

3.8 3.9 Lesson 1

HW Solutions

$$1. \quad y = \cos^{-1}(x^2) \quad \frac{dy}{dx} = \frac{-1}{\sqrt{1-(x^2)^2}} \cdot 2x$$

$$3. \quad y = \sin^{-1}(\sqrt{2}t) \quad y' = \frac{1}{\sqrt{1-(\sqrt{2}t)^2}} \cdot \sqrt{2}$$

$$5. \quad y = \sin^{-1}\left(\frac{3}{t^2}\right) \quad y' = \frac{1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2}} \cdot -6t^{-3} = \frac{1}{\sqrt{1-\frac{9}{t^4}}} \cdot \frac{-6}{t^3} = \frac{-6}{t^3 \sqrt{t^4-9}} = \frac{-6}{t \sqrt{t^4-9}}$$

$$7. \quad y = x \sin^{-1}x + \sqrt{1-x^2}$$

$$\frac{dy}{dx} = x \cdot \frac{1}{\sqrt{1-x^2}} + \sin^{-1}x + \frac{1}{2}(1-x^2)^{-1/2}(-2x)$$

$$9. \quad v(t) = x'(t) = \frac{1}{\sqrt{1-\left(\frac{t}{4}\right)^2}} \cdot \frac{1}{4} \quad x'(3) = \frac{1}{\sqrt{1-\frac{9}{16}}} \cdot \frac{1}{4} = \frac{4}{\sqrt{7}} \cdot \frac{1}{4} = \boxed{\frac{1}{\sqrt{7}}}$$

$$11. \quad x'(t) = \frac{1}{1+t^2} \quad x'(2) = \frac{1}{1+4} = \frac{1}{5}$$

$$13. \quad \frac{dy}{ds} = \frac{1}{|2s+1|\sqrt{(2s+1)^2-1}} \cdot 2$$

$$15. \quad \frac{dy}{dx} = \frac{-1}{(x^2+1)\sqrt{(x^2+1)^2-1}} \cdot 2x$$

$$17. \quad \frac{dy}{dx} = \frac{\frac{1}{\frac{1}{t}} \cdot \frac{-1}{t^2}}{\left|\frac{1}{t}\right| \sqrt{\left(\frac{1}{t}\right)^2-1}} = \frac{-1}{\sqrt{\frac{1}{t^2}-1}} \cdot t = \frac{-1}{\sqrt{1-t^2}}$$

$$19. \quad y' = \frac{\frac{-1}{1+t-1} \cdot \frac{1}{2\sqrt{t-1}}}{1 + (\sqrt{t-1})^2} = \frac{-1}{1+t-1} \cdot \frac{1}{2\sqrt{t-1}} = \frac{-1}{2t\sqrt{t-1}}$$

$$21. \quad y' = \frac{\frac{1}{1+(\sqrt{x^2-1})^2} \cdot \frac{1}{2\sqrt{x^2-1}} \cdot 2x}{\frac{1}{x\sqrt{x^2-1}}} = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{1}{1+x^2-1} \cdot \frac{1}{\sqrt{x^2-1}} \cdot x = \frac{1}{x\sqrt{x^2-1}}$$

$$= \frac{1}{x\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}} = 0 \quad [\text{interesting}]$$

#35 TRUE

#36. FALSE

$$\#37, \quad \frac{d}{dx} \sin^{-1}\left(\frac{x}{2}\right) = \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2}$$

$$= \frac{1}{\sqrt{1-\frac{x^2}{4}}} \cdot \frac{1}{2}$$

$$= \frac{1}{\sqrt{\frac{4-x^2}{4}}} \cdot \frac{1}{2} = \frac{1}{\frac{\sqrt{4-x^2}}{2}} \cdot \frac{1}{2} = \frac{1}{\sqrt{4-x^2}} \quad (E)$$

$$\#38. \quad \frac{d}{dx} \tan^{-1}(3x) = \frac{1}{1+(3x)^2} \cdot 3 = \frac{3}{1+9x^2} \quad (D)$$

$$\#39. \quad \frac{d}{dx} \sec^{-1}(x^2) = \frac{1}{x^2 \sqrt{x^4-1}} \cdot 2x = \frac{2}{x \sqrt{x^4-1}} \quad (A)$$

$$\#40. \quad y' = \frac{1}{1+(2x)^2} \cdot 2 = \frac{2}{1+4x^2} \quad y' \Big|_{x=1} = \frac{2}{1+4} = \frac{2}{5} \quad (C)$$

$$5. \quad y' = e^{2x/3} \cdot \frac{2}{3}$$

$$7. \quad y' = e^2 - e^x$$

$$9. \quad y' = e^{\sqrt{x}} \cdot \frac{1}{2} x^{-1/2}$$

$$11. \quad y' = 8^x \ln 8$$

$$13. \quad y' = 3^{\csc x} \ln 3 \cdot (-\csc x \cot x)$$

$$15. \quad y' = \frac{1}{x^2} \cdot 2x$$

$$17. \quad y' = \left[\frac{1}{1/x} \cdot \frac{-1}{x^2} \right] = -\frac{1}{x} \quad x > 0$$

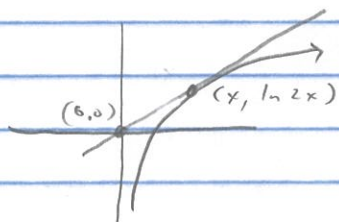
$$19. \quad y' = \frac{1}{\ln x} \cdot \frac{1}{x}$$

$$21. \quad y = \frac{\ln x^2}{\ln 4} \quad y' = \frac{1}{\ln 4} \cdot \frac{1}{x^2} \cdot 2x$$

$$23. \quad y = \frac{\ln(\frac{1}{x})}{\ln(2)} \quad y' = \frac{1}{\ln(2)} \cdot \frac{1}{1/x} \cdot \frac{-1}{x^2} \quad x > 0$$

$$25. \quad y' = \ln 2 \cdot \frac{1}{x} \cdot \frac{1}{\ln 2} \quad x > 0$$

31.



$$m = \frac{\Delta y}{\Delta x} = \frac{\ln 2x}{x}$$

$$m = \frac{dy}{dx} = \frac{1}{2x} \cdot 2 = \frac{1}{x}$$

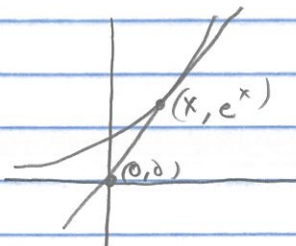
$$\frac{\ln 2x}{x} = \frac{1}{x} \quad \therefore \ln 2x = 1$$

$$2x = e$$

$$x = e/2$$

$$m = \frac{1}{x} = \frac{1}{e/2} = \boxed{\frac{2}{e}}$$

49.



$$m = \frac{dy}{dx} = \frac{e^x}{x}$$

$$\frac{e^x}{x} = e^x$$

$$m = \frac{dy}{dx} = e^x$$

$$xe^x - e^x = 0$$

$$e^x(x-1) = 0 \quad \therefore x = 1$$

$$m = e$$

$$\boxed{y - e = e(x - 1)} \text{ or } \boxed{y = ex}$$