

## Lesson 2 3.8, 3.9 HW SOLUTIONS

#43  $y = (\sin x)^x$

$$\ln y = x \ln(\sin x)$$

$$\frac{1}{y} \frac{dy}{dx} = x \cdot \frac{1}{\sin x} \cdot \cos x + \ln(\sin x)$$

$$\frac{dy}{dx} = (\sin x)^x \left[ x \frac{\cos x}{\sin x} + \ln(\sin x) \right]$$

or

$$\frac{dy}{dx} = (\sin x)^x \left[ x \cot x + \ln(\sin x) \right]$$

#45  $y = \sqrt[5]{\text{stuff}}$

$$\ln y = \frac{1}{5} [4 \ln(x-3) + \ln(x^2+1) - 3 \ln(2x+5)]$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{4}{5} \cdot \frac{1}{x-3} + \frac{1}{5} \frac{1}{x^2+1} (2x) - \frac{3}{5} \frac{1}{2x+5} \cdot 2$$

$$\frac{dy}{dx} = \sqrt[5]{\text{stuff}} \left[ \frac{4}{5} \frac{1}{x-3} + \frac{2}{5} \frac{x}{x^2+1} - \frac{6}{5} \frac{1}{2x+5} \right]$$

#46  $y = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}}$

$$\ln y = \ln x + \frac{1}{2} \ln(x^2+1) - \frac{2}{3} \ln(x+1)$$

$$\frac{1}{y} \frac{dy}{dx} = \frac{1}{x} + \frac{1}{2} \cdot \frac{1}{x^2+1} \cdot 2x - \frac{2}{3} \frac{1}{x+1}$$

$$\frac{dy}{dx} = \frac{x\sqrt{x^2+1}}{(x+1)^{2/3}} \left[ \frac{1}{x} + \frac{x}{x^2+1} - \frac{2}{3} \frac{1}{x+1} \right]$$

$$\#47 \quad y = x^{\ln x}$$

$$\ln y = \ln x \ln x$$

$$\frac{1}{y} \frac{dy}{dx} = \ln x \cdot \frac{1}{x} + \ln x \cdot \frac{1}{x}$$

$$\frac{dy}{dx} = x^{\ln x} \left[ \frac{2 \ln x}{x} \right]$$

MC #2

$$\begin{aligned} \frac{dy}{dx} &= 3 \cos^2(3x-2) (-\sin(3x-2)) \cdot 3 \\ &= -9 \cos^2(3x-2) \sin(3x-2) \end{aligned}$$

[A]

MC #3

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 = \frac{2}{\sqrt{1-4x^2}}$$

[C]

$$\#28 \quad A] \quad f'(x) = 5x^4 + 6x^2 + 1 > 0 \quad \forall x$$

$\therefore f$  is always increasing

$\therefore f$  is a one-to-one function

$\therefore$  The inverse of  $f$  is also a function

$$B] \quad f(1) = 3 \quad f'(1) = 12$$

$$C] \quad f^{-1}(3) = 1 \quad (f^{-1})'(3) = \frac{1}{f'(1)} = \frac{1}{12}$$

$$\#29 \quad A] \quad f'(x) = -\sin x + 3 > 0 \quad \forall x$$

$$\text{since } -1 \leq -\sin x \leq 1$$

$$2 \leq -\sin x + 3 \leq 4$$

$\therefore f$  is always increasing

$\therefore f$  is one to one function

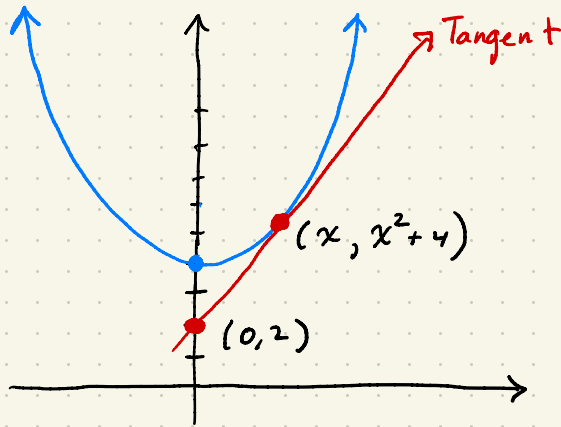
$\therefore$  The inverse of  $f$  is also a function

$$B] \quad g'(1) = \frac{1}{f'(0)} = \frac{1}{-\sin(0) + 3} = \frac{1}{3}$$

$$\cos a + 3a = 1 \rightarrow \text{This happens at } a=0$$

Extra Problem

$$f(x) = x^2 + 4$$



$$m = \frac{\Delta Y}{\Delta X} = \frac{(x^2 + 4) - (2)}{x - 0} = \frac{x^2 + 2}{x}$$

$$m = f'(x) = 2x$$

$$\frac{x^2 + 2}{x} = 2x$$

$$2x^2 = x^2 + 2$$

$$x^2 = 2$$

$$x = \pm\sqrt{2}$$

question calls for positive x-value

$$x = +\sqrt{2}$$

CHECK

$$m|_{\sqrt{2}} = \frac{(\sqrt{2})^2 + 2}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2} \checkmark$$

$$f'(\sqrt{2}) = 2\sqrt{2} \checkmark$$