqrt{1-y^2} + 1 - 2y\sqrt{1-y^2}} = \boxed{\frac{y\sqrt{1-y^2}}{2y\sqrt{1-y^2} - x\sqrt{1-y^2} - 1}}$$

$$(19) \quad e^{xy} \left(x \frac{dy}{dx} + y \right) - \sin y \frac{dy}{dx} = \frac{1}{x+y} \left(1 + \frac{dy}{dx} \right)$$

$$x e^{xy} \frac{dy}{dx} + y e^{xy} - \sin y \frac{dy}{dx} = \frac{1}{x+y} + \frac{1}{x+y} \frac{dy}{dx}$$

$$\frac{dy}{dx} \left(x e^{xy} - \sin y - \frac{1}{x+y} \right) = \frac{1}{x+y} - y e^{xy}$$

$$\frac{dy}{dx} = \frac{\frac{1}{x+y} - y e^{xy}}{x e^{xy} - \sin y - \frac{1}{x+y}}$$

$$\dots e^0 = 1$$

$$(20) \quad x=1: \quad e^y + y + 3 = 4 \quad e^y + y = 1$$

$$\therefore y = 0$$

$$e^y \frac{dy}{dx} + \frac{dy}{dx} + 3 = 0 \rightarrow \frac{dy}{dx} = \frac{-3}{e^y + 1}$$

$$\frac{dy}{dx} (e^y + 1) = -3 \quad \left. \frac{dy}{dx} \right|_{y=0} = \frac{-3}{1+1} = -\frac{3}{2}$$

$$\boxed{y = -\frac{3}{2}(x-1)}$$